Axial Magnification 2020

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Reference:

Isaac Barrows Optical Lectures, 1667;
Translated by H.C. Fay
Edited by A.G. Bennett
Publisher: The Worshipful Company of Spectacle Makers;
London, England; 1987
ISBN # 0-951-2217-0-1

Friedrich Schiller, in his 27 letters on the Aesthetic Education of Man(kind), stated that *play* is the act of balancing abstract thoughts regarding what *should be*, with our perceptions of what actually *is.* He stated that it is necessary for the determination of beauty, defined as the connection between the actual, and the ideal which is unknowable in its entirety. It is with this sense of play that William Brown, PhD. introduced geometrical optics during my freshman year of optometry school in 1979. This aesthetic education provided for the continued construction of context out of the free interplay of content and form, as well as over three decades of fun.

The playground on which I present the results is that of the circle. I begin there because our heads are already full of ideas about how its pieces fit. For example, we may believe that parallel lines intersect it across equal arcs, since that makes sense to us. From there we can see that equal arcs along a circle subtend equal angles, and that certain triangles within a circle can therefore be shown to have the same shape, with their sides forming ratio equalities. Quadrilaterals with corners along the same circle can then describe equalities with multiple ratios. In 1667 Isaac Barrow used this to find triangles using other triangles, and describe tangential refraction along a line and at a circle. This approach requires no math beyond plane geometry, and encourages a spatial understanding devoid of sign convention and jargon.

For those clinicians wishing to have more than a working knowledge of axial magnification, I have drawn geometric figures to cover the necessary preliminary concepts. Axial magnification is presented only after a thorough spatial representation of tangential refraction along a line and a circle. In order to visualize the relevant axial ratio equalities involved using triangles, the optic axis is then represented as a circle of infinite radius, and the sign convention remains unnecessary.

Section 1

Geometry of the Circle

Figure 1:

Given a circle with diameter EU and center N:

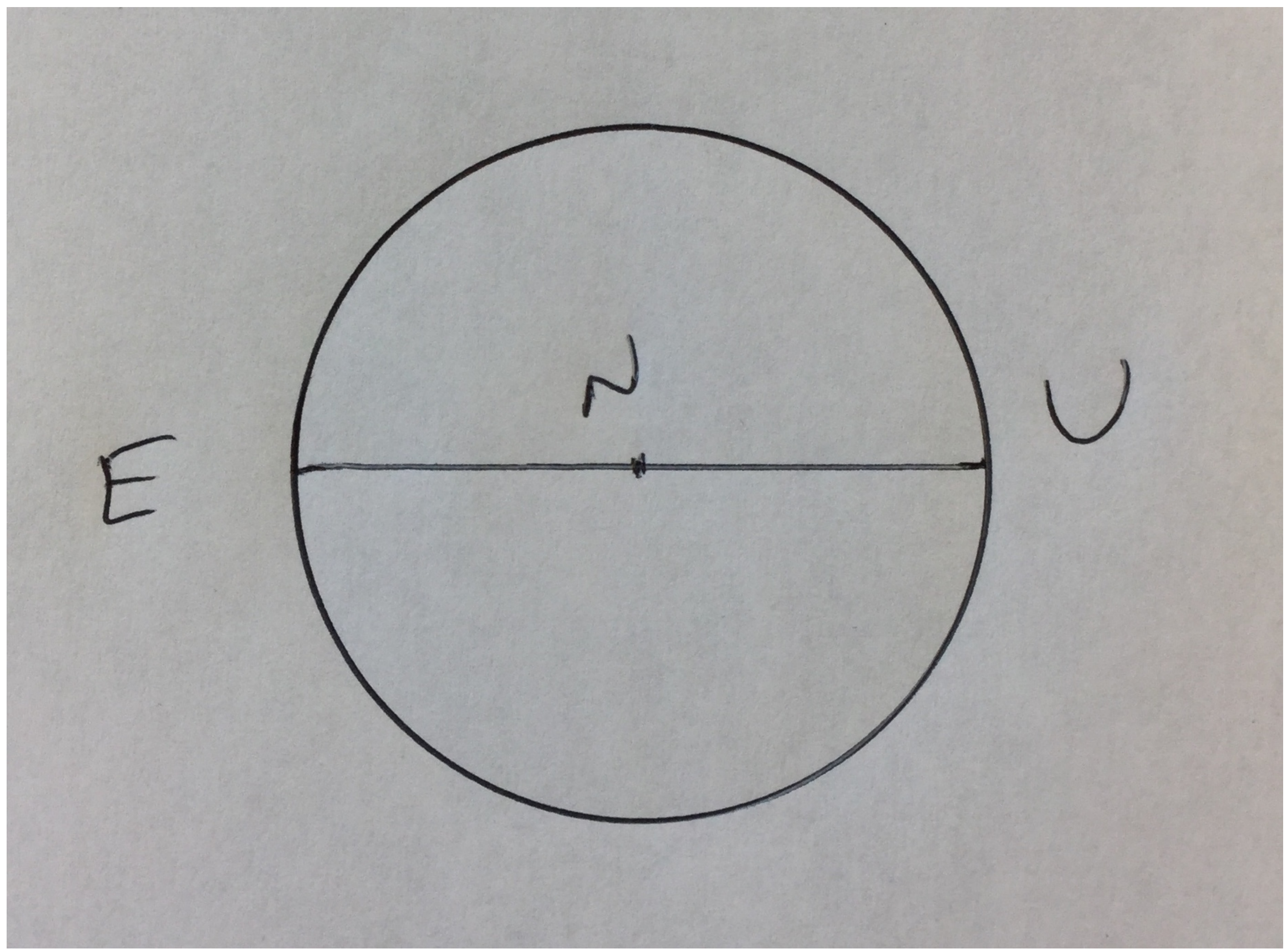
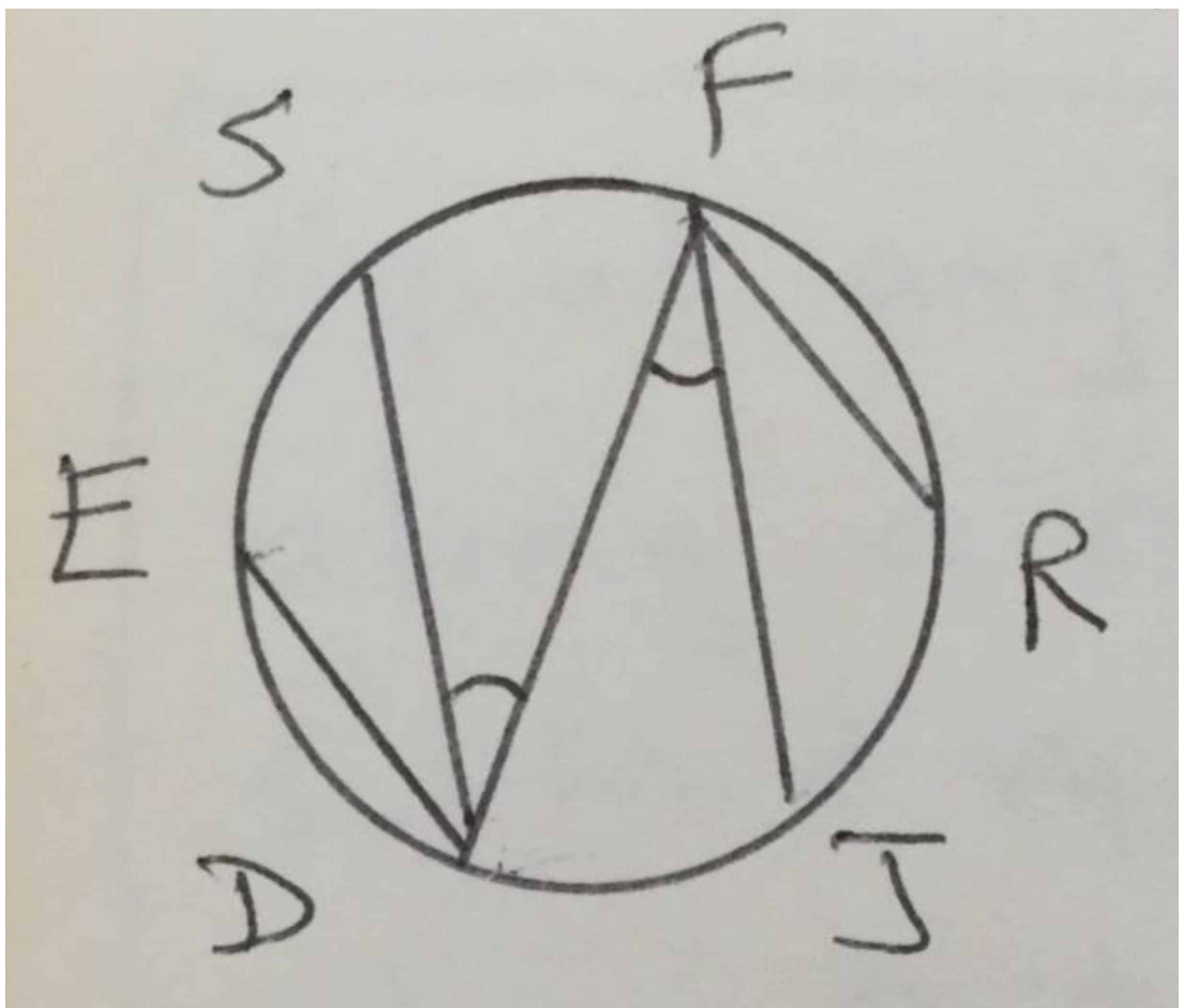


Figure 2:

Any two equal arcs ~ES and ~RJ can be shown to subtend equal angles by drawing any two parallel lines SD and JF:



~SF = ~JD

~ES + ~SF = ~RJ + ~JD

~EF = ~RD

ED || RF

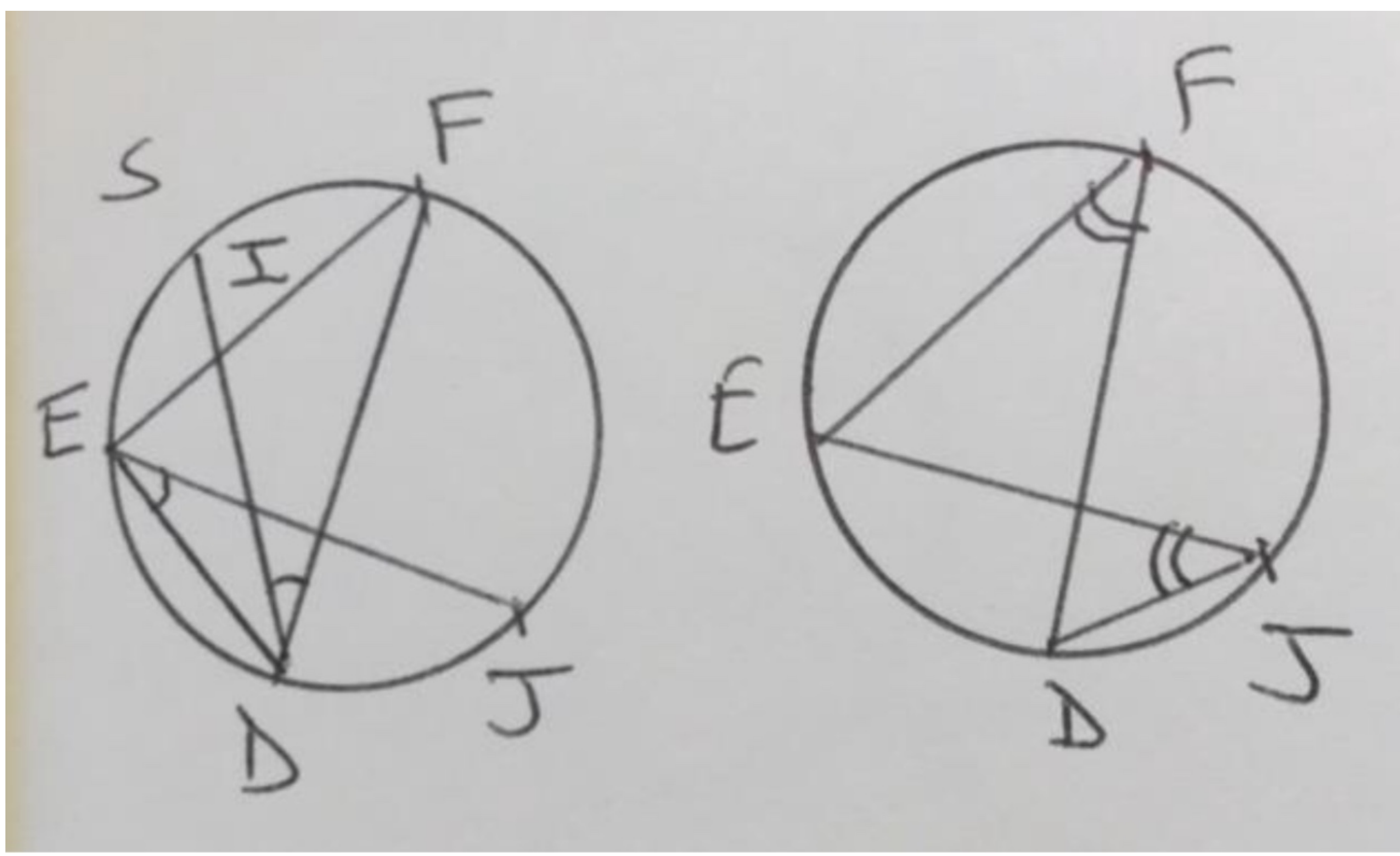
Since equal angles along a circle therefore subtend equal arcs, any angle along any circle can be defined in terms of its subtended arc and the circle's diameter. For example:

$$\angle RFJ = \sim\frac{RJ}{EU}$$

Figure 3:

Triangles need only two equal angles to be the same shape, (or ≅).

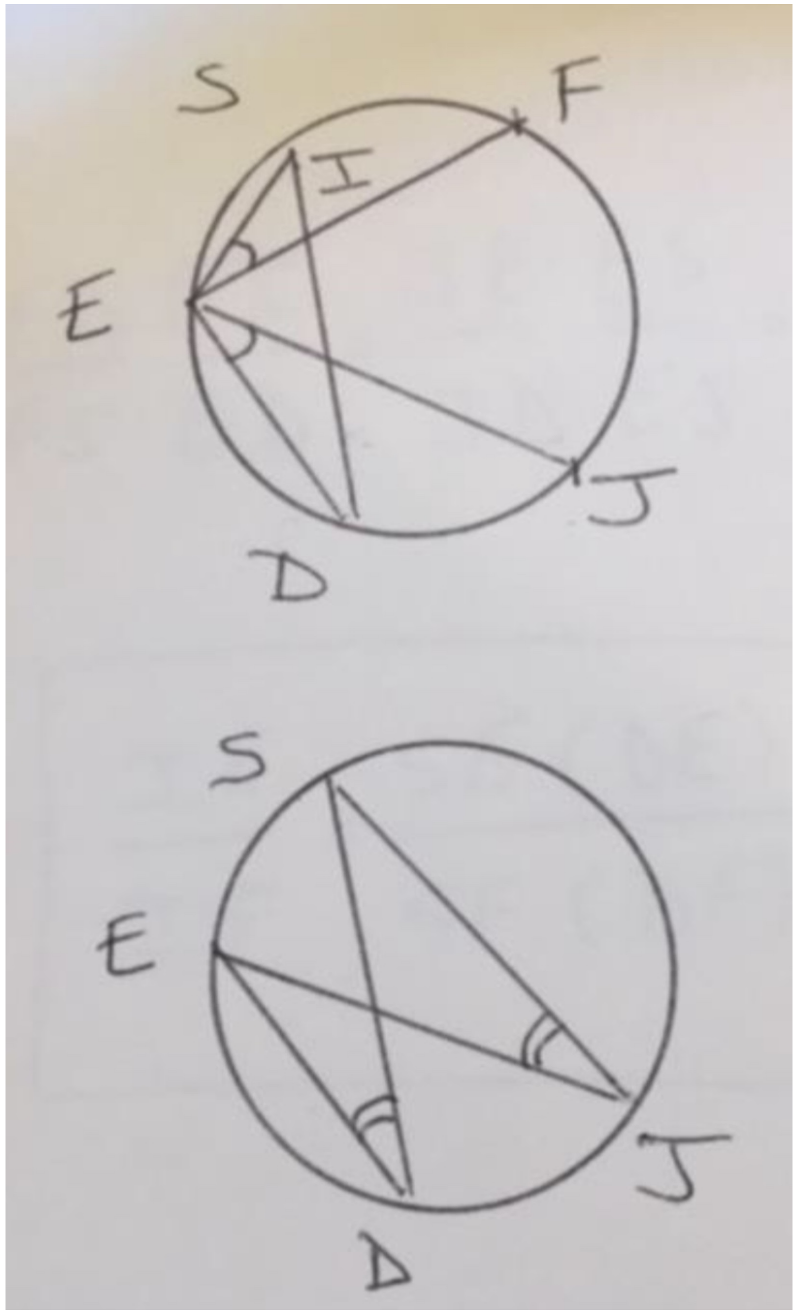
Since equal arcs subtend equal angles along a circle:



$$\Delta EJD \cong \Delta DFI$$

$$\frac{FD}{FI} = \frac{JE}{JD}$$

Figure 4:



$$\sim SJ = \sim FD$$

$$\Delta EJS \cong \Delta EDI$$

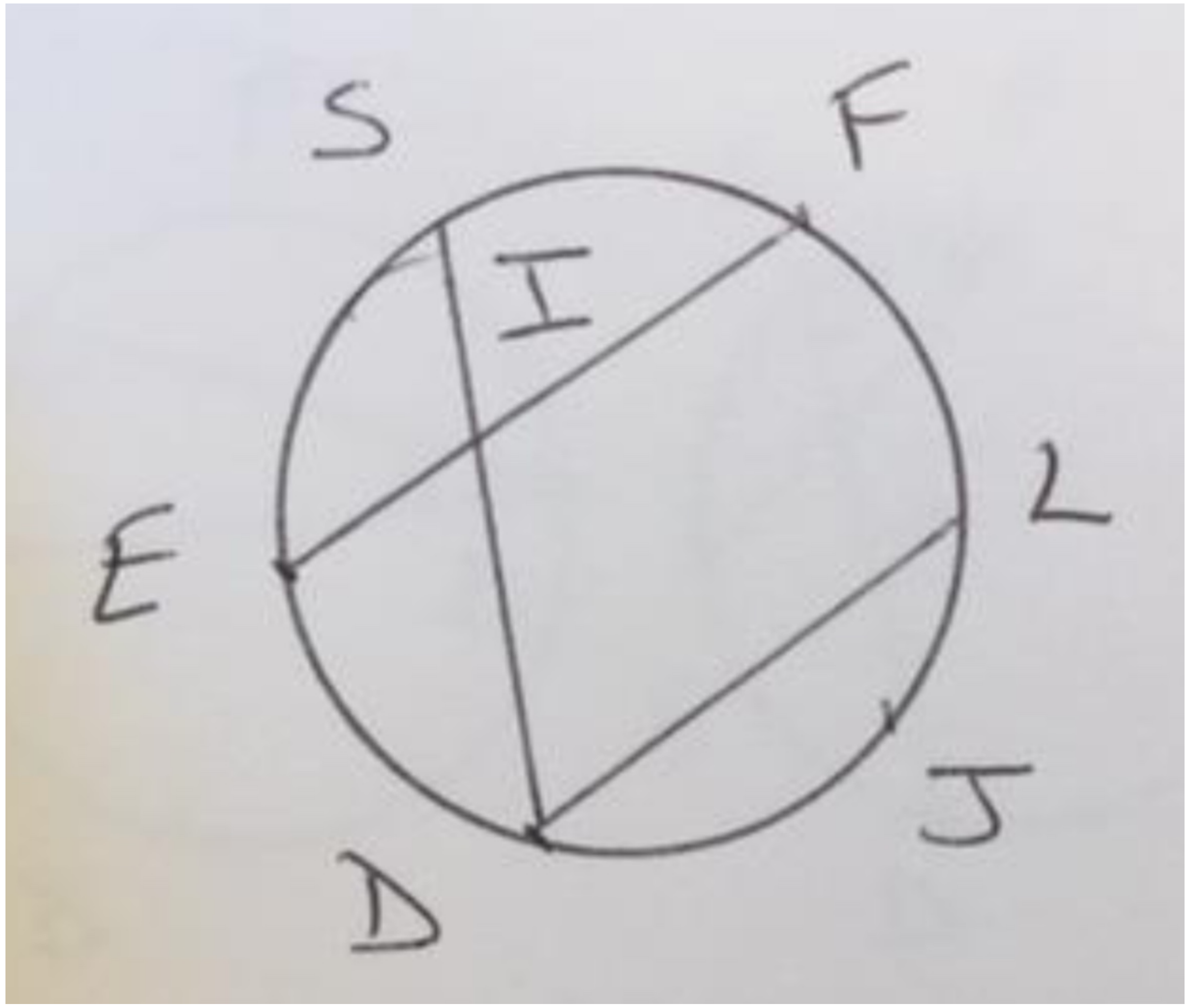
$$\frac{EI}{ED} = \frac{ES}{EJ}$$

$$\frac{FD.EI}{FI.ED} = \frac{JE.ES}{JD.EJ} = \frac{SE}{SF}$$

$$\mathbf{\frac{IE}{IF} = \frac{SE.DE}{SF.DF}}$$

which describes an important property of any cyclic quadrilateral SEDF.

Figure 5:



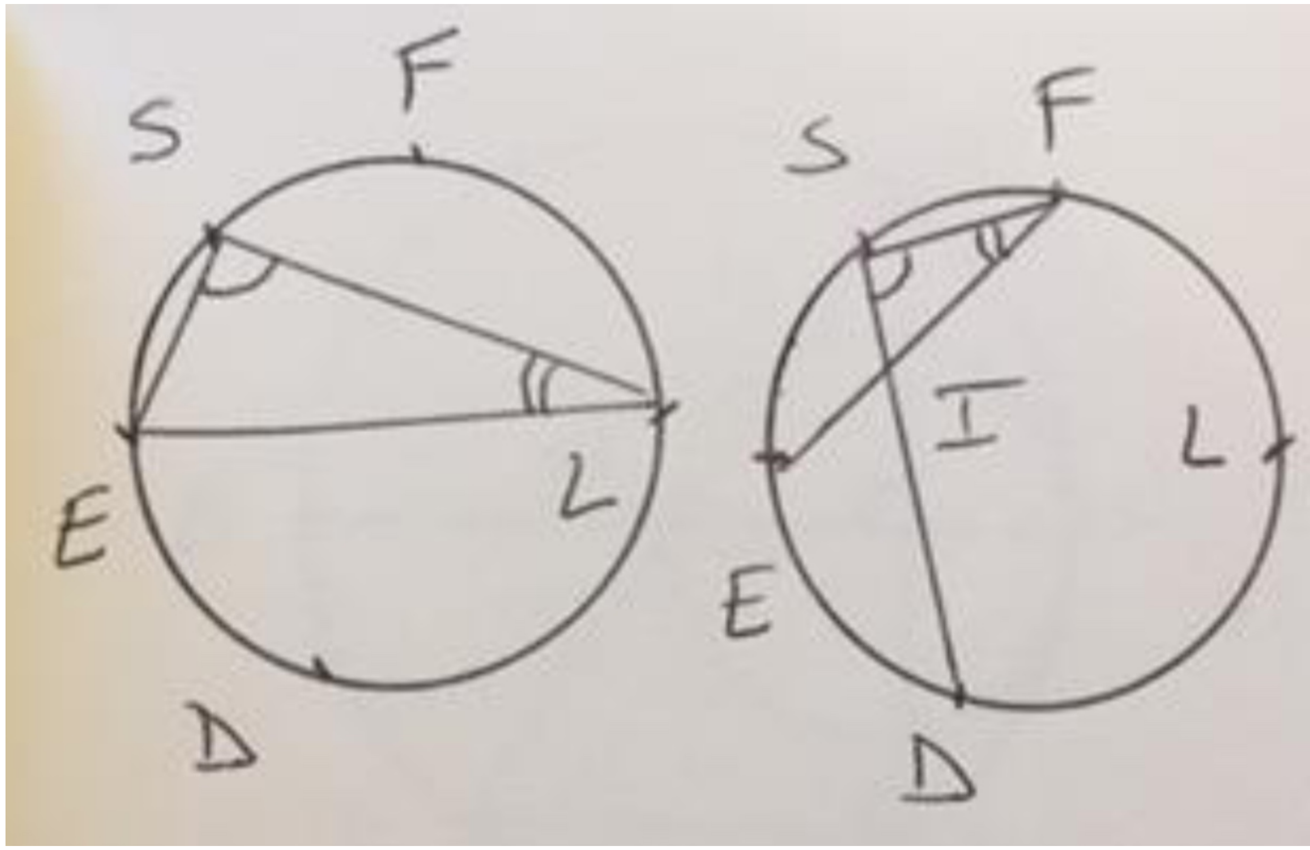
$$LD \parallel FE$$

$$\frac{DE}{DF} = \frac{LF}{LE}$$

$$\frac{IE}{IF} = \frac{SE.LF}{SF.LE}$$

$$\frac{FE}{FI} = \frac{SE.LF + SF.LE}{SF.LE}$$

Figure 6:



LD || FE

~EL = ~FD

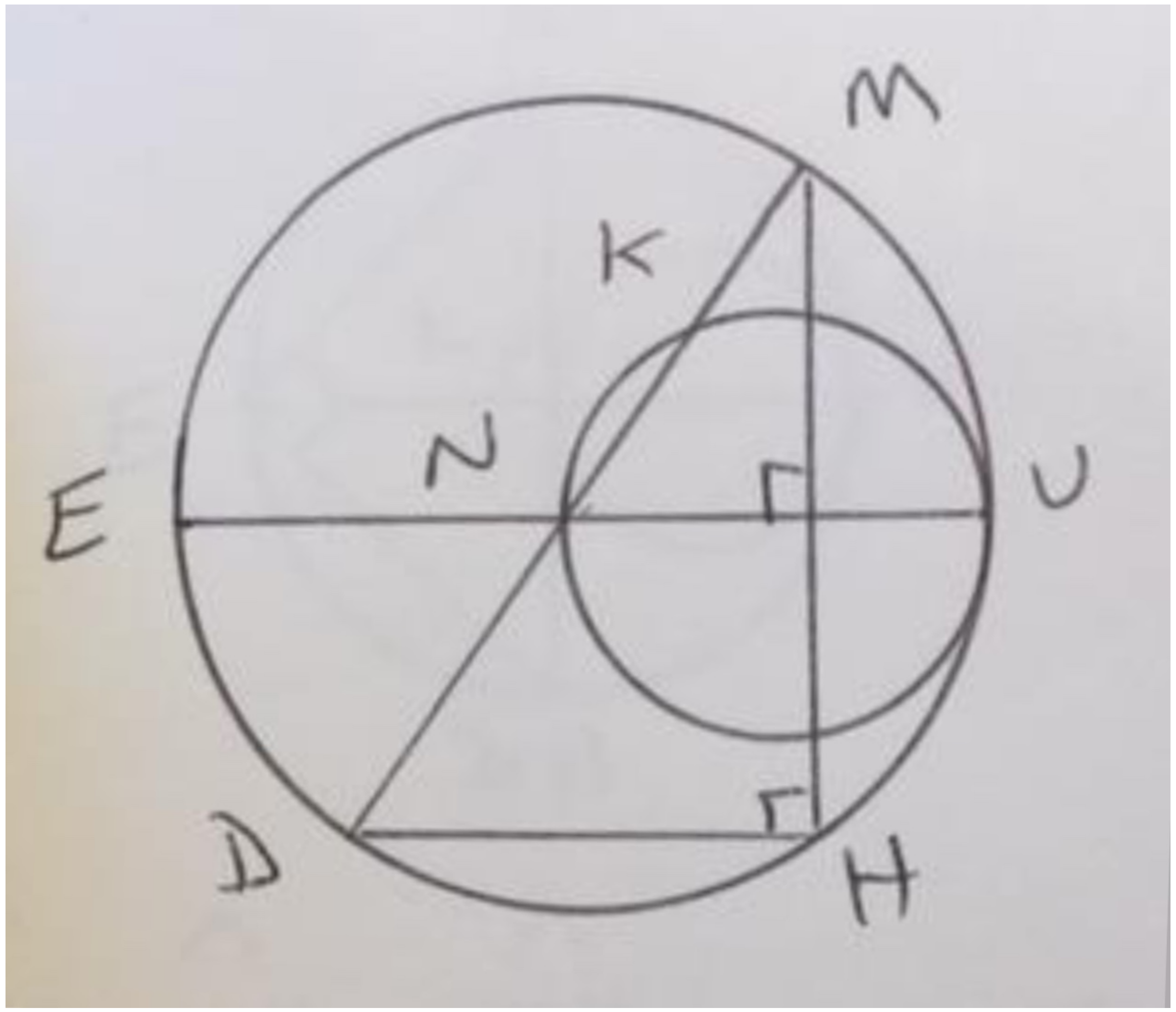
ΔLSE ≅ ΔFSI

$$LS = \frac{FS.LE}{FI}$$

FE.LS = SE.LF + SF.LE

which describes an important property of any cyclic quadrilateral SELF.

Figure 7:



$$\angle KNU = \angle MDH$$

$$\frac{\sim UK}{UN} = \frac{\sim MH}{MD} = \frac{\sim MH}{UE}$$

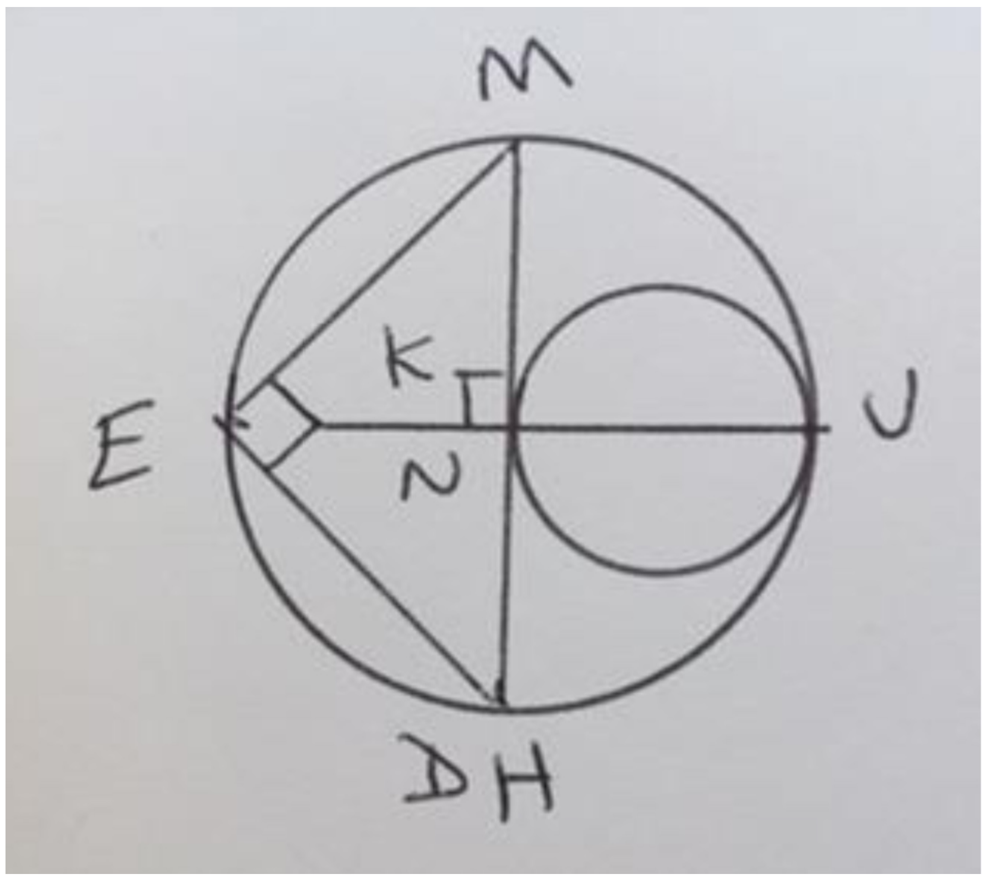
$$= \frac{2(\sim UM)}{UE} = \frac{2(\sim UM)}{2(UN)}$$

$$\mathbf{\angle KNU = 2\angle MEU}$$

$$\mathbf{\sim UK = \sim UM}$$

Figure 8:

Let K ⇒ N and D ⇒ H:



$$\frac{\sim UK}{UN} = \frac{\sim MH}{MD} = \frac{\sim MH}{UE} = \angle MEH$$

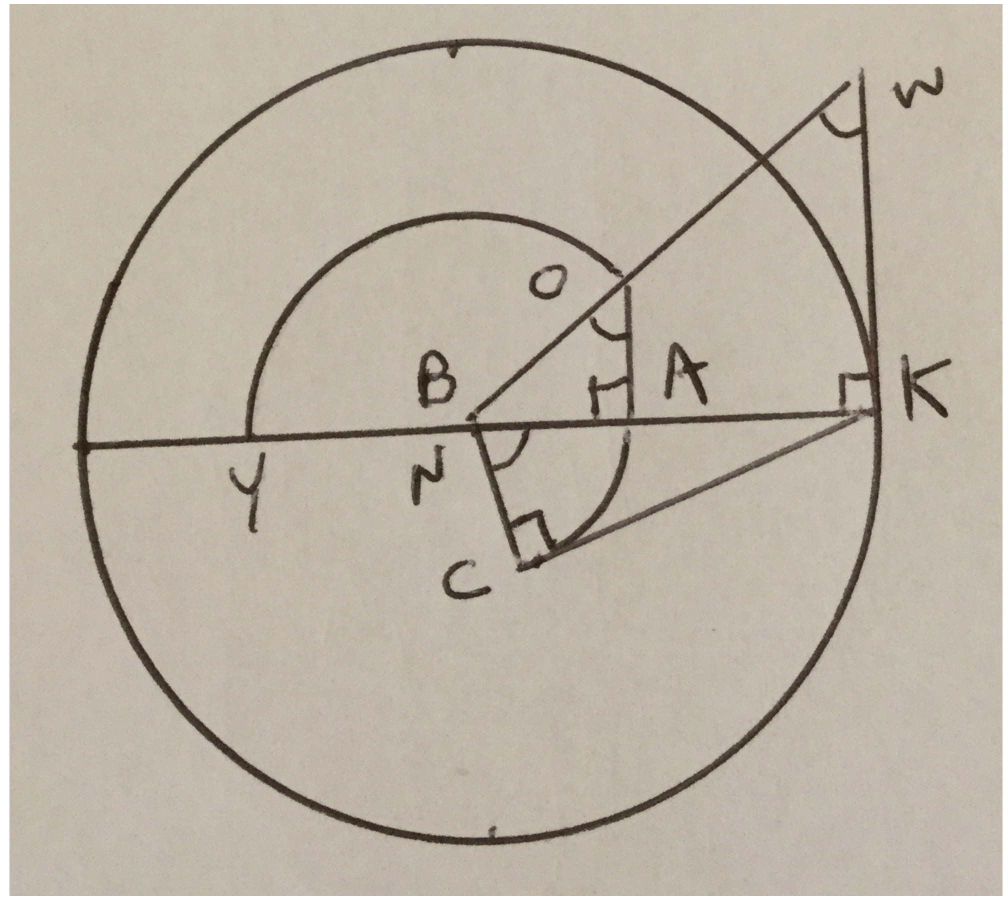
$$\frac{\sim UK}{UN} = \angle MNU$$

$$\mathbf{\frac{2(\sim UK)}{UN} = \angle MNH = \pi}$$

Section 2

Refraction Along a Line

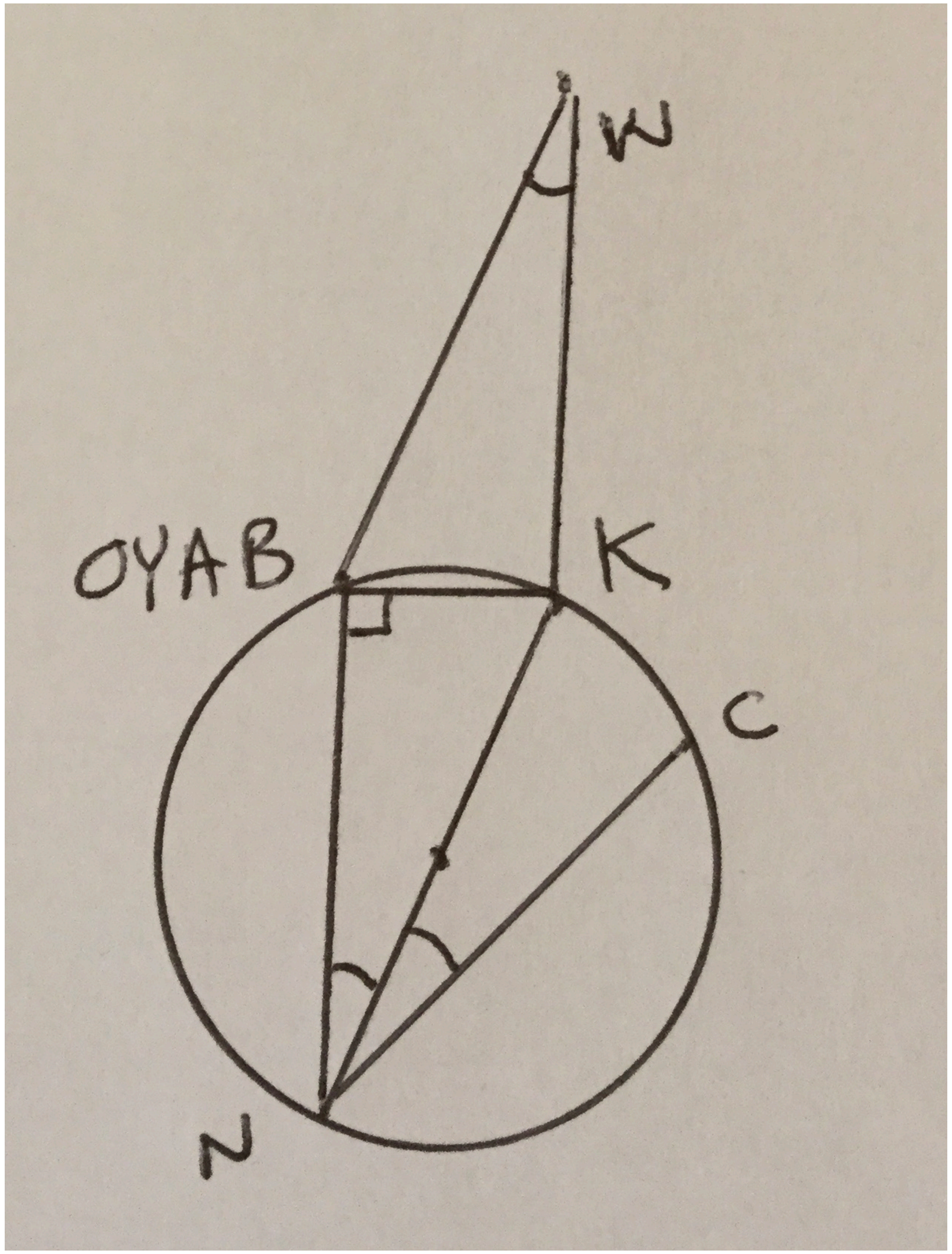
Figure 9:



$$\frac{(KW)}{(OA)} = \frac{NK}{NA} = \frac{NK}{NC} = \frac{OB}{OA} = \frac{WB}{WK}$$

KW (=OB) = YN

Figure 10:



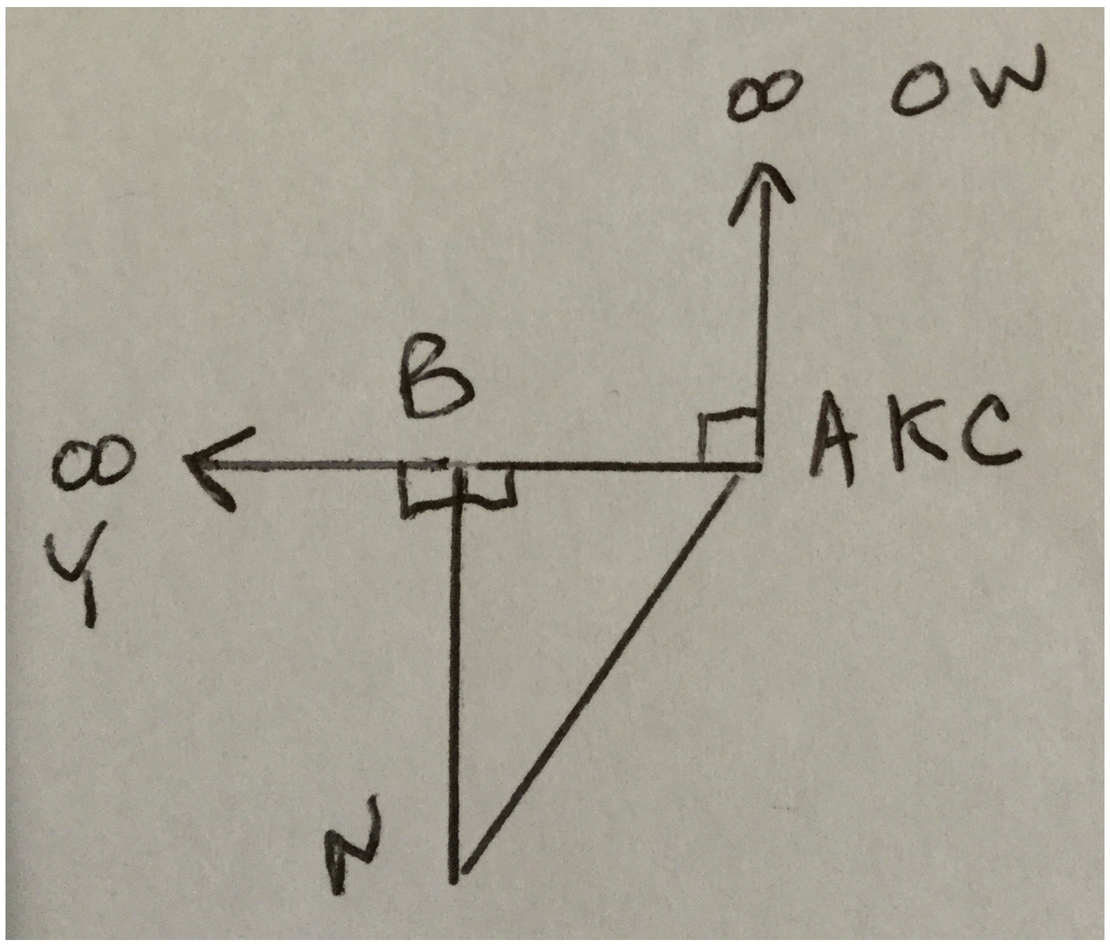
Create right triangle NBK.

When A lies at B:

$$\frac{NK}{NA} = \frac{NK}{NC} = \frac{(OB)}{(OA)} = \frac{WB}{WK}$$

$$KW = YN$$

Figure 11:

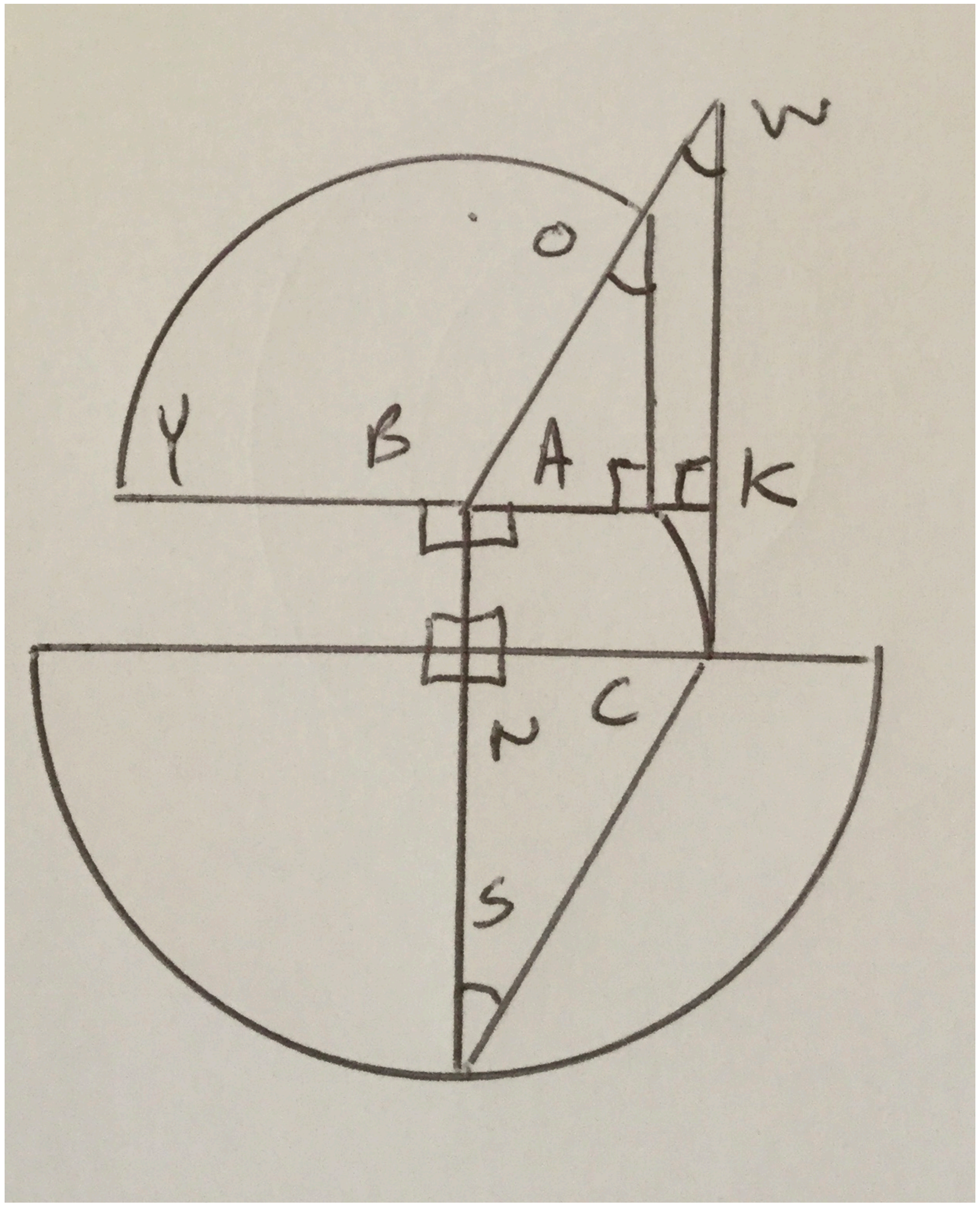


When A lies at K:

$$\frac{NK}{NA} = \frac{NK}{NC} = \frac{(OB)}{(OA)} = \frac{WB}{WK}$$

KW = YN = infinity

Figure12:



when:

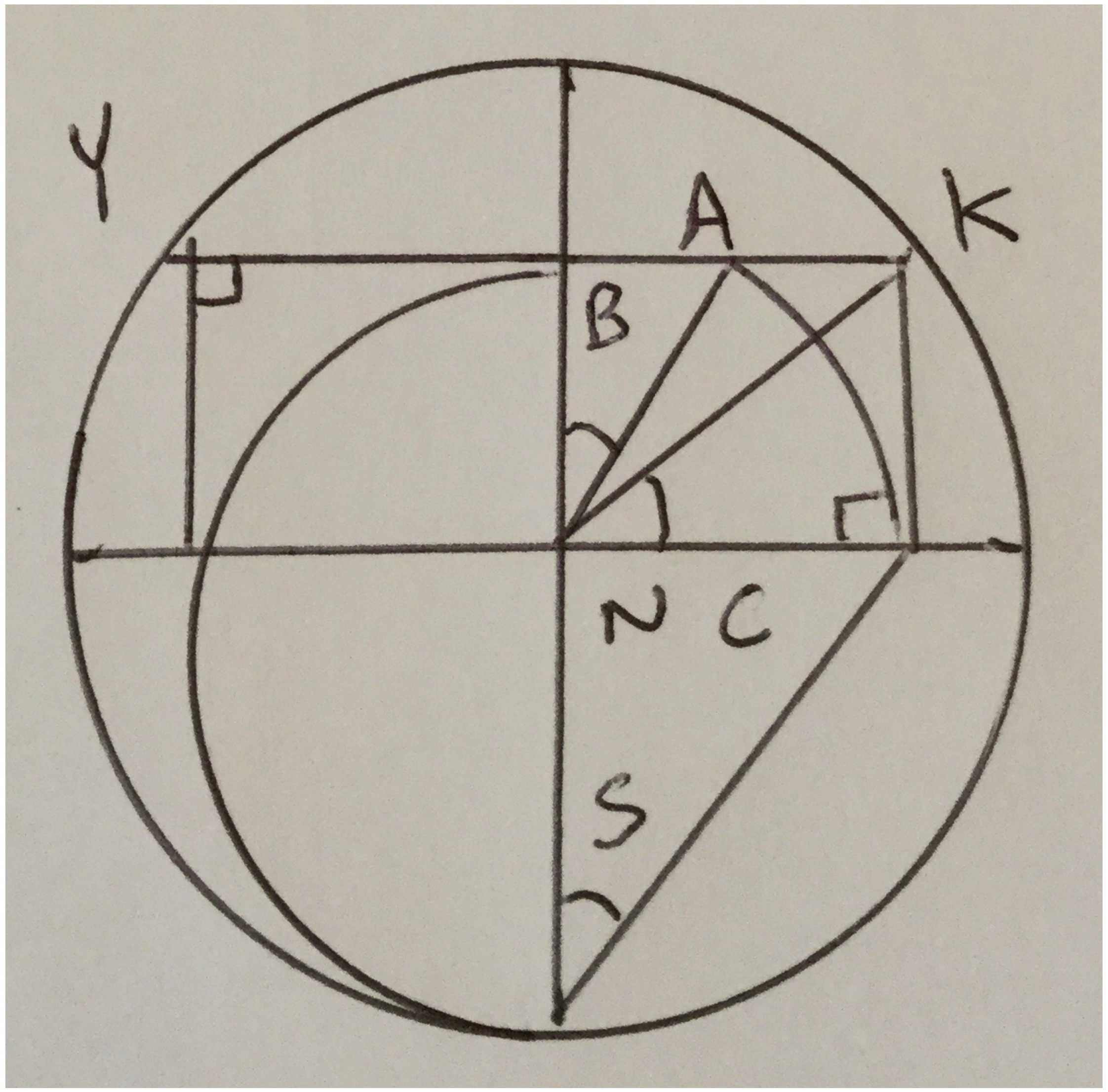
SC = BW II SC

KW = NS

$\frac{NS}{NC} = \frac{NS}{NA}$

$\frac{NC}{NB} = \frac{NA}{NB}$

Figure 13:



if: $\frac{NS}{NC} = \frac{NC}{NB}$

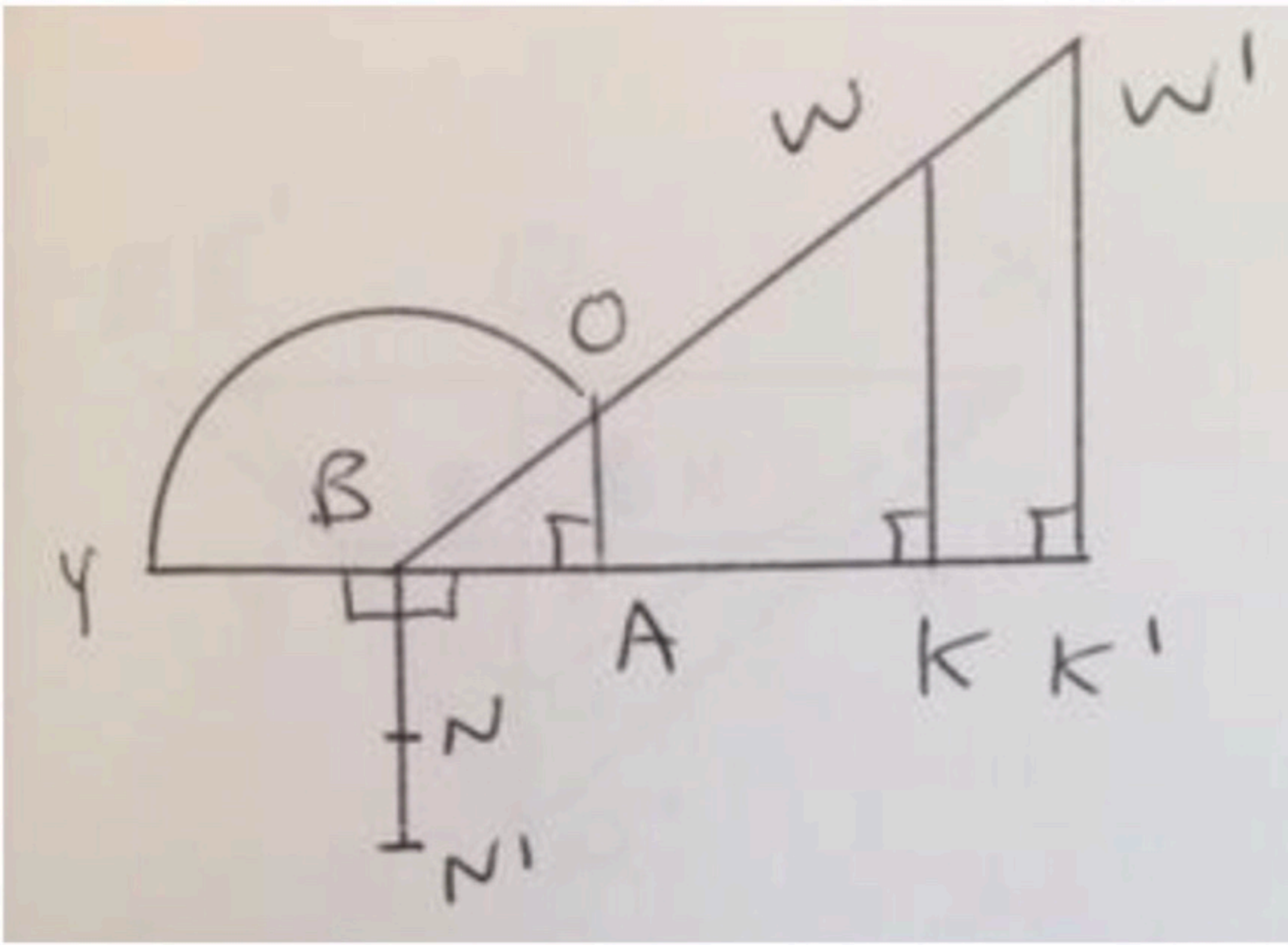
then: $\frac{NK}{NC} = \frac{NA}{NB}$

NA II SC

KW (= NS) = YN

It is obvious that as A approaches K from B, the relative rate that YN and KW approach infinity does not plateau, peak, or dip. Since we have shown that YN = KW when A lies at a point along BK other than B as well as at B, we have shown that YN = KW for all points A along BK.

Figure 14:



$$\frac{OB}{OA} = \frac{NK}{NA} = \frac{N'K'}{N'A}$$

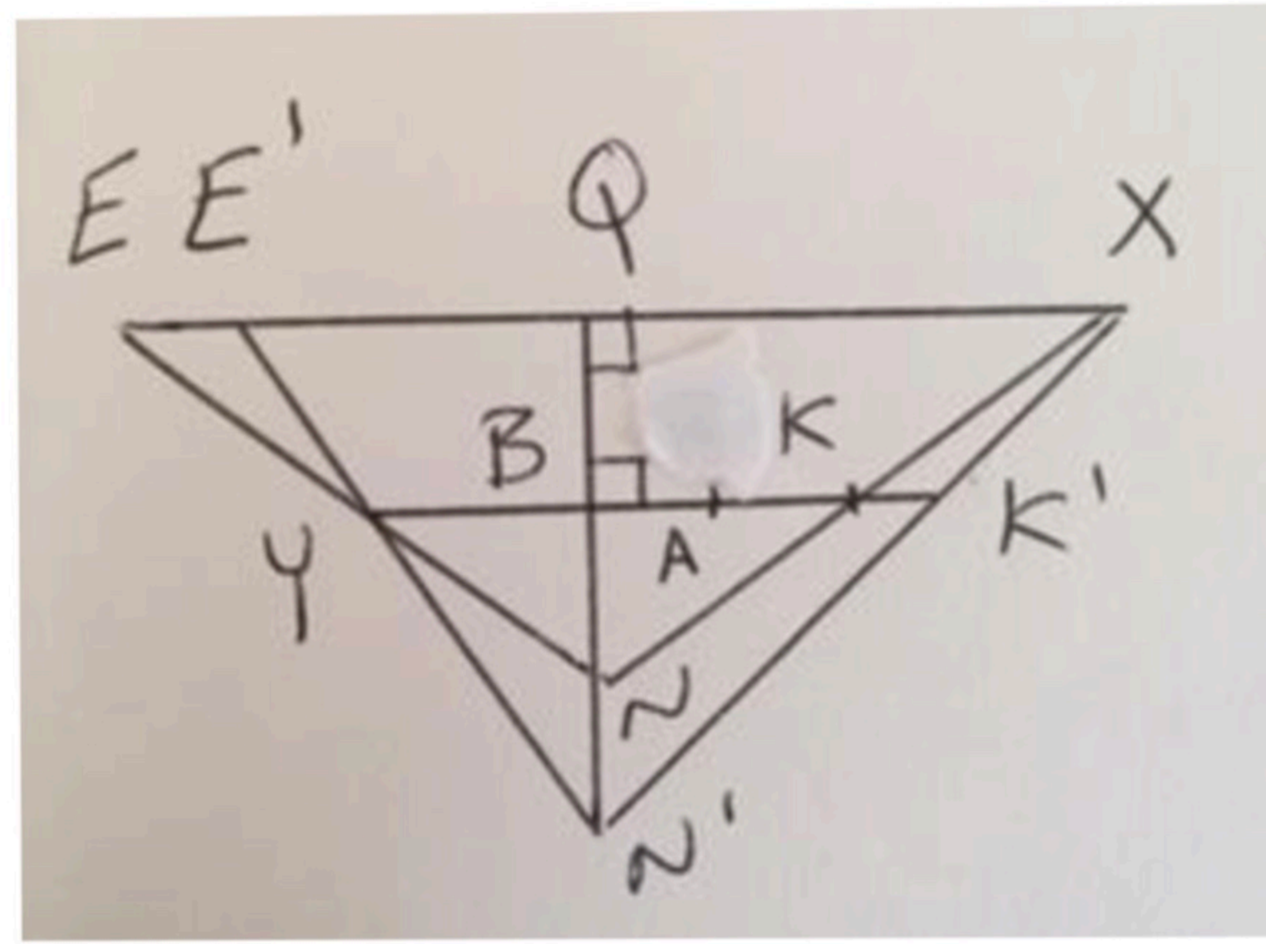
$$KW = YN$$

$$K'W' = YN'$$

$$\frac{KB}{YN} = \frac{K'B}{YN'}$$

Figure 15:

As the equal lengths of EN and E’N’ rotate about Y until they overlap, they approach their minimum which also occurs when N’K’X’ and NKX overlap.

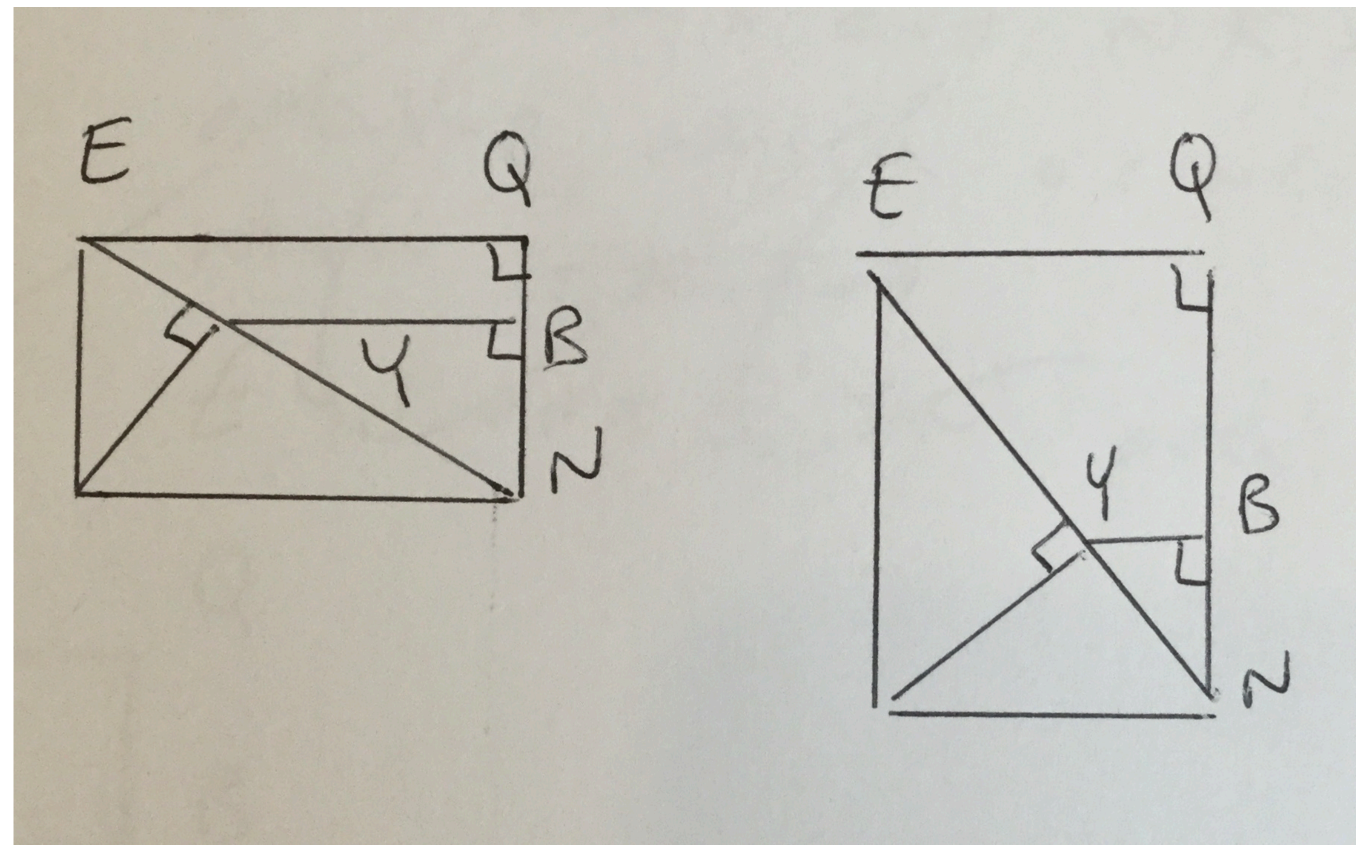


$$\frac{QX}{EN} = \frac{KB}{YN} = \frac{K'B}{YN'} = \frac{QX}{E'N'}$$

only one N’K’X exists for NKX because only one E’N’ equals EN

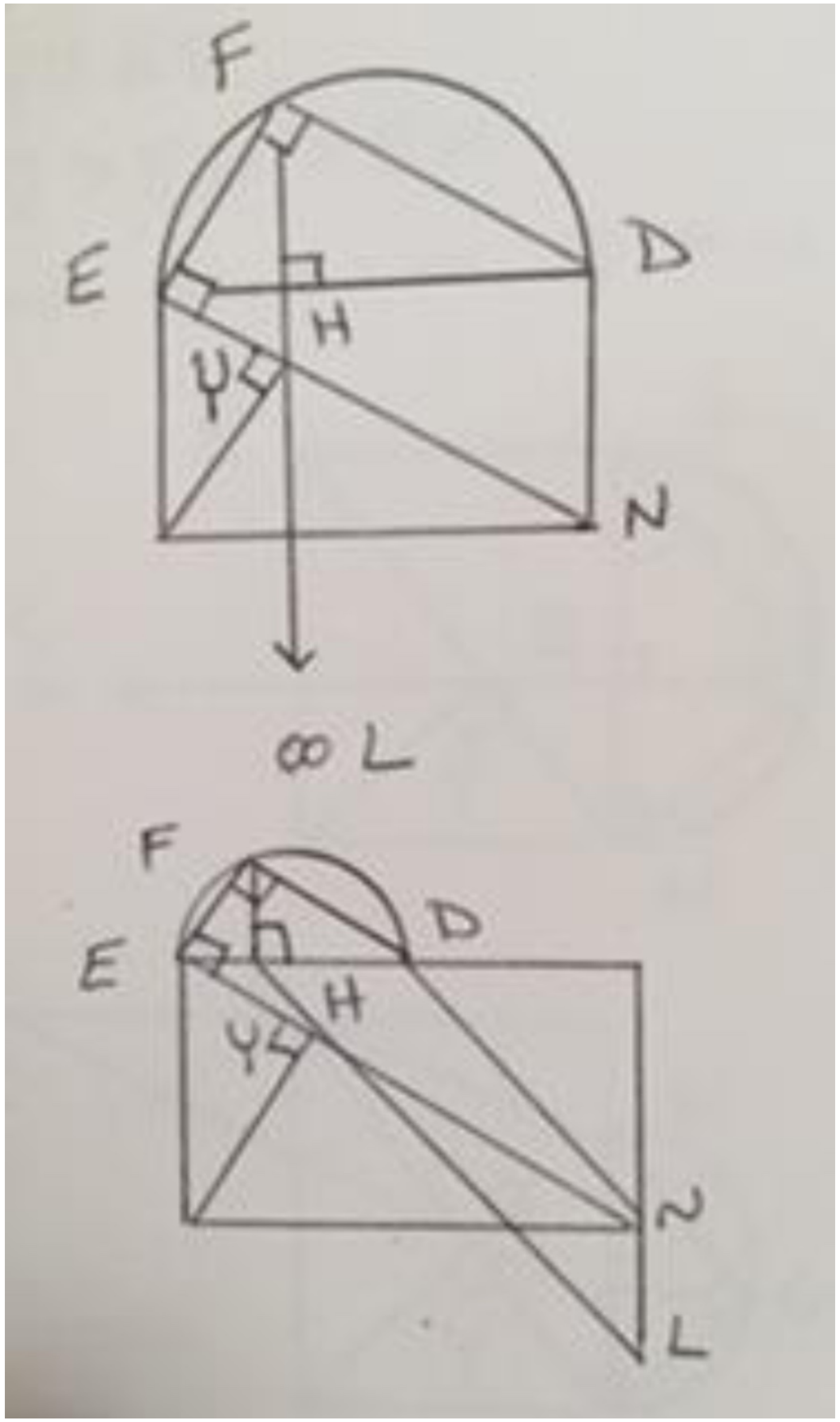
Figure 16:

Let X = Z when both NKX and N’K’X overlap, which occurs when EYN is the shortest line segment through Y connecting line QB to its perpendicular at Q. This occurs when:



because:

Figure 17:



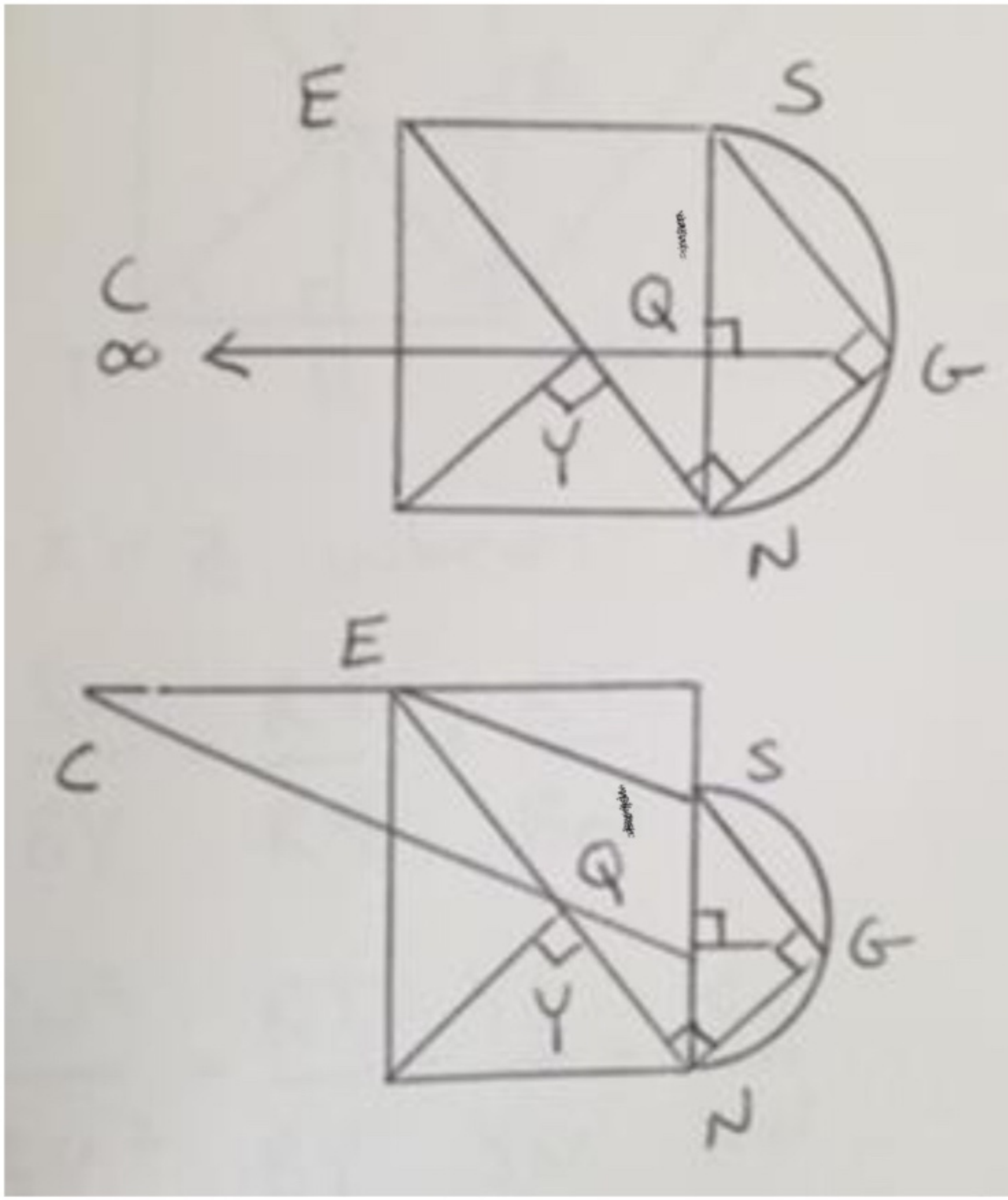
LH || ND

LH > NF > NE

holds true as:

H ⇒ E

Figure 18:



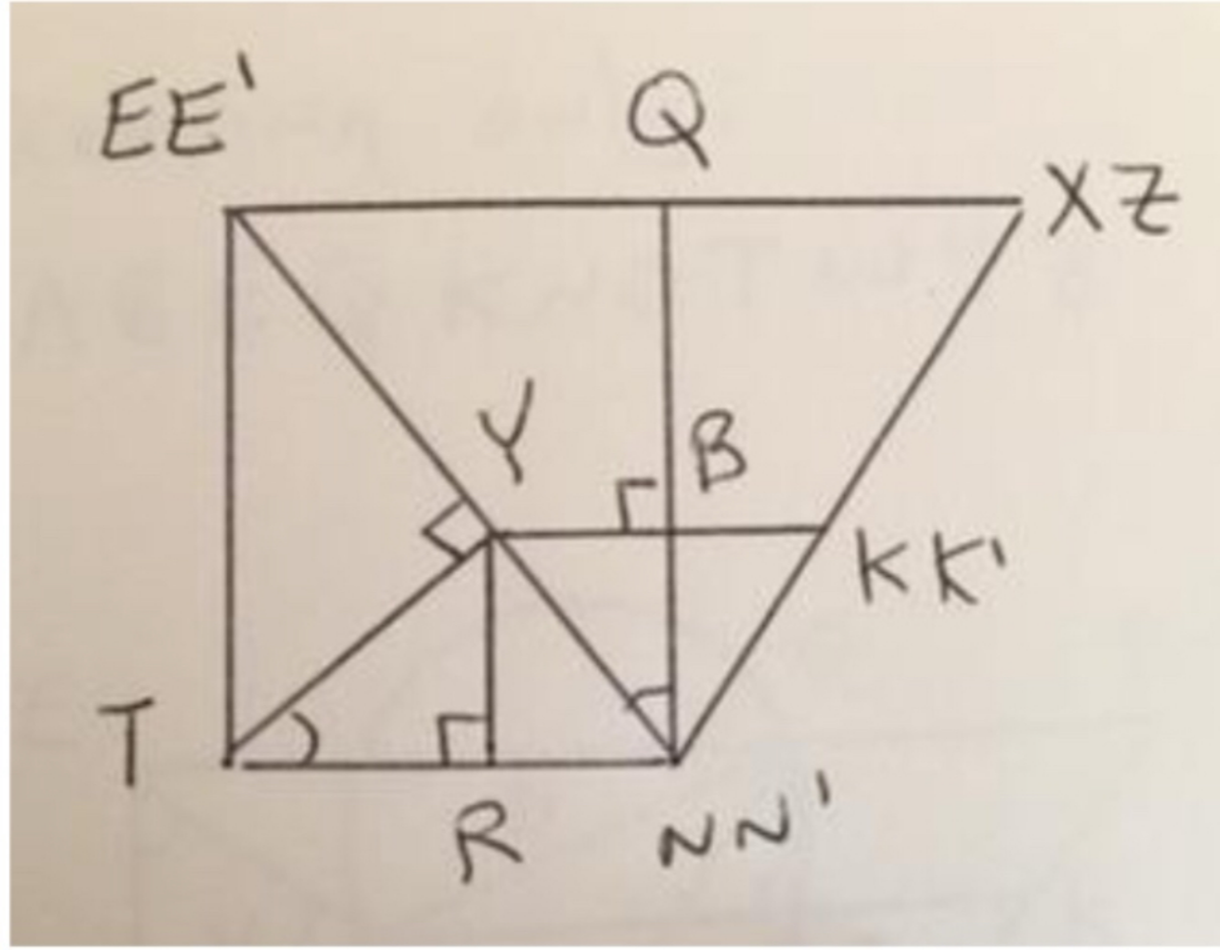
CQ' || ES

CQ' > EG > EN

holds true as

Q' ⇒ N

Figure 19:

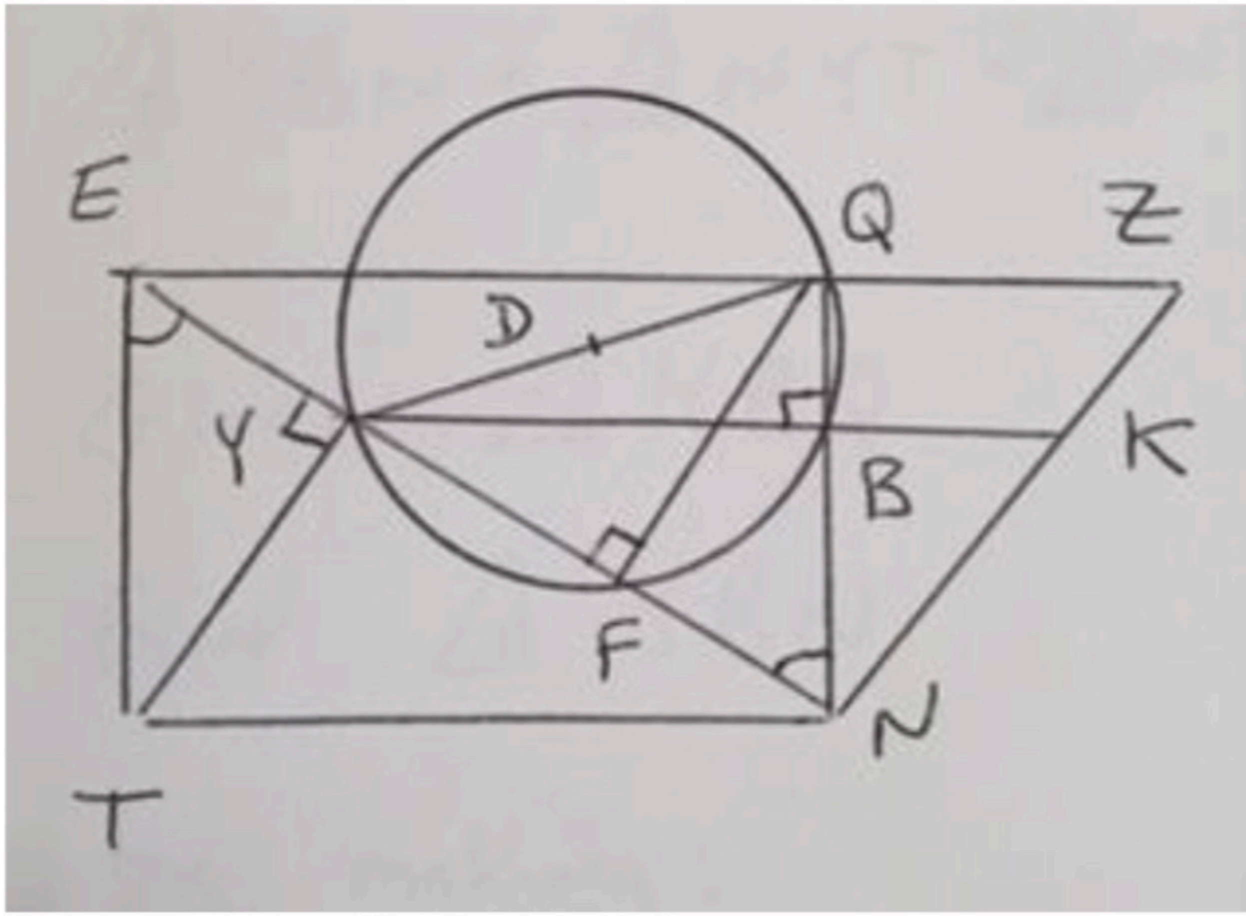


X = Z when:

$$\frac{BN}{BY} = \frac{RT}{RY} = \frac{RT}{BN}$$

$$\frac{BN^2}{BY^2} = \frac{RT}{BY} = \frac{YE}{YN} = \frac{KX}{KN}$$

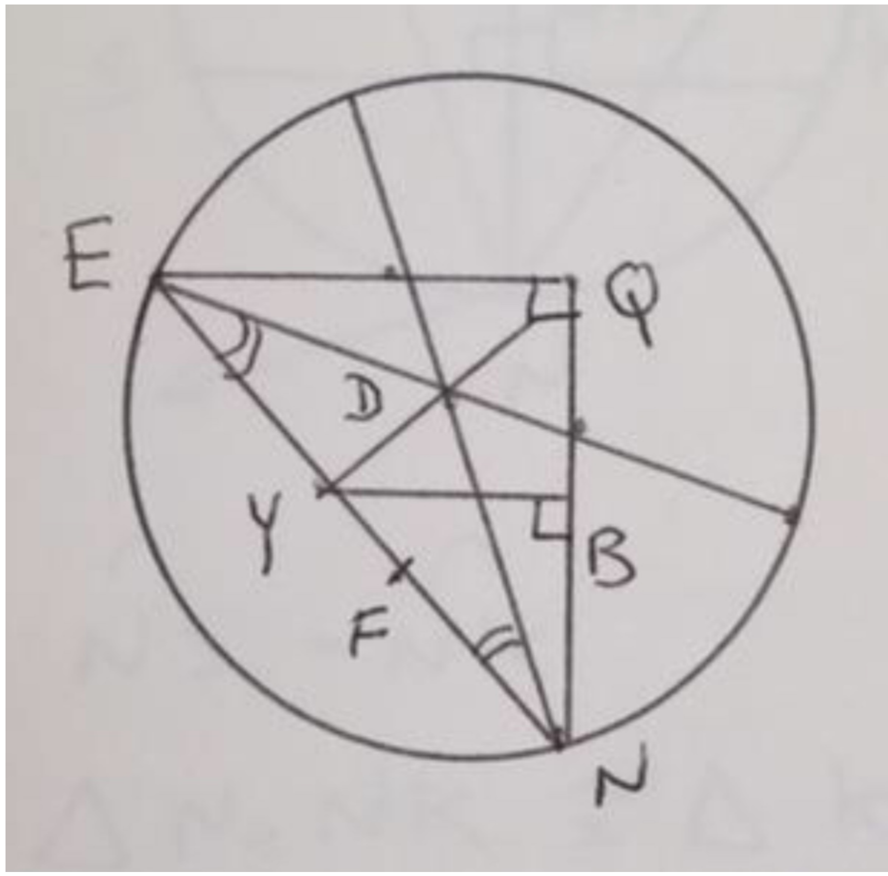
Figure 20:



given ΔYBN, find ΔYBQ using:

$$\Delta YBN \cong \Delta NYT \cong \Delta NTE$$

Figure 21:



given ΔYBQ, find ΔYBN by making:

EY = NF

which occurs when ~EN lies on a circle concentric with circle YFBQ

because:

DY = DF

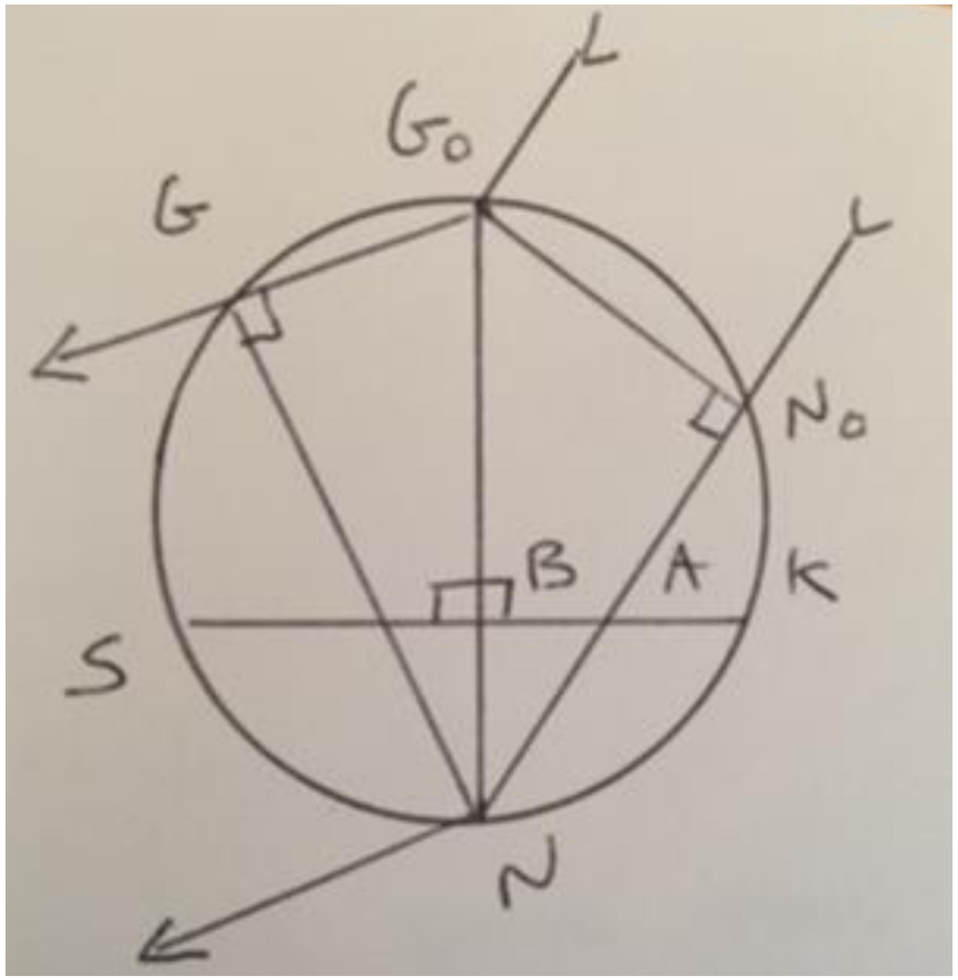
ΔEDY = ΔNDF

EY = NF

Before considering refraction along a line, picture yourself sitting on the beach watching waves roll in. Notice that even when wavefronts far out in the ocean are traveling perpendicular to the beach, they become closer to parallel with the beach as they crash. On beaches that are long and sloped, or have many sandbars, these wavefronts all crash parallel to the beach, regardless of their orientation in the open ocean.

Now picture yourself in a car applying brakes while driving. If the brakes on the front right wheel grip harder, the car will turn to the right. This is intuitive. For the same reason, when a wavefront hits a sandbar at an angle, one side of the wavefront will slow before the other, and this will tend to turn the wavefront parallel to the beach. This essentially represents refraction along a line.

Figure 22:



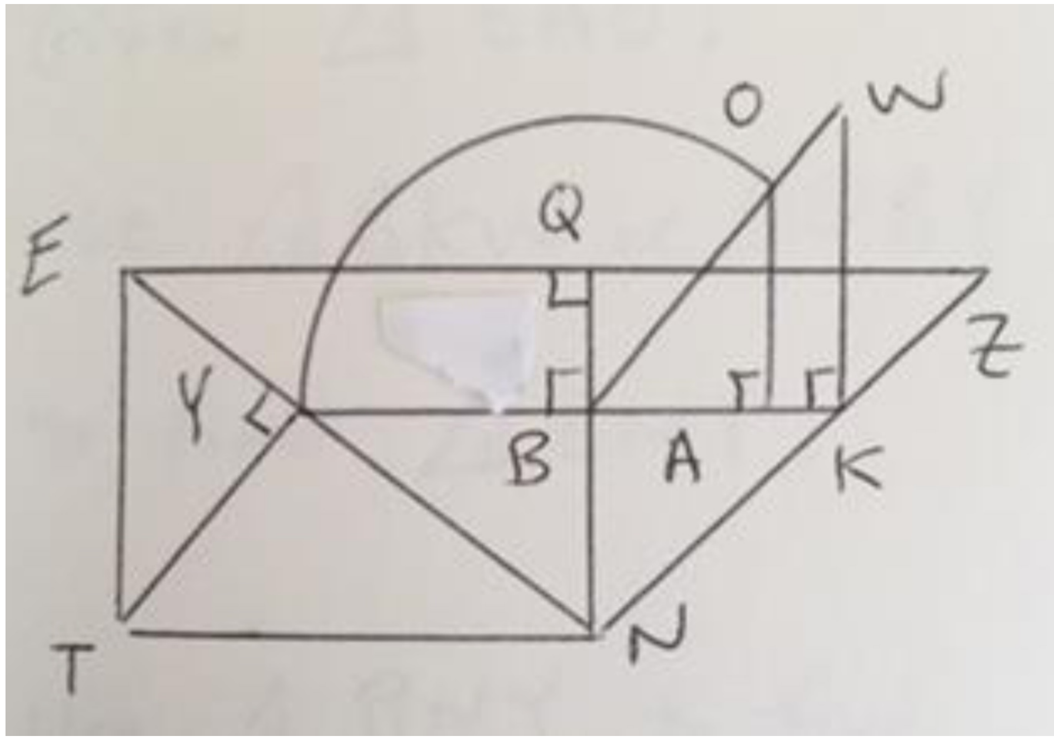
$\sim NS \;=\; \sim NK$

$\Delta N_\circ NK \;\cong\; \Delta KNA$

$$\mathbb{R} \;=\; \frac{NN_\circ}{GG_\circ} \;=\; \frac{NN_\circ}{NK} \;=\; \frac{NK}{NA}$$

wavefront $G_\circ N_\circ$ refracts into
wavefront GN along $G_\circ N$,
because it travels $G_\circ G$
in the same time it travels $N_\circ N$

Figure 23:



If $\mathbb{R} = \dfrac{OB}{OA}$ and $KW = YN$:

$\mathbb{R} = \dfrac{NK}{NA}$

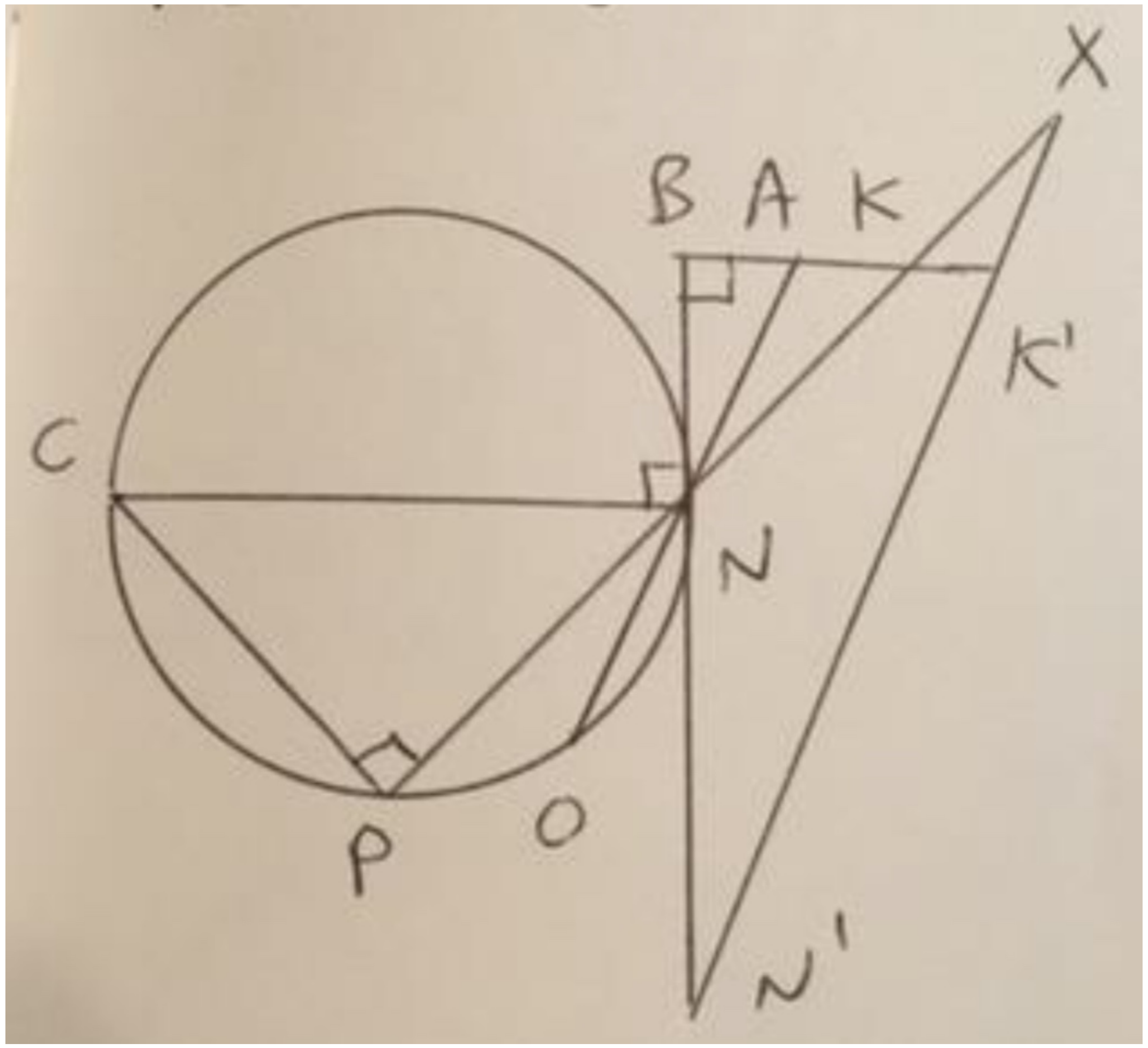
and Z is the clear image of object A refracted at N along BN.

given ΔBAO:
use ΔBKW or ΔQBY to find ΔBNY.
use ΔBNY to find ΔBKW or ΔQBY.

Section 3

Refraction Along a Circle

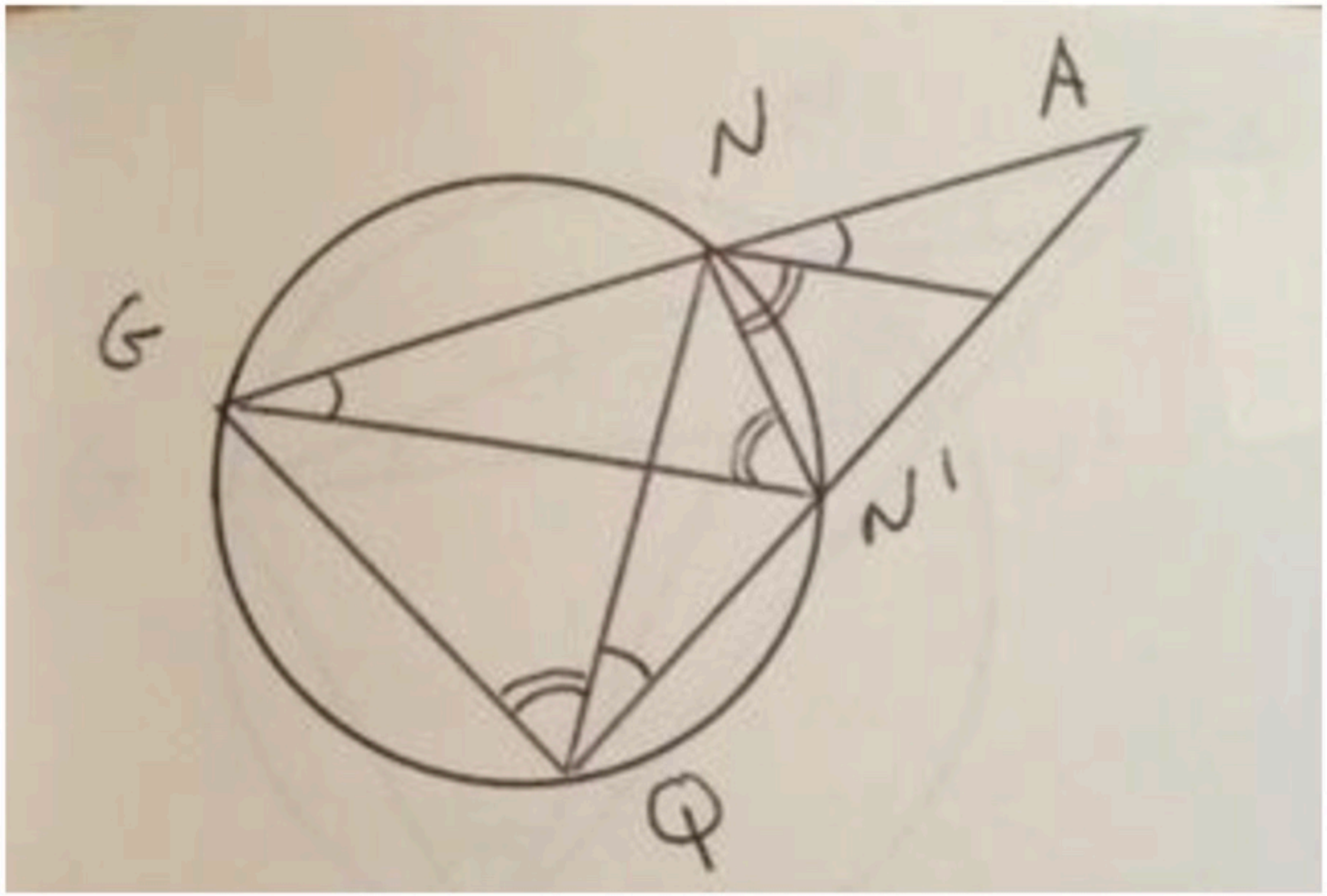
Figure 24:



$$\Delta KNA \cong \Delta OCP$$

$$\mathbb{R} = \frac{NK}{NA} = \frac{N'K'}{N'A} = \frac{CO}{CP}$$

Figure 25:



$\Delta ANN' \cong \Delta AQG$

Figure 26:

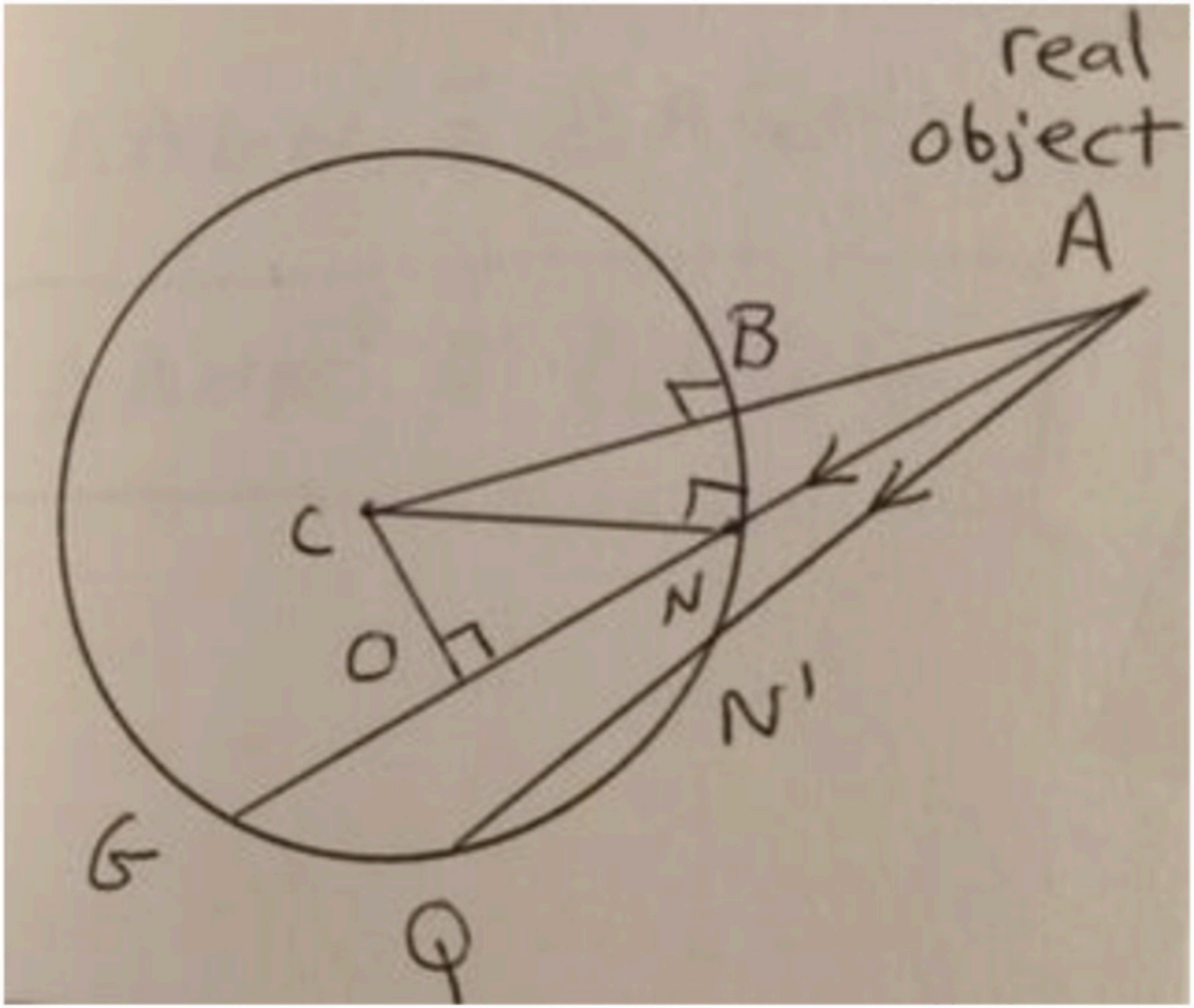
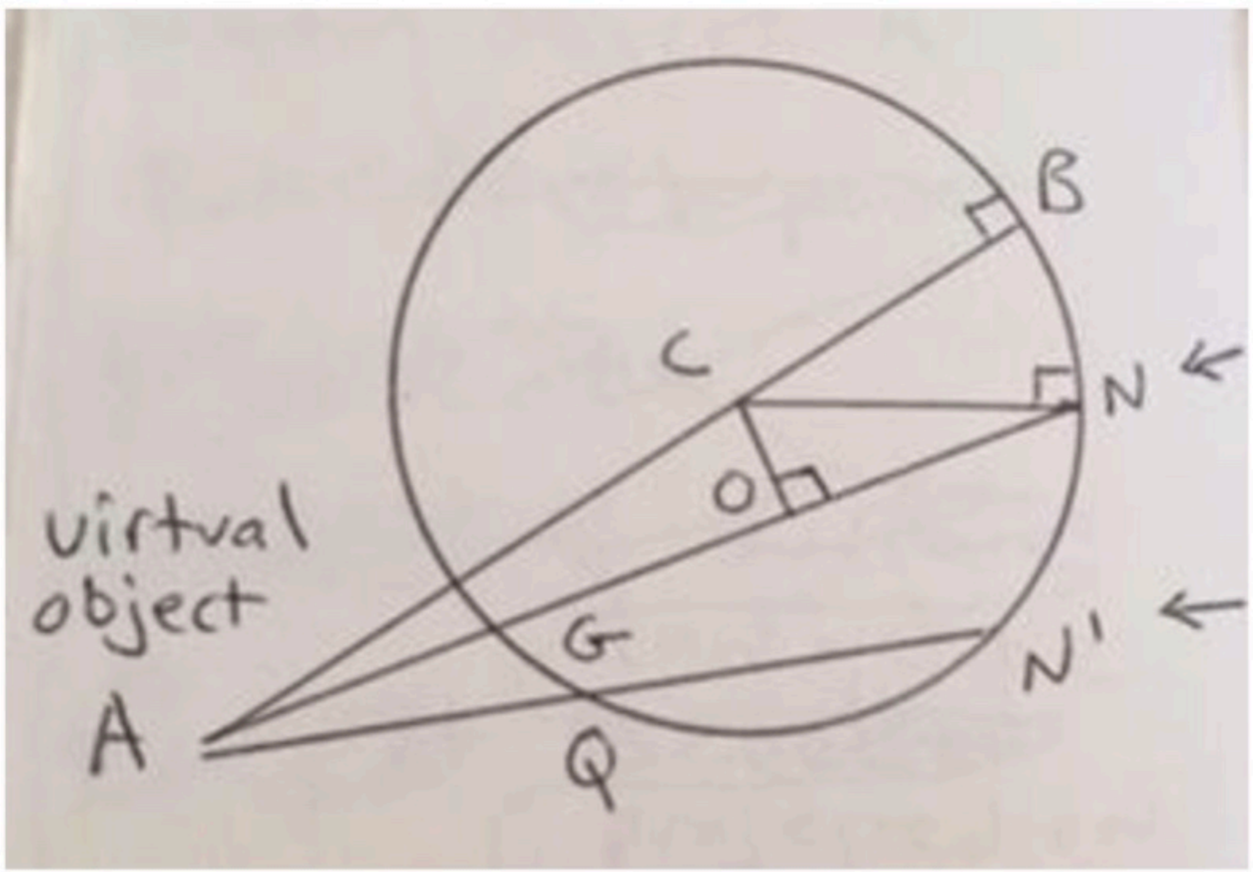
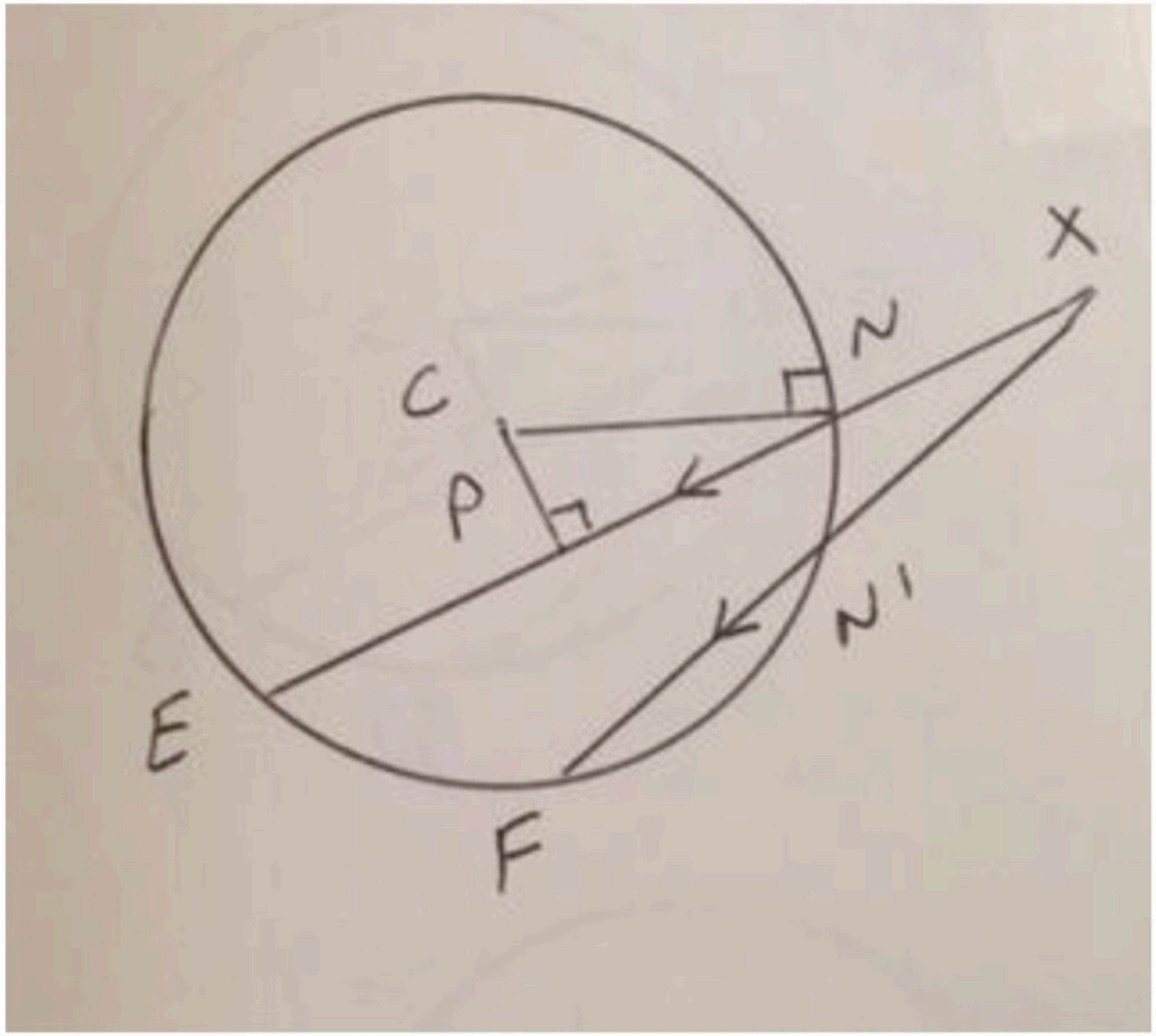


Figure 27:



the virtual object A can not be projected on a screen
due to refraction at BN

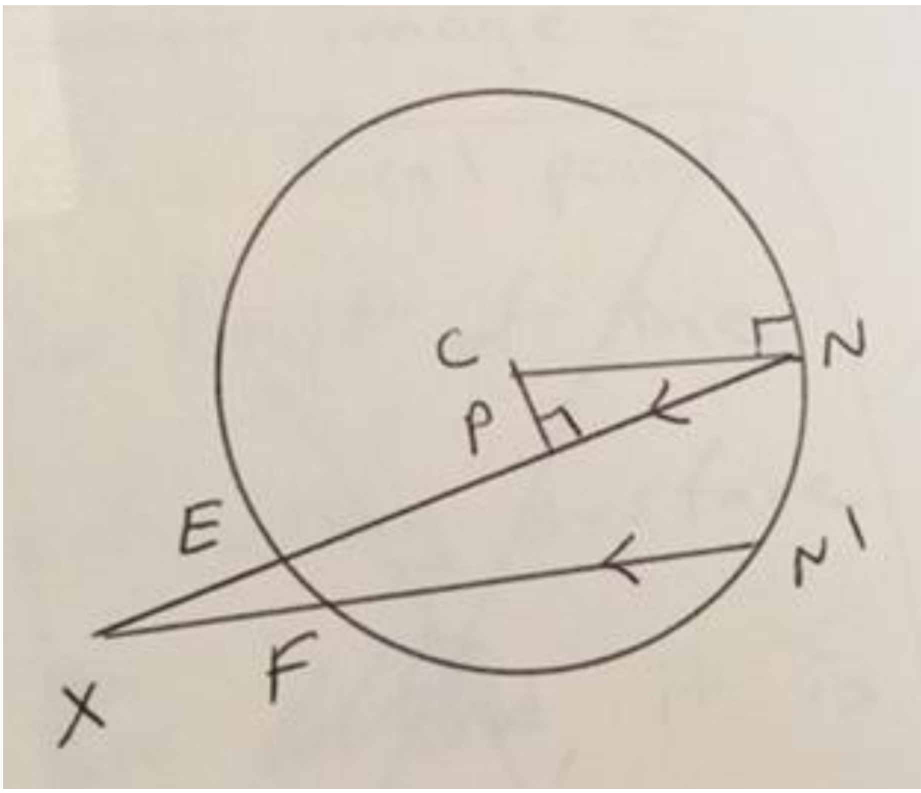
Figure 28:



$\Delta XNN' \cong \Delta XFE$

the virtual image (Z) can not be projected on a screen

Figure 29:



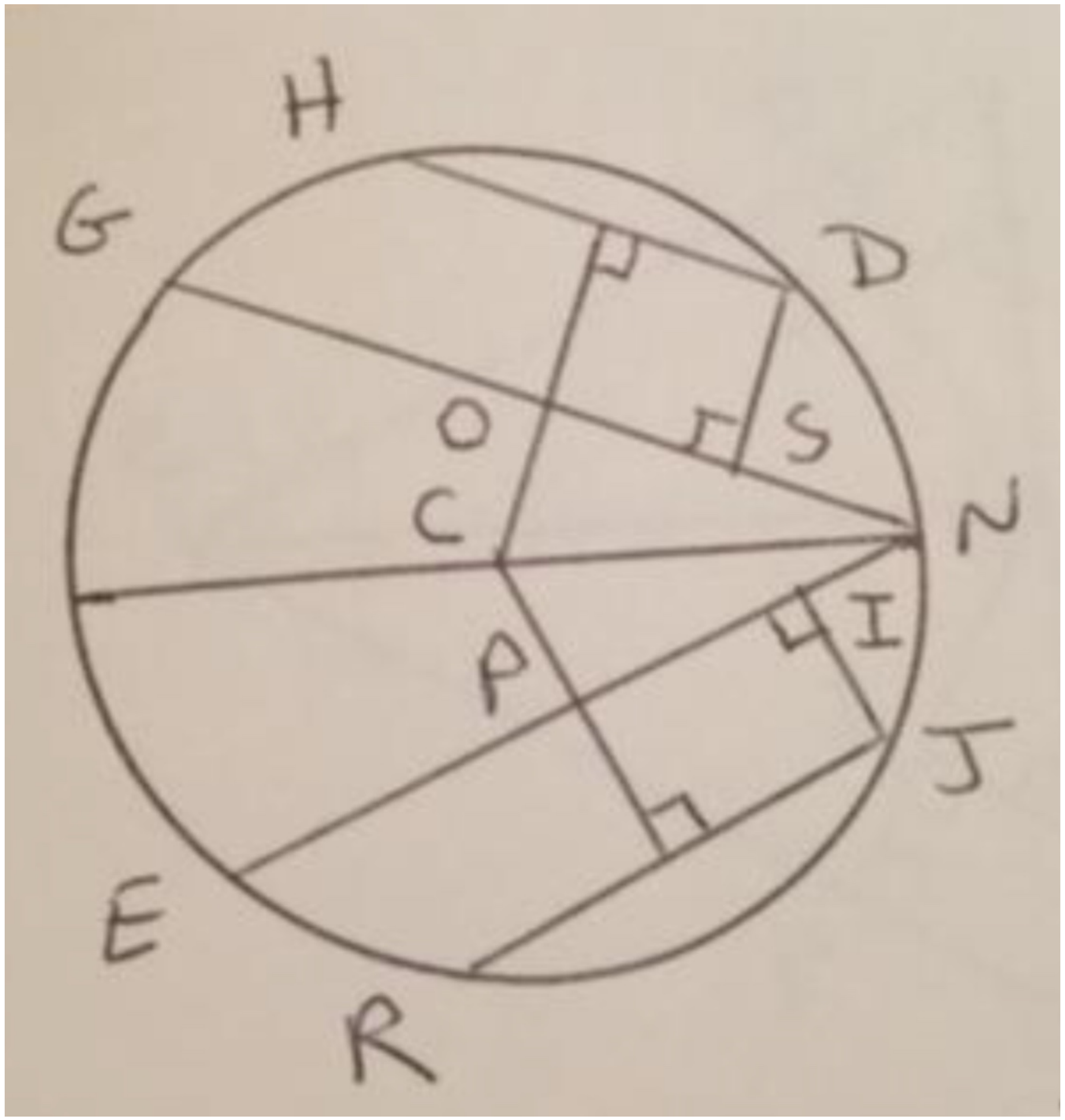
the real image (Z) can be projected on a screen

$$\frac{AG + AN'}{2AN'} = \frac{QG + NN'}{2NN'}$$

$$\frac{XE + XN'}{2XN'} = \frac{EF + NN'}{2NN'}$$

$$\frac{QG + NN'}{EF + NN'} = \frac{(AG + AN')}{2AN'} \frac{2XN'}{(XE + XN')}$$

Figure 30:



HD = QN’

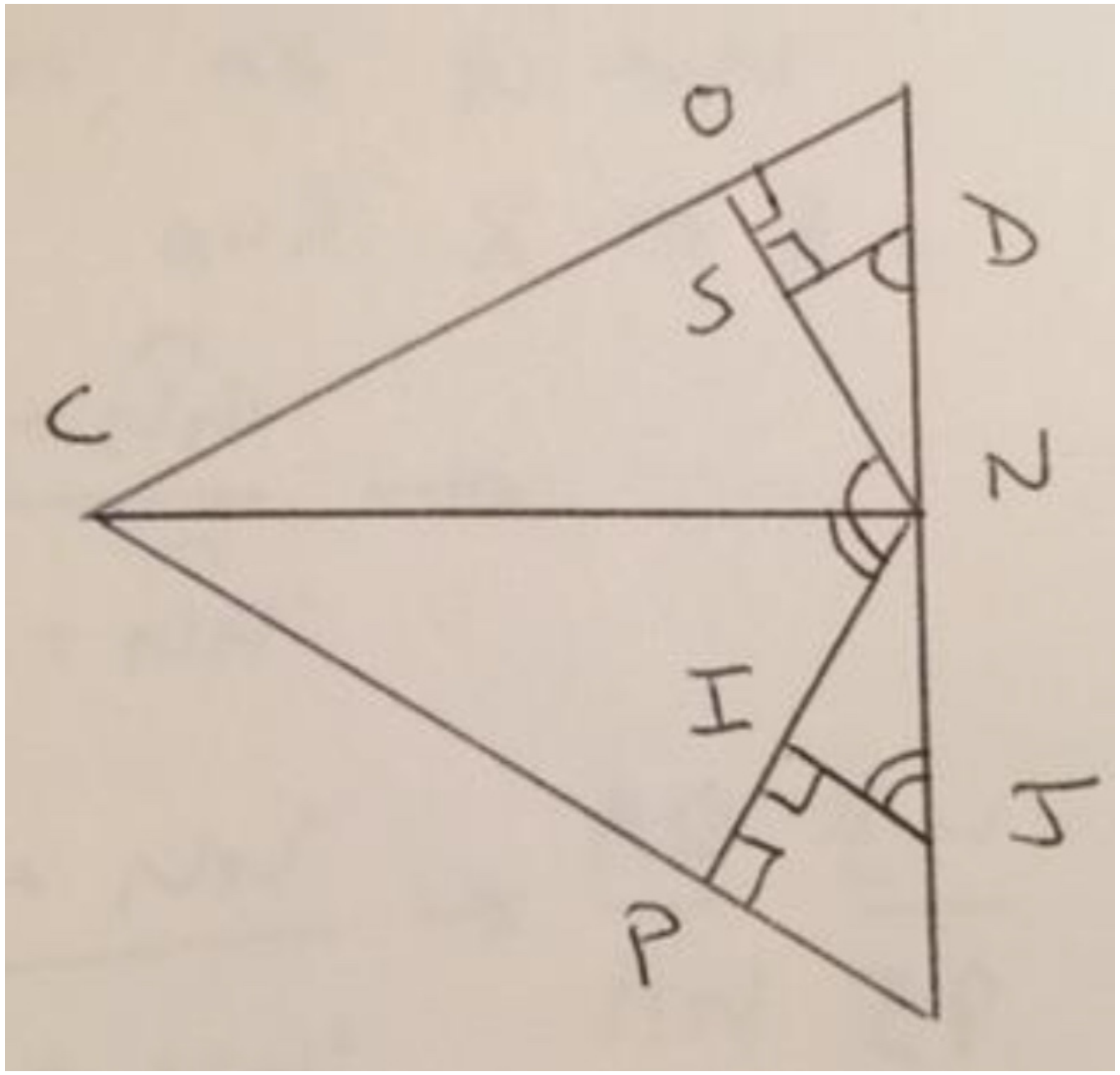
RJ = FN’

as N’⇒ N:

X⇒ Z, and ~DJ⇒ DJ

so that:

Figure 31:



$$\frac{DS}{JI} \Rightarrow \frac{CO}{CP}$$

$$\frac{JI}{JN} \Rightarrow \frac{NP}{NC}$$

$$\frac{DN}{DS} \Rightarrow \frac{NC}{NO}$$

$$\frac{ND}{NJ} \Rightarrow \frac{NP}{NO} \quad \frac{CO}{CP}$$

thus, as N’ $\Rightarrow$ N and X $\Rightarrow$ Z:

$$\frac{\sim QG + \sim NN'}{\sim EF + \sim NN'} \Rightarrow \frac{QG + NN'}{EF + NN'} \Rightarrow$$

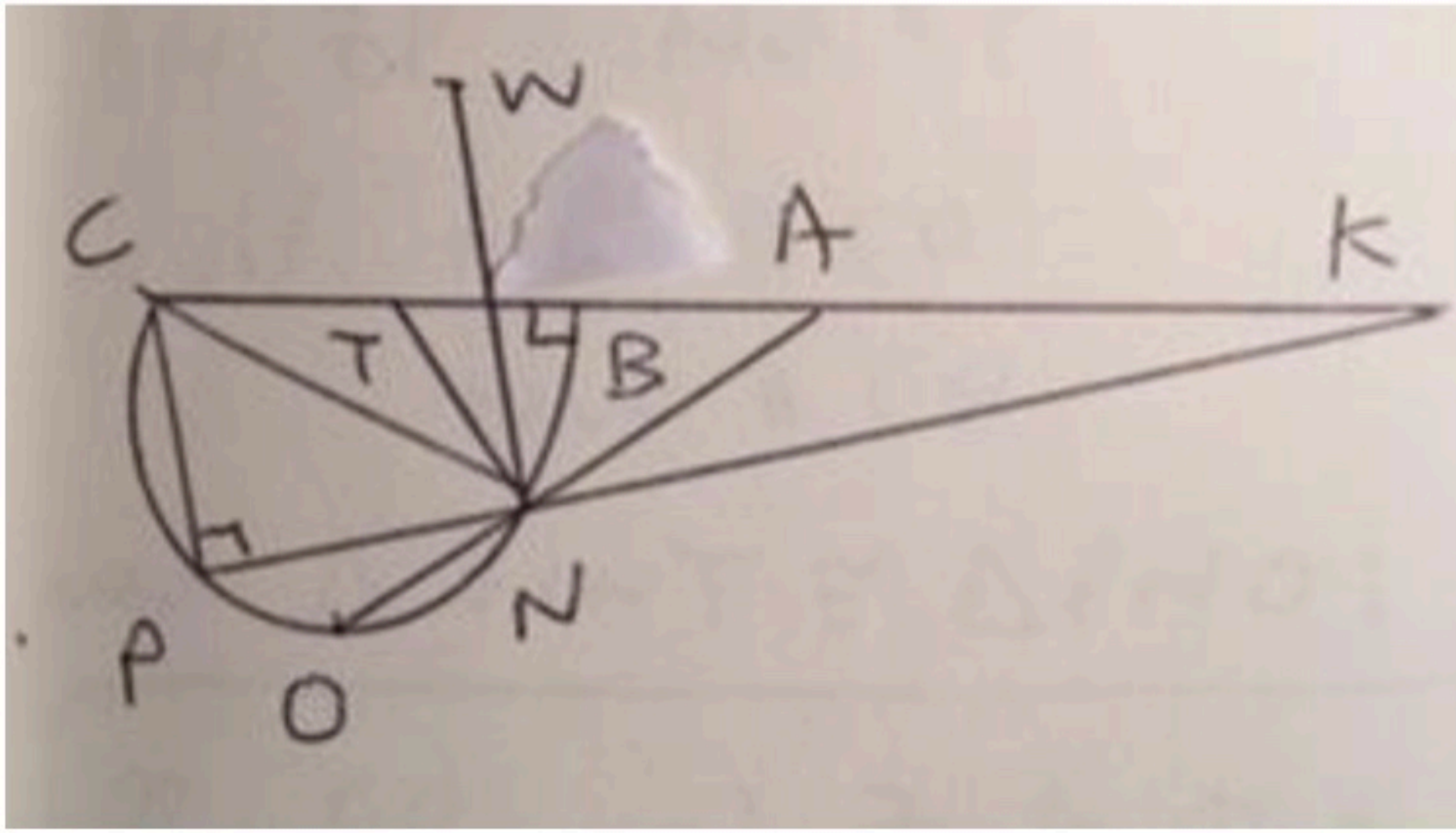
$$\frac{AO}{AN}\ \frac{ZN}{ZP}$$

and:

$$\frac{\sim QG + \sim NN'}{\sim EF + \sim NN'} = \frac{2\,(\sim ND)}{2\,(\sim NJ)} \Rightarrow$$

$$\frac{ND}{NJ} \Rightarrow \frac{NP}{NO}\ \frac{CO}{CP}$$

Figure 32:



NT || CO
NW || CP
when X = Z lies along
both NP and CW:

$$\frac{AO}{AN}\,\frac{ZN}{ZP} = \frac{CO}{NT}\,\frac{NW}{CP}$$

when $\Delta WNT \cong \Delta PNO$, $NW > NT$

and

$$\frac{AO}{AN}\,\frac{ZN}{ZP} = \frac{NP}{NO}\,\frac{CO}{CP}$$

so if:

$NT \parallel CO$

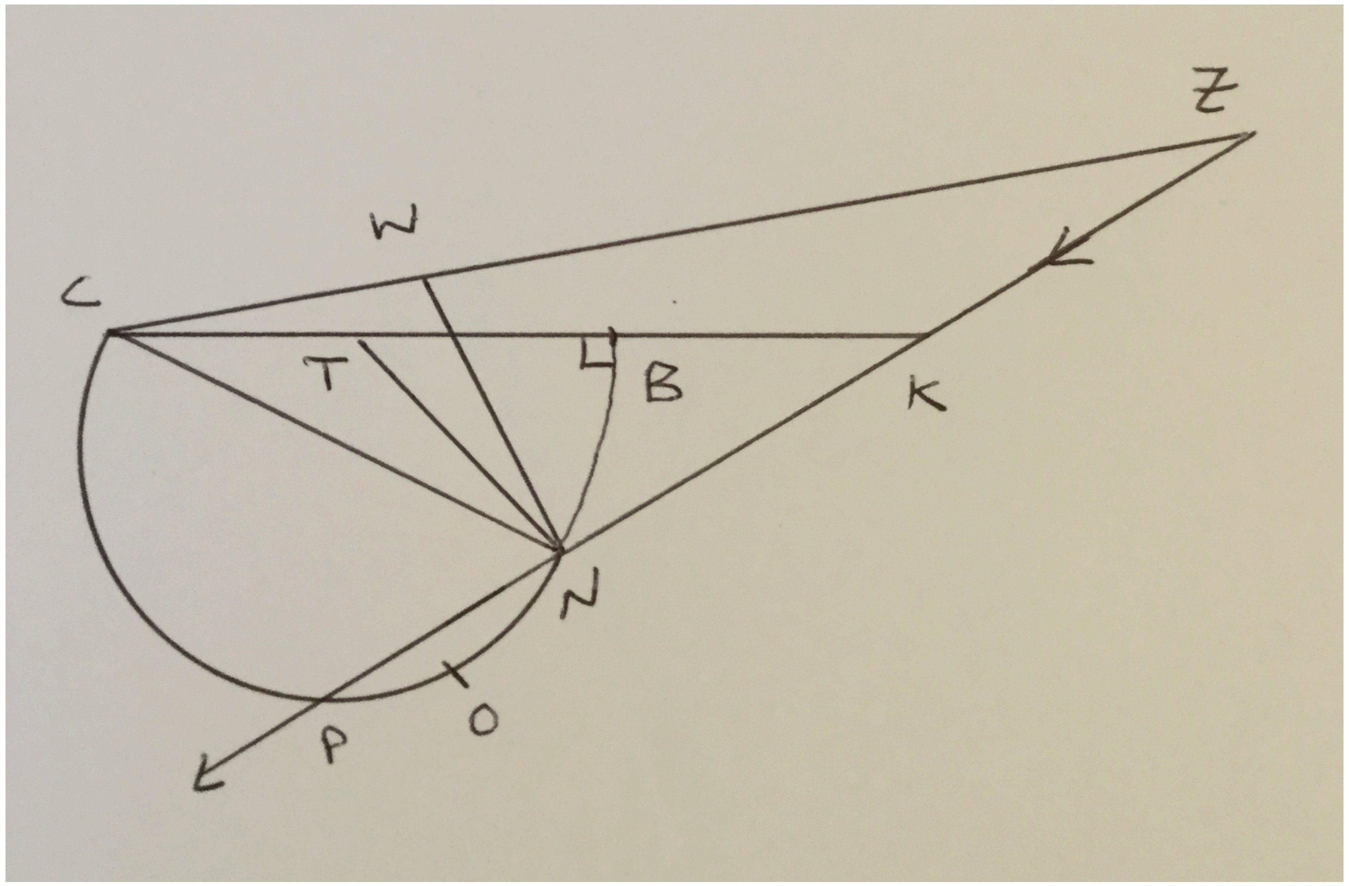
$NW \parallel CP$

and $\Delta WNT \cong \Delta PNO$:

$$\mathbb{R} = \frac{CO}{CP}$$

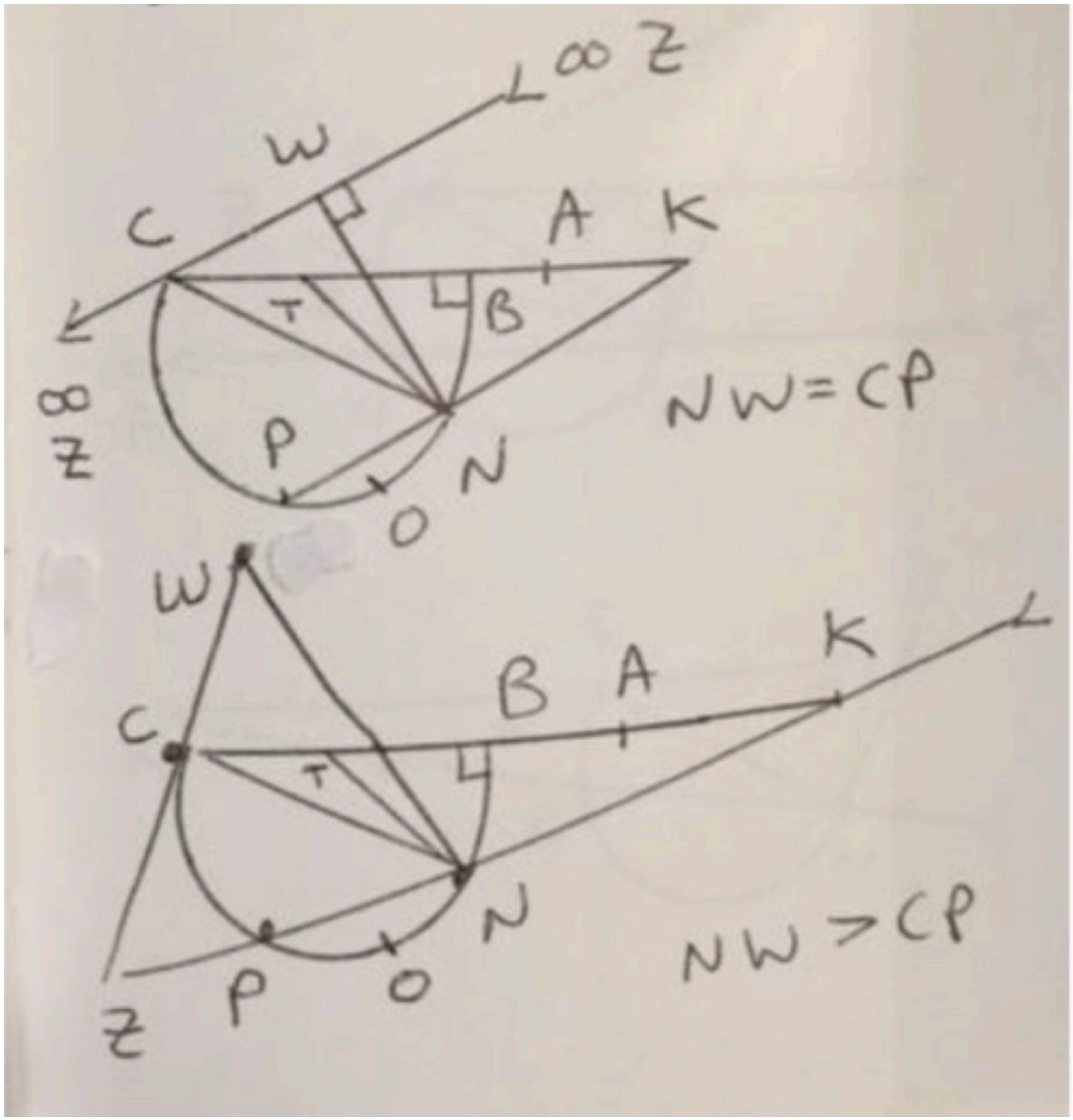
and Z is the clear image of object A refracted at N along ~BN

Figure 33:



Off-axis rays from any on-axis object A, (real or virtual), can not form a virtual on-axis image Z because NW must be less than CP for Z to be virtual; but NW must also be greater than NT.

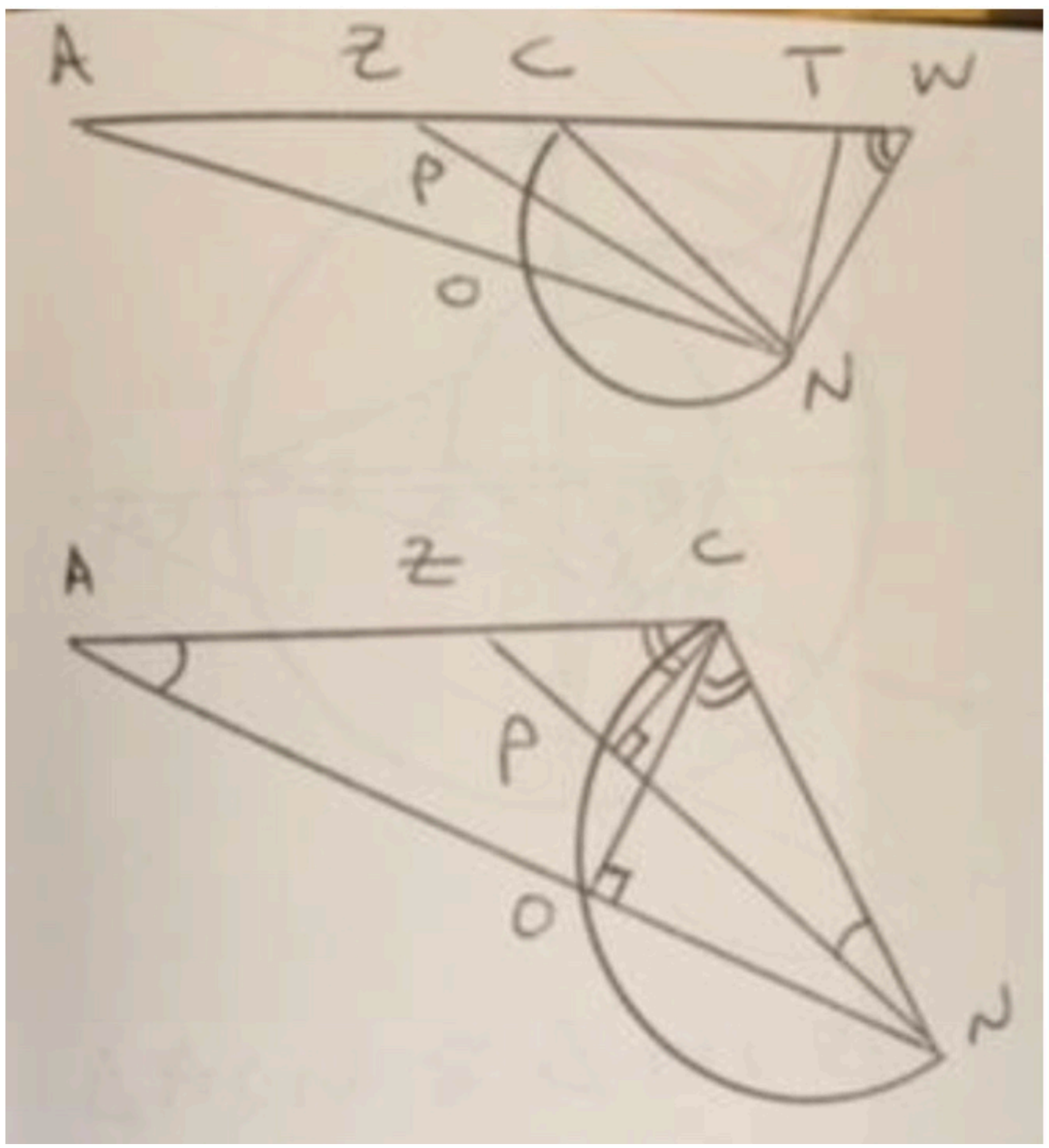
Figure 34:



Off-axis rays from any real on-axis object A can not form a real on-axis image Z because NW must be greater than (or equal to) CP for Z to be real; but NW must also be greater than NT.

Figure 35:

Off-axis rays from a virtual on-axis object A ***can*** form a real on-axis image Z because NW must be greater than or equal to CP for Z to be real; and NW must also be greater than NT. When WT lies along the axis, so does Z. This occurs when:



NT $\parallel$ CO

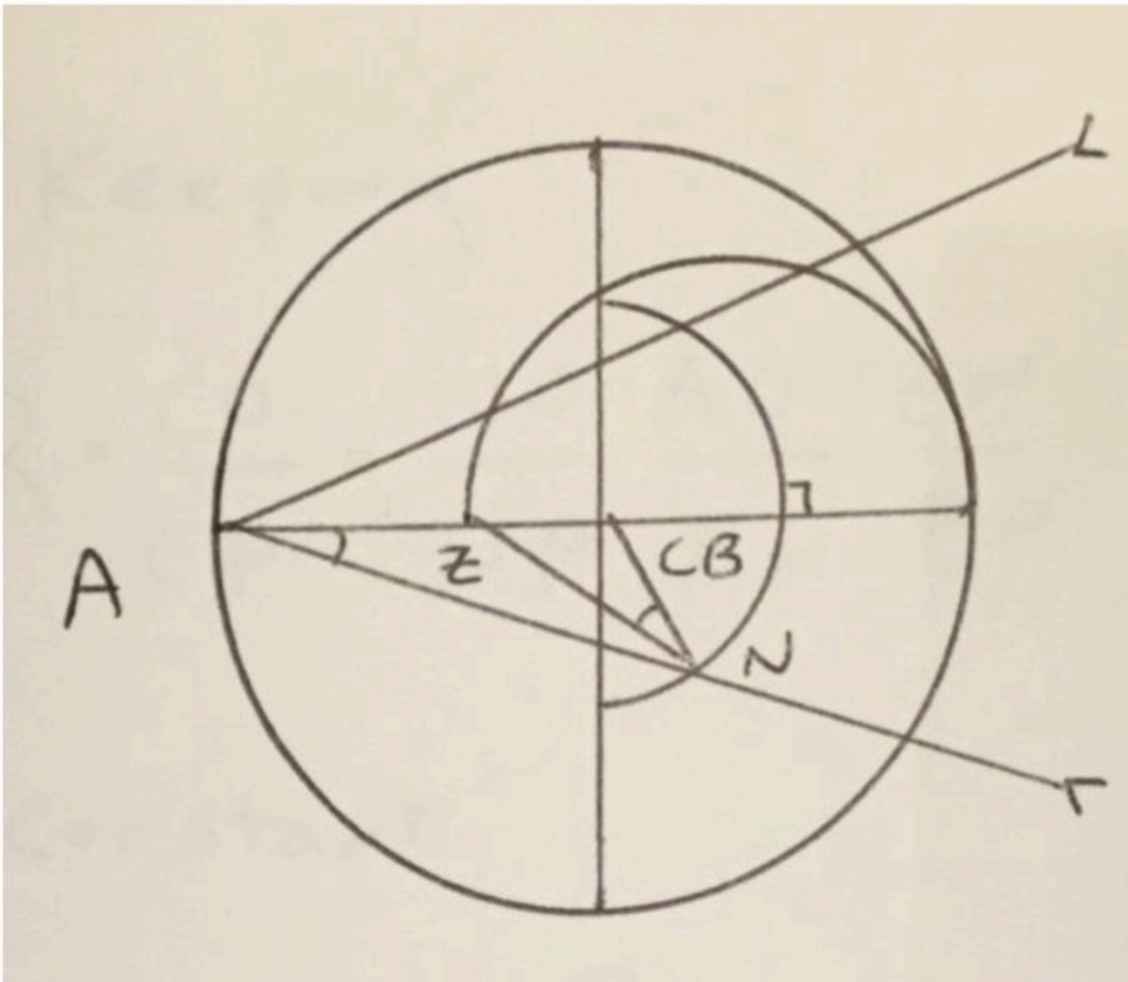
NW $\parallel$ CP

ΔWNT $\cong$ ΔPNO

$\angle$NWT $=$ $\angle$NPO $=$ $\angle$NCO

ΔCPN $\cong$ ΔCOA

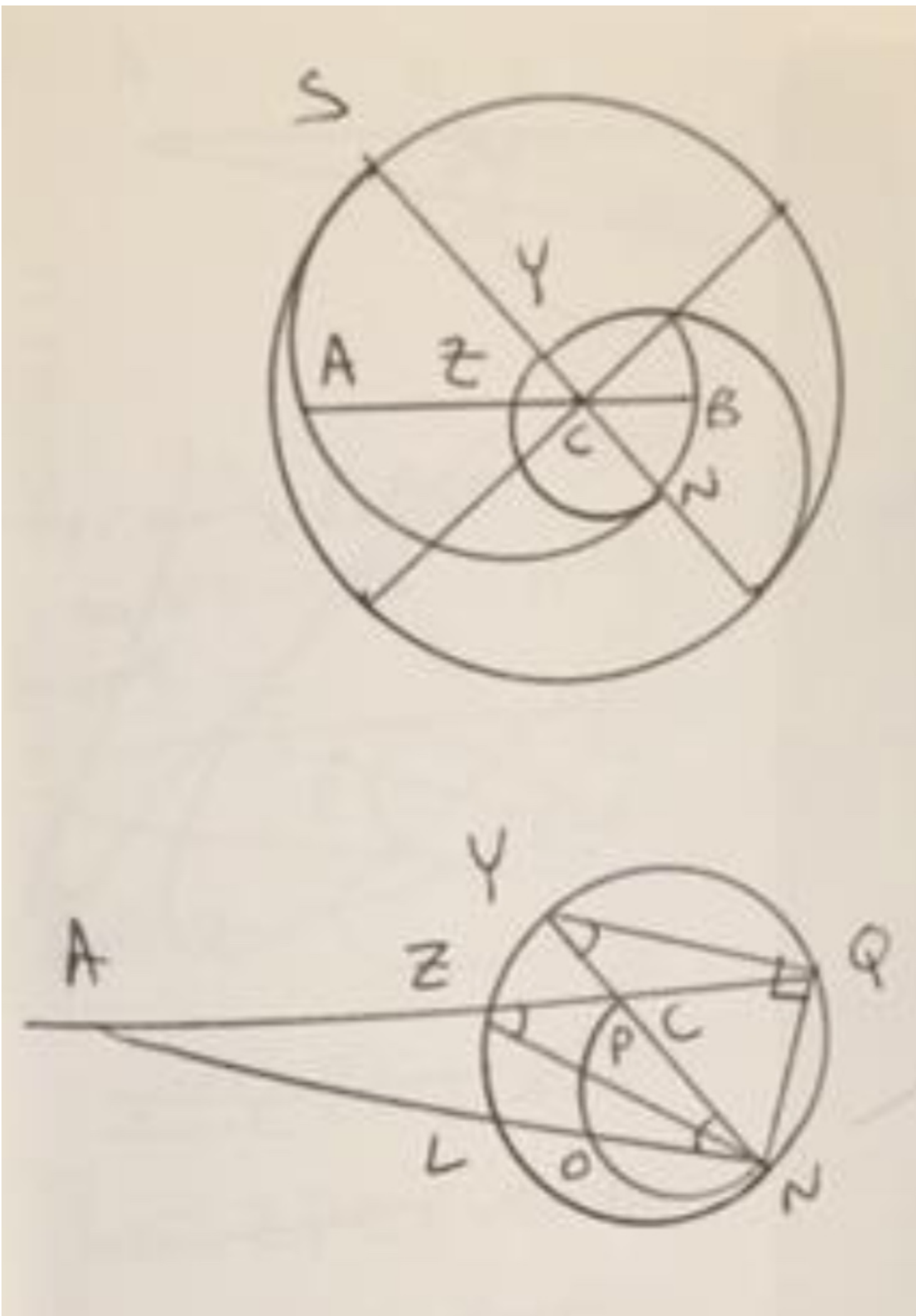
Figure 36:



When off-axis rays from a virtual on-axis object A form a real on-axis image Z, this is the on-axis real image of the on-axis virtual object A at all points N because:

$$\Delta ACN \;\cong\; \Delta NCZ \quad \text{for all } N$$

Figure 37:



This can also be demonstrated by constructing:

SC/CN = CN/CY

$$\frac{CY}{CN} = \frac{CN}{CS} = \frac{CY + CN}{CN + CS} = \frac{NY}{NS}$$

$$\frac{AO}{AN}\,\frac{ZN}{ZP} = \frac{SC}{SN}\,\frac{ZN}{ZP} = \frac{NC}{NY}\,\frac{ZN}{ZP} =$$

$$\frac{NC}{NY}\,\frac{YN}{YC} = \frac{CN}{CY}$$

$$\frac{CO}{CP}\,\frac{NP}{NO} = \frac{LY}{LN}\,\frac{PN}{PC} = \frac{QN}{QY}\,\frac{PN}{PC} =$$

$$\frac{QN\,(ZN)}{QY\,(ZY)} = \frac{CN}{CY}$$

Section 4

Axial Refraction at a Circle

keeping:

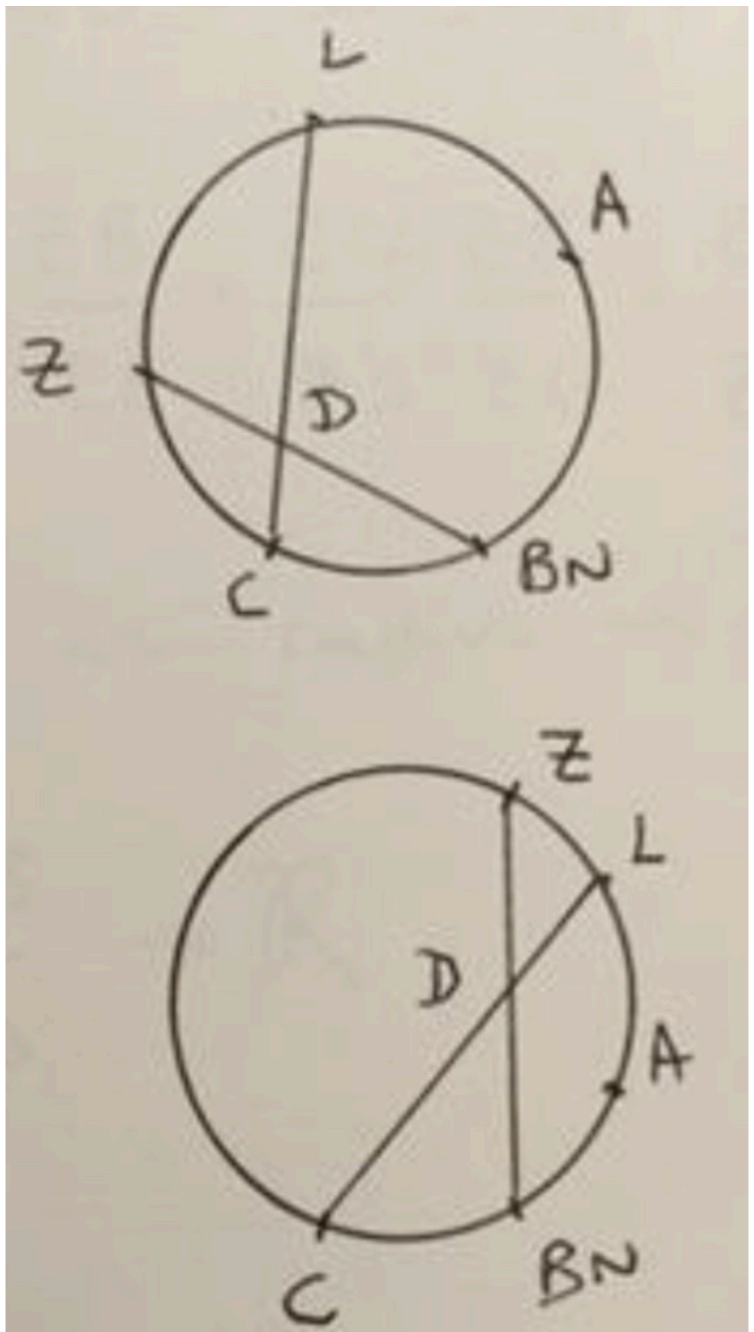
$$\mathbb{R} = \frac{CO}{CP} = \frac{NO}{NP}\,\frac{AO}{AN}\,\frac{ZN}{ZP}$$

constant as $N \Rightarrow B$:

$$\frac{BC}{BC}\,\frac{AC}{AB}\,\frac{ZB}{ZC} \Rightarrow \mathbb{R}$$

Figure 38:

“axial” refraction can be described along a circle of infinite radius



draw CDL so:

AL || ZB so:

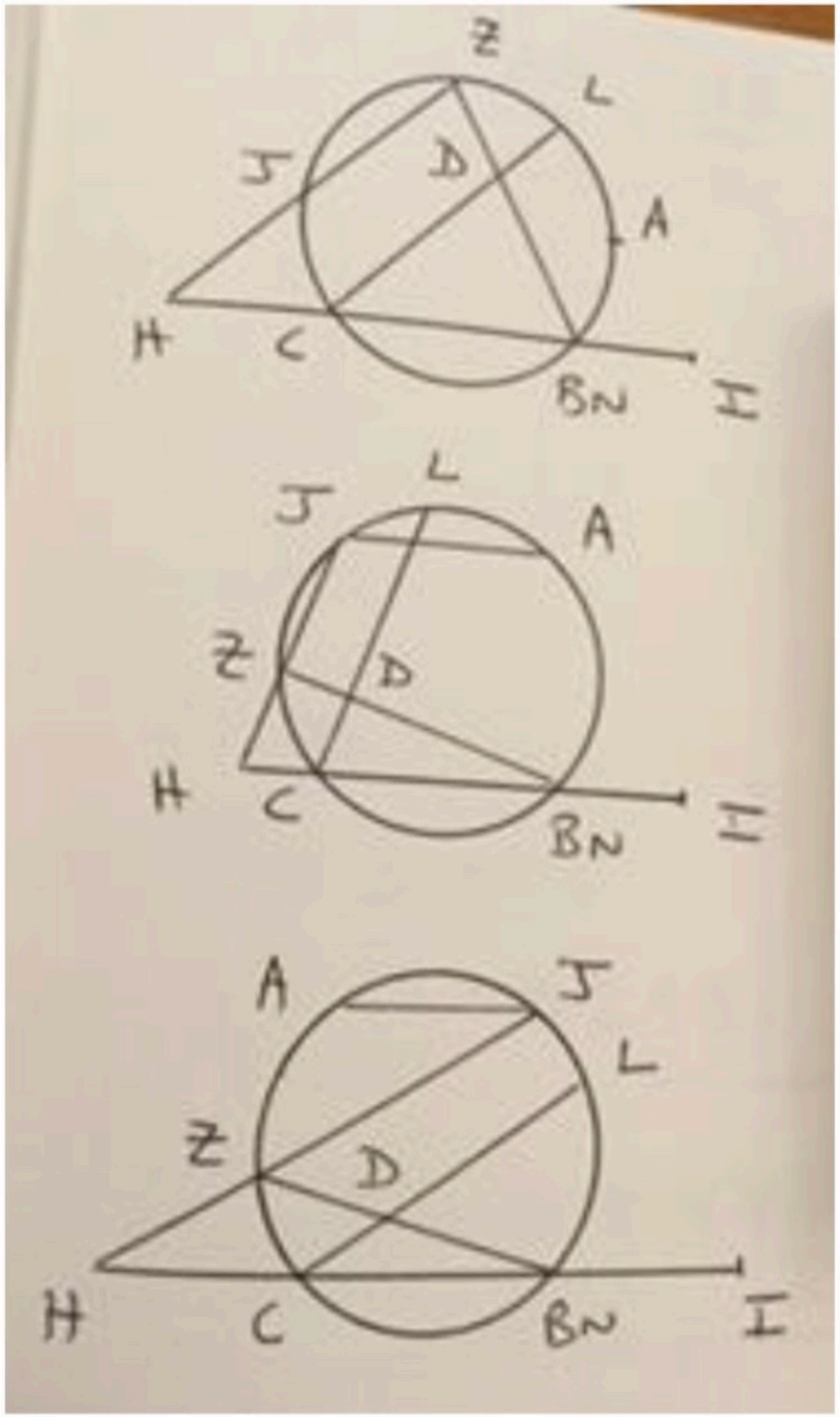
$\Delta ACB \cong \Delta ZCD$ and:

$$\frac{AC}{AB}\,\frac{ZB}{ZC} = \frac{ZC}{ZD}\,\frac{ZB}{ZC} = \frac{ZB}{ZD}$$

so as the radius $\Rightarrow \infty$

$$\frac{ZB}{ZD} \Rightarrow \mathbb{R}$$

Figure 39:



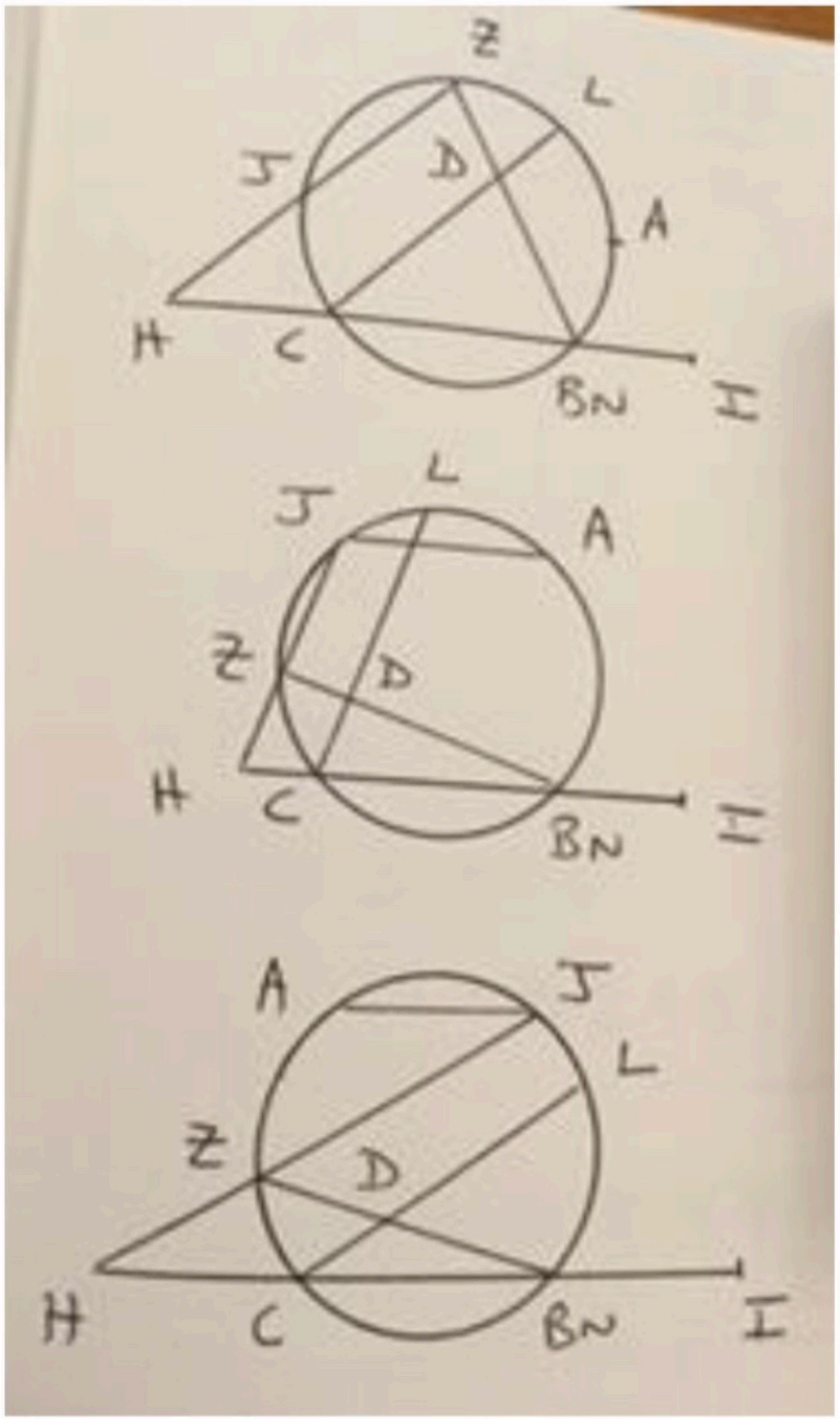
AL ll ZB
AZ = BL
~AZ = ~BL

HZ ll CL
ZC = LJ
~ZC = ~LJ

~AZ + ~ZC = ~AZC
~BL + ~LJ = ~BLJ

~AZC = ~BLJ
AJ ll CB

Figure 40:



HZ II CL

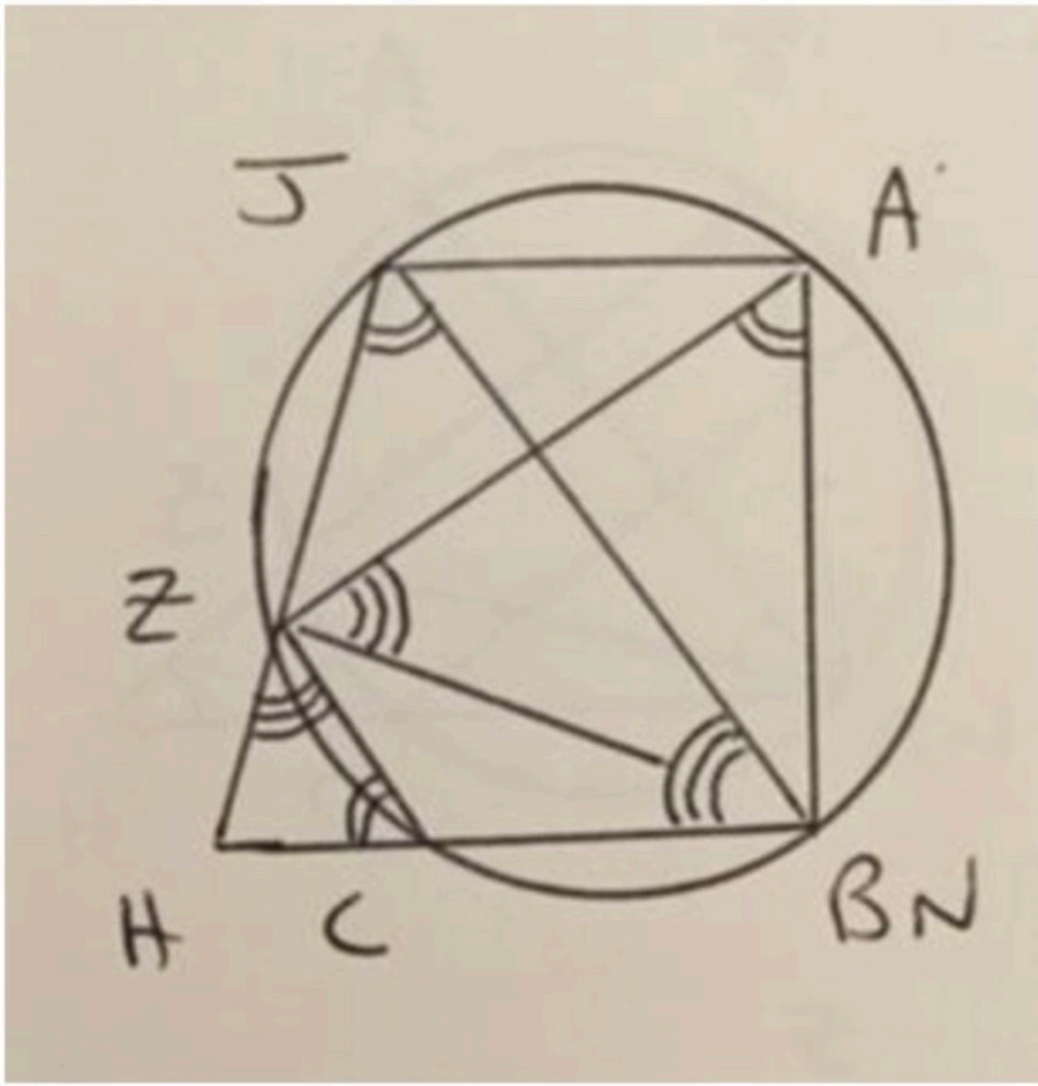
$$\frac{ZB}{ZD} = \frac{HB}{HC}$$

ΔHBZ ≅ ΔHJC

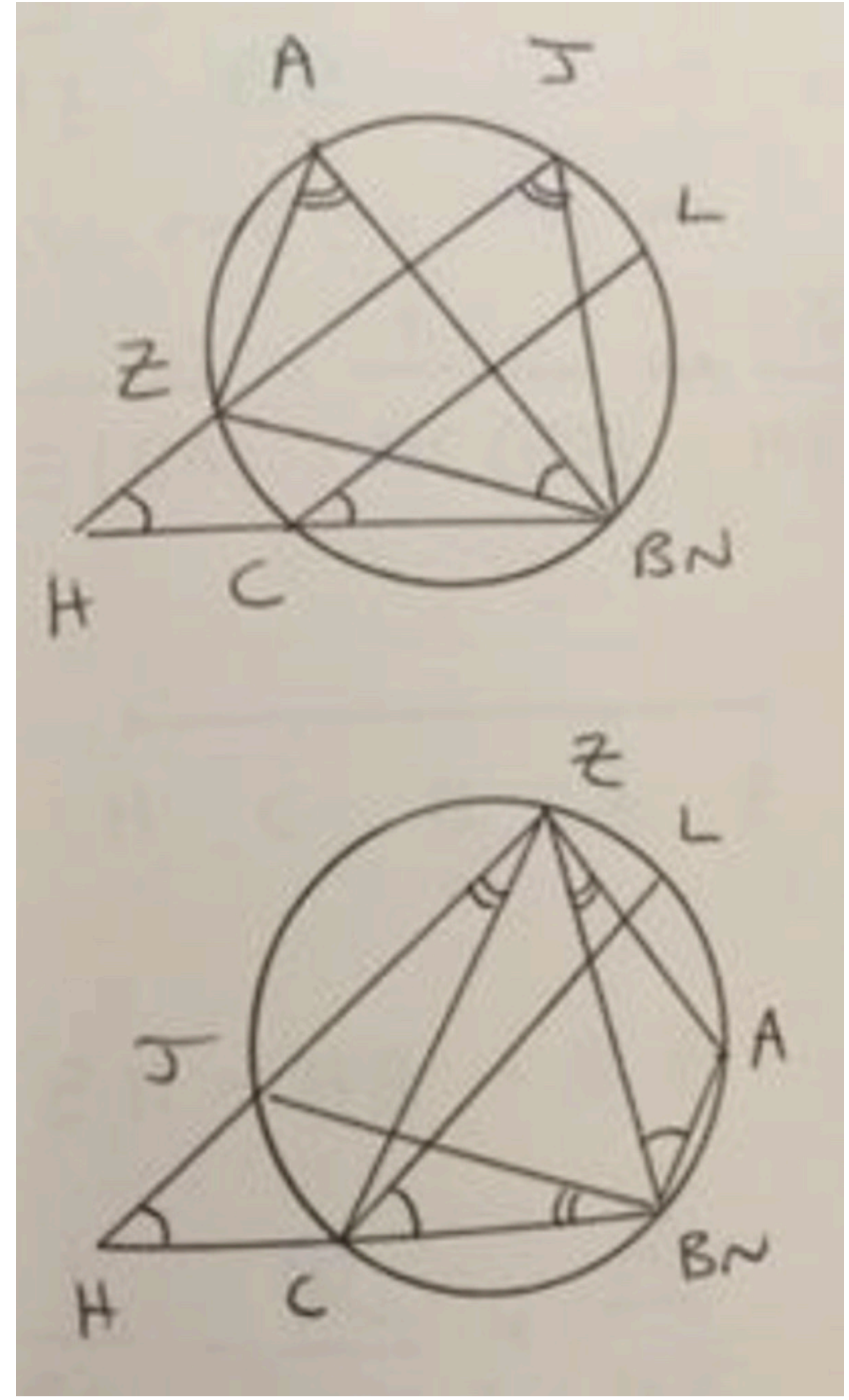
when ΔHJC = ΔIAB:

$$\frac{IB}{IA} = \frac{HZ}{HB}$$

Figure 41:



$\Delta HCZ \cong \Delta HJB \cong \Delta BAZ$



$$\Delta HCZ \cong \Delta HJB \cong \Delta BAZ$$

$$\frac{HC}{HZ} = \frac{BA}{BZ}$$

as the radius $\Rightarrow \infty$

$$\frac{1}{HZ\,(BA)} = \frac{1}{HC\,(BZ)} \Rightarrow \frac{\mathbb{R}}{HB\,(BZ)}$$

These equalities are used with the following possible sums resulting from the circle with infinite radius, to produce the conjugate foci equations:

HZ= HB + BZ or
HB = HZ + BZ or
BZ = HZ + HB

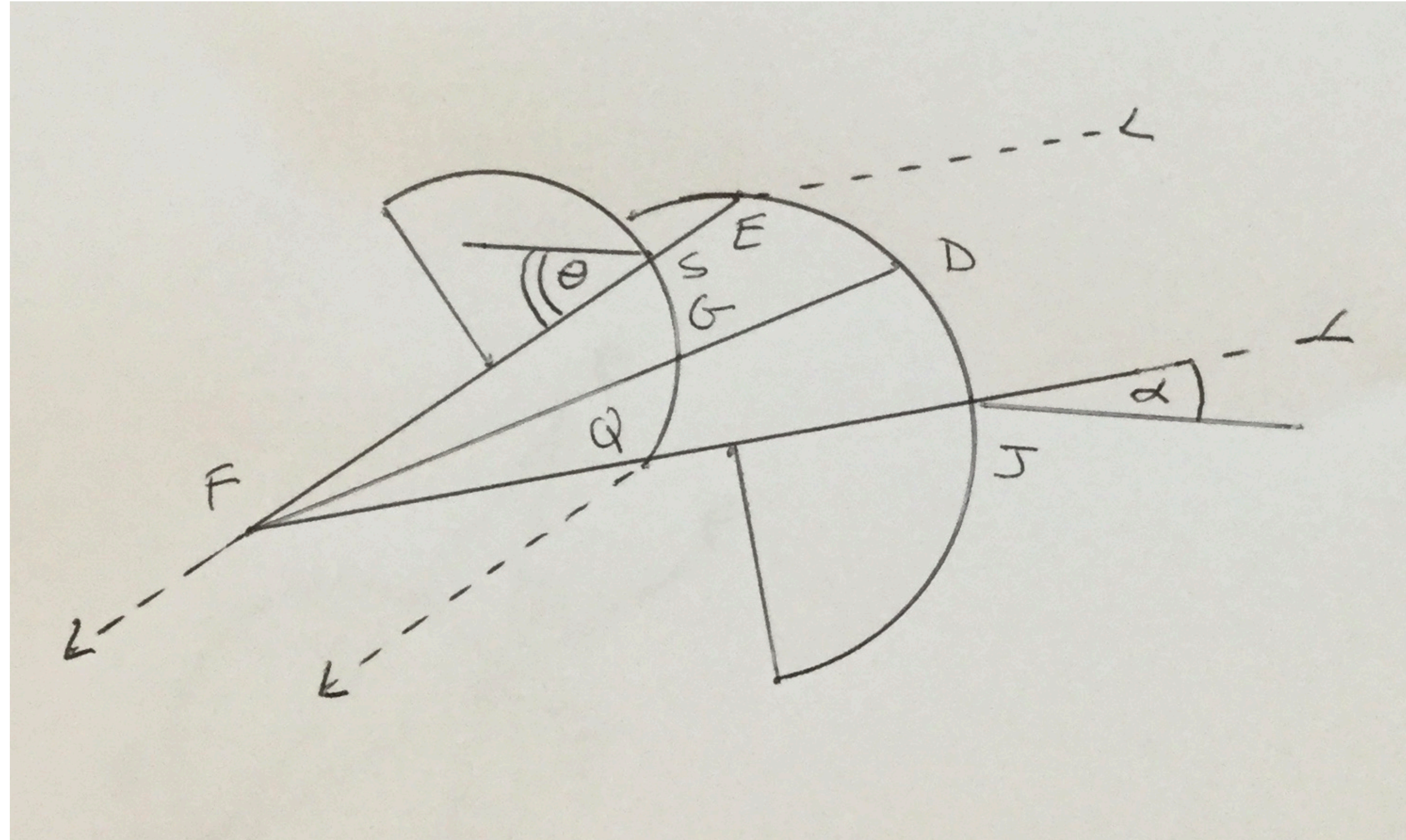
Section 5

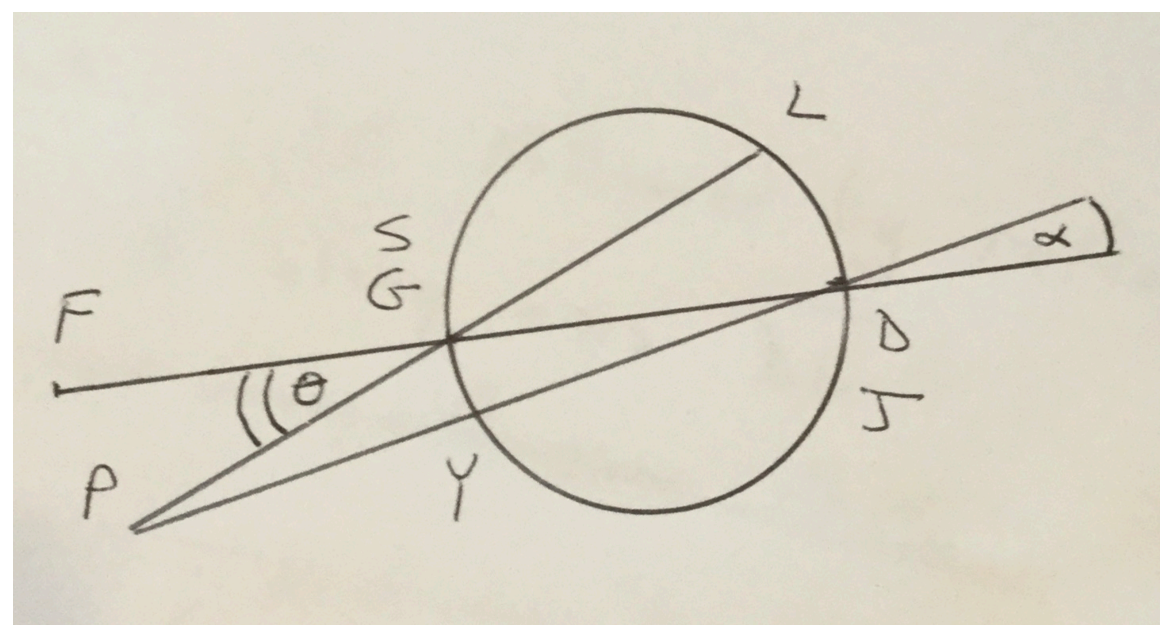
Afocal Axial Angular Magnification

Before considering afocal axial angular magnification, imagine two cars driving down the same street. When one car passes a sign post, it speeds up until it reaches the next sign post, then slows back down to its original speed, which is the same speed of the other car. Not only will the car that sped up be further down the road, it will also have had a greater average speed during the trip. This effect depends on two factors. The first is the degree to which the car speeds up between the sign posts, and the second is the distance between those sign posts.

This metaphor can be used to illustrate afocal axial angular magnification, which simply depends on two factors. The first is the degree to which light rays change between two lenses or refracting surfaces. The second is the separation of those two lenses or refracting surfaces. This is why a collapsible telescope no longer magnifies a distant object when it is “collapsed,” and its lenses are no longer separated.

Figure 42:





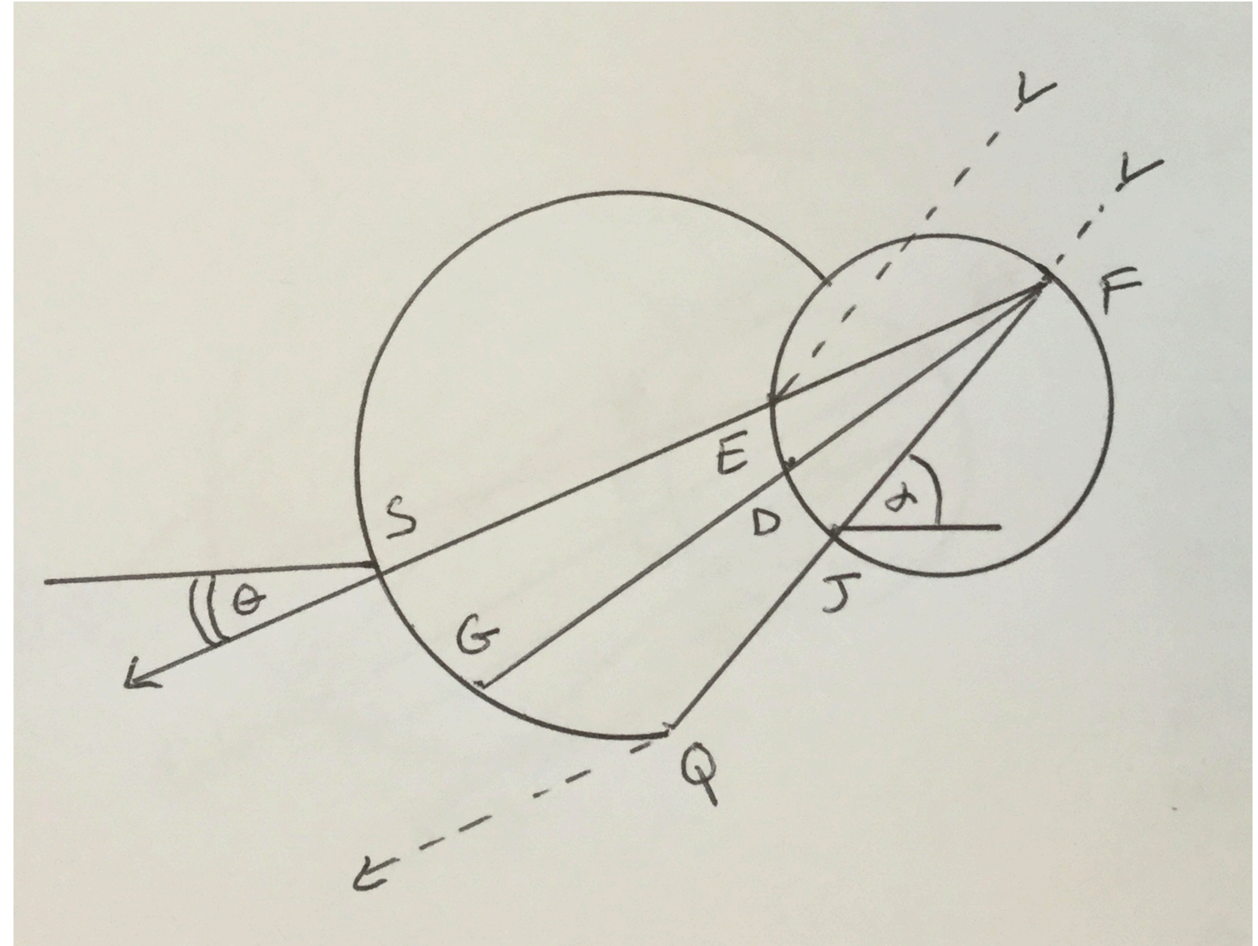
In figure 41, given distance refraction at ~JDE followed by refraction into distance at ~QGS along axis DGF:

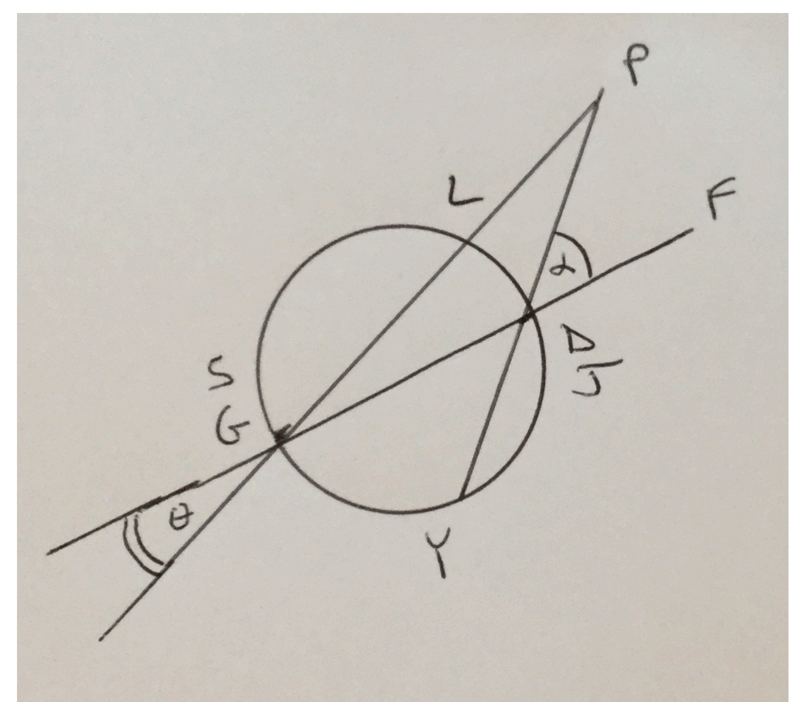
as angle JFD = angle SFG, and both approach zero,

$$\frac{\theta}{\alpha} \Rightarrow \frac{\sim LD/GD}{\sim YG/GD} \quad \text{as} \quad P \Rightarrow F$$

$$\frac{\theta}{\alpha} \Rightarrow \frac{FD}{FG} \quad \text{as} \quad P \Rightarrow F$$

Figure 43:





In figure 42, given distance refraction at ~JDE followed by refraction into distance at ~QGS along axis FDG:

as angle JFD = angle SFG, and both approach zero,

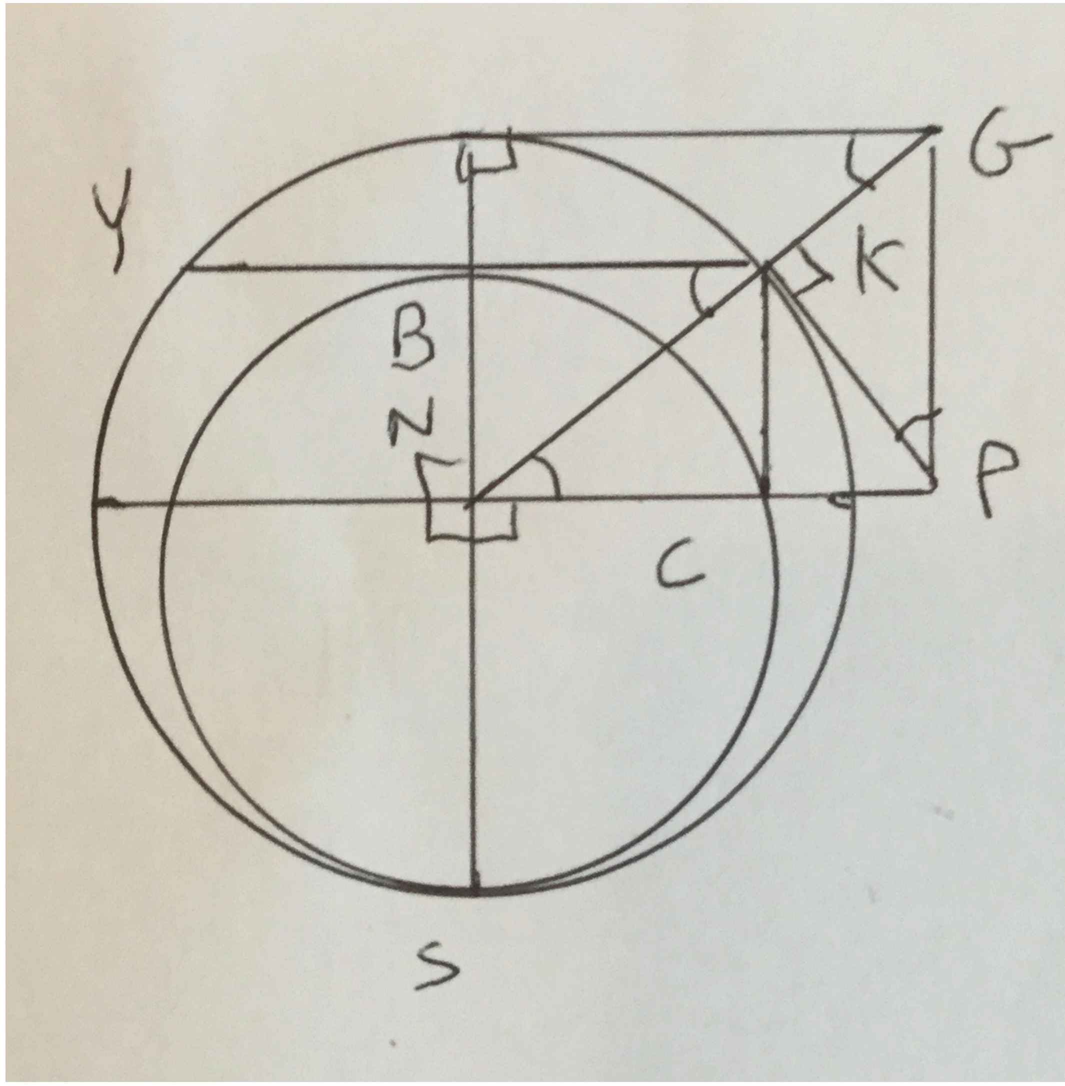
$$\frac{\theta}{\alpha} \Rightarrow \frac{\sim LD/GD}{\sim YG/GD} \quad \text{as} \quad P \Rightarrow F$$

$$\frac{\theta}{\alpha} \Rightarrow \frac{FD}{FG} \quad \text{as} \quad P \Rightarrow F$$

Section 6

Clinical Determination of Axial Retinal Image Size Magnification

Figure 44:



$\Delta NBK = \Delta GKP$

From figure 13, recall the "continued proportion"

$$\frac{NS}{NC} = \frac{NC}{NB}$$

and notice that:

$$\frac{NK}{NB} = \frac{KN + KG}{GP}$$

which equals:

$$\frac{NK + NB}{NK}$$

We have just shown that:

$$\frac{NK}{NB} = 1 + \frac{NB}{NK} = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}}}$$

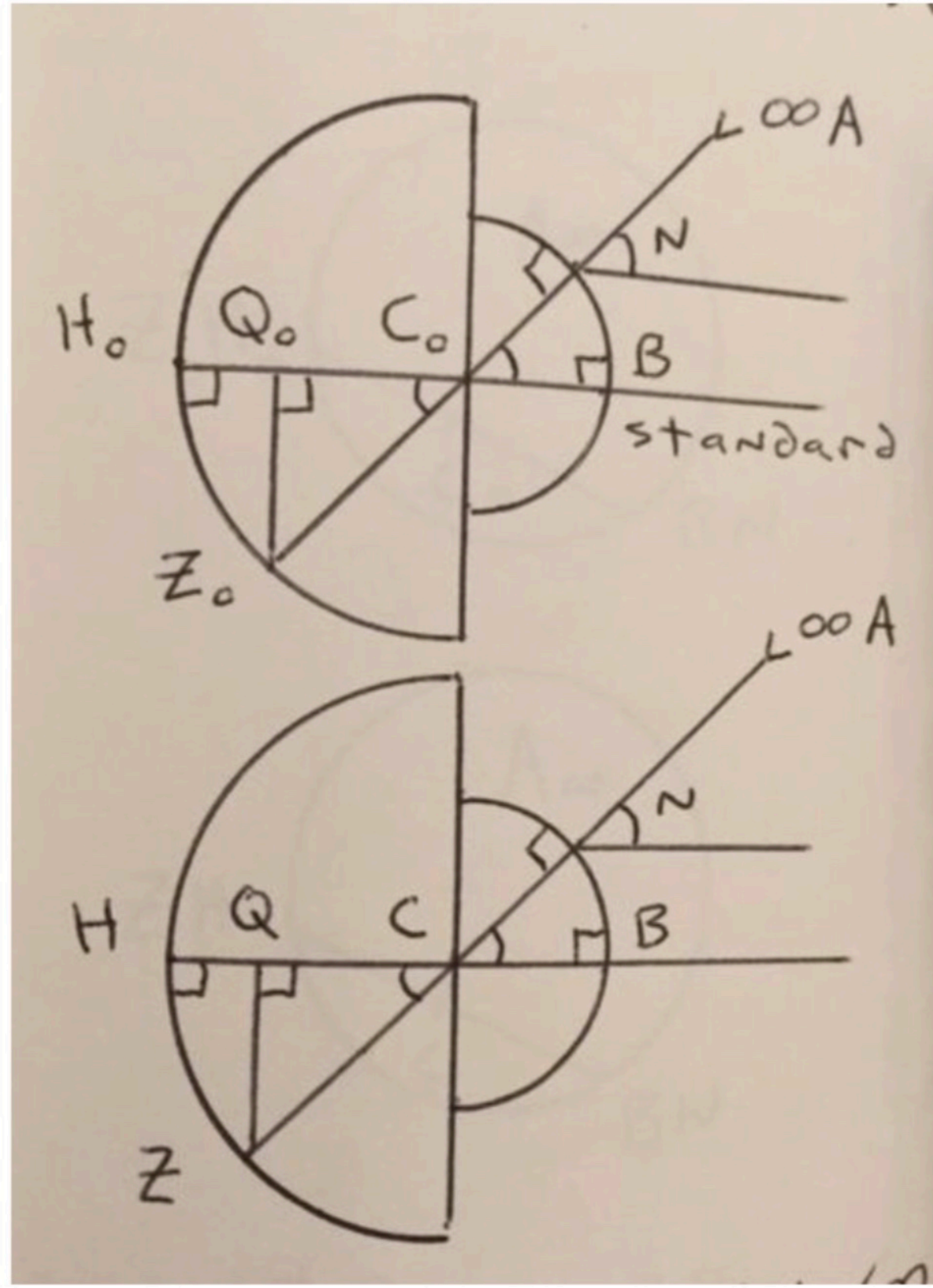
Since we have shown that neither NK or NB can measure the other length, we have shown that there is no length relative to itself, ("unit length"), that will measure all finite lengths.

This is relevant in any discussion of magnification. We can either consider such non-measurable distances to be irrational numbers, which are continuing fractions, or we can consider “number theory” itself to be irrational, along with the presumption that anything, even a unit measurement, can be real defined by itself.

Axial retinal image size magnification is not a number, but rather a ratio. It therefore requires a standard retinal image size for comparison. It is fair to call any such magnification using a standard, which is by definition arbitrary, meaningless in and of itself. However, it is simply a tool to use for ***comparing*** magnifications. Such comparisons are meaningful and not arbitrary, because arbitrary standards factor out when comparing ratios.

Figure 45:

The top diagram references the standard eye. The bottom diagram references any eye used for comparison, with the retinal image size designated as HZ.

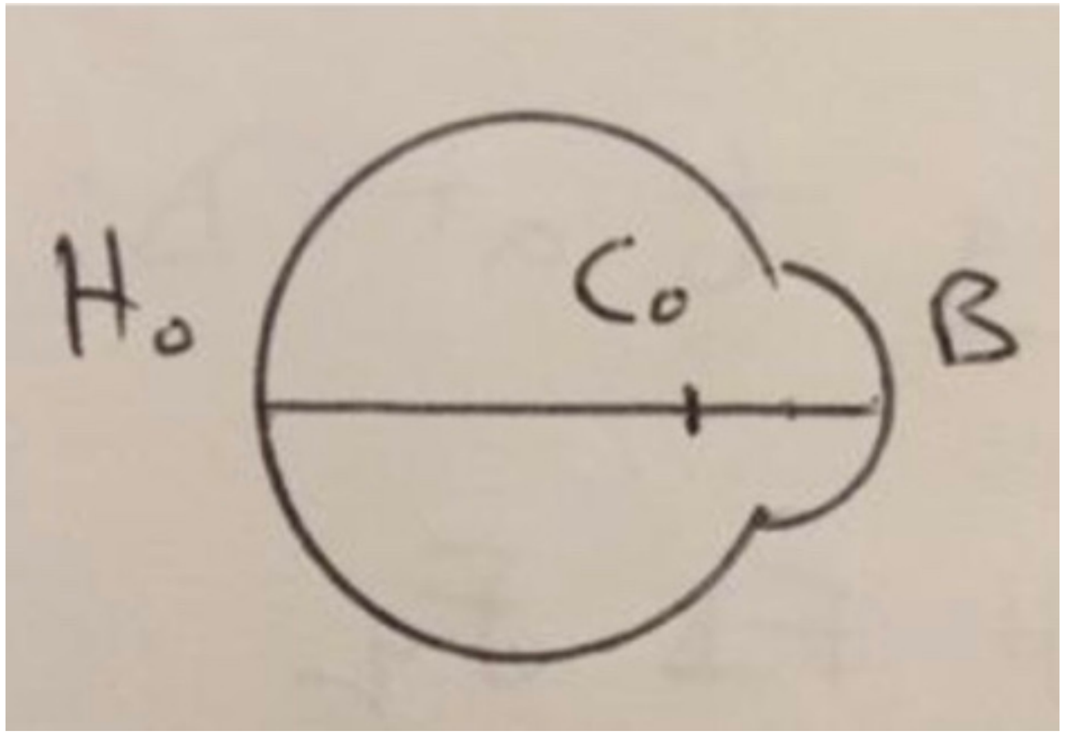


$$\frac{ZQ}{Z_\circ Q_\circ} = \frac{ZC}{Z_\circ C_\circ} = \frac{HC}{H_\circ C_\circ} = \frac{BH/\mathbb{R}}{BH_\circ/\mathbb{R}}$$

as N $\Rightarrow$ B:

$$\boldsymbol{M} \Rightarrow \frac{ZQ}{Z_\circ Q_\circ} = \frac{BH}{BH_\circ}$$

Figure 46:



In order to find the magnification ***M***, (in this case that of retinal image size magnification), we need to know both the standard BH_o, as well as BH for the eye in question. When a distant object is focused at Z, and a distance refractive error exists, Z lies at E rather than at H.

using $BH_{\circ}$ as the chosen ocular standard where:

$$\mathbb{R} = \frac{H_{\circ}B}{H_{\circ}C_{\circ}} = \frac{HB}{HC} = \frac{EB}{EL} = \frac{4}{3}$$

and $$\frac{\mathbb{R}}{BH_{\circ}} = 60 \text{ diopters}$$

(where a diopter is a unit of inverse meter length)

Measure BL to find:

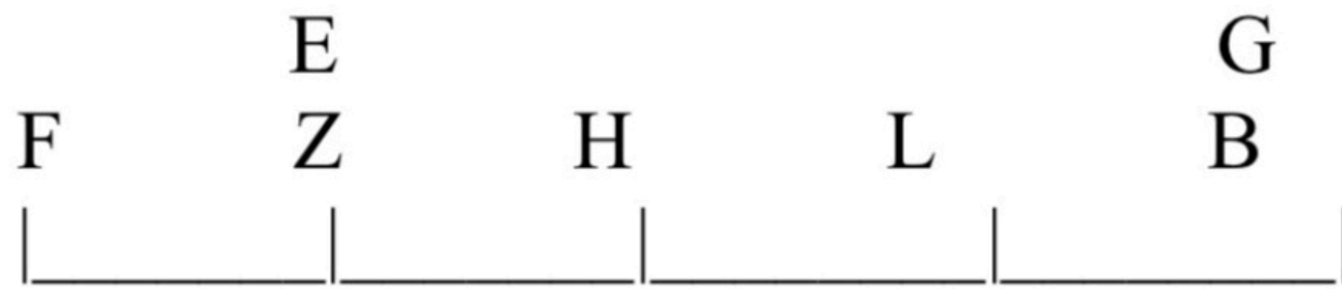
$$\frac{\mathbb{R}}{BE} = \frac{1}{EL} = \frac{\mathbb{R} - 1}{BL}$$

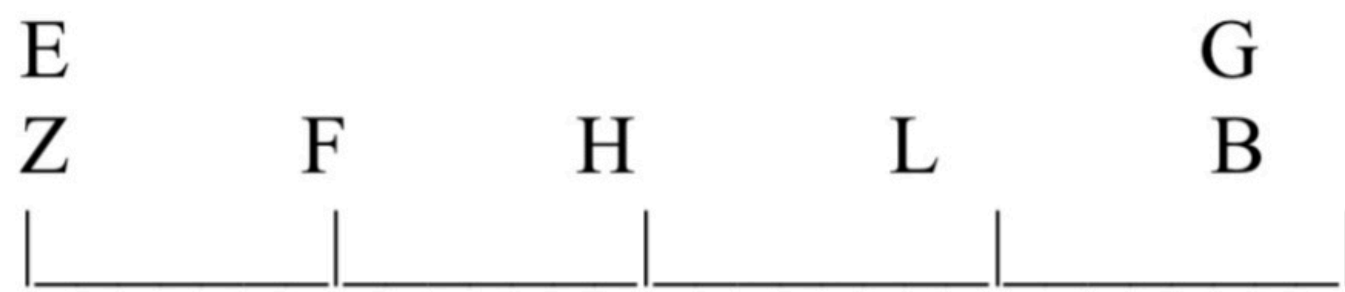
in order to calculate BH using:

```
          E                  B
|________|________|________|________|
H        Z        L        G        F
```

$$\frac{\mathbb{R}}{BE} = \frac{1}{BF} + \frac{\mathbb{R}}{BH}$$

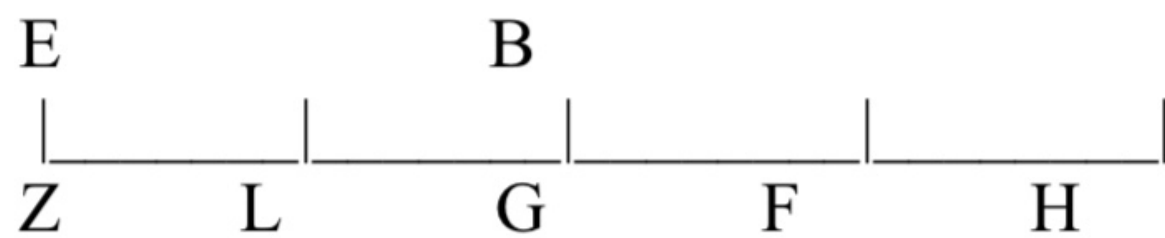
or





$$\frac{\mathbb{R}}{BH} = \frac{1}{BF} + \frac{\mathbb{R}}{BE}$$

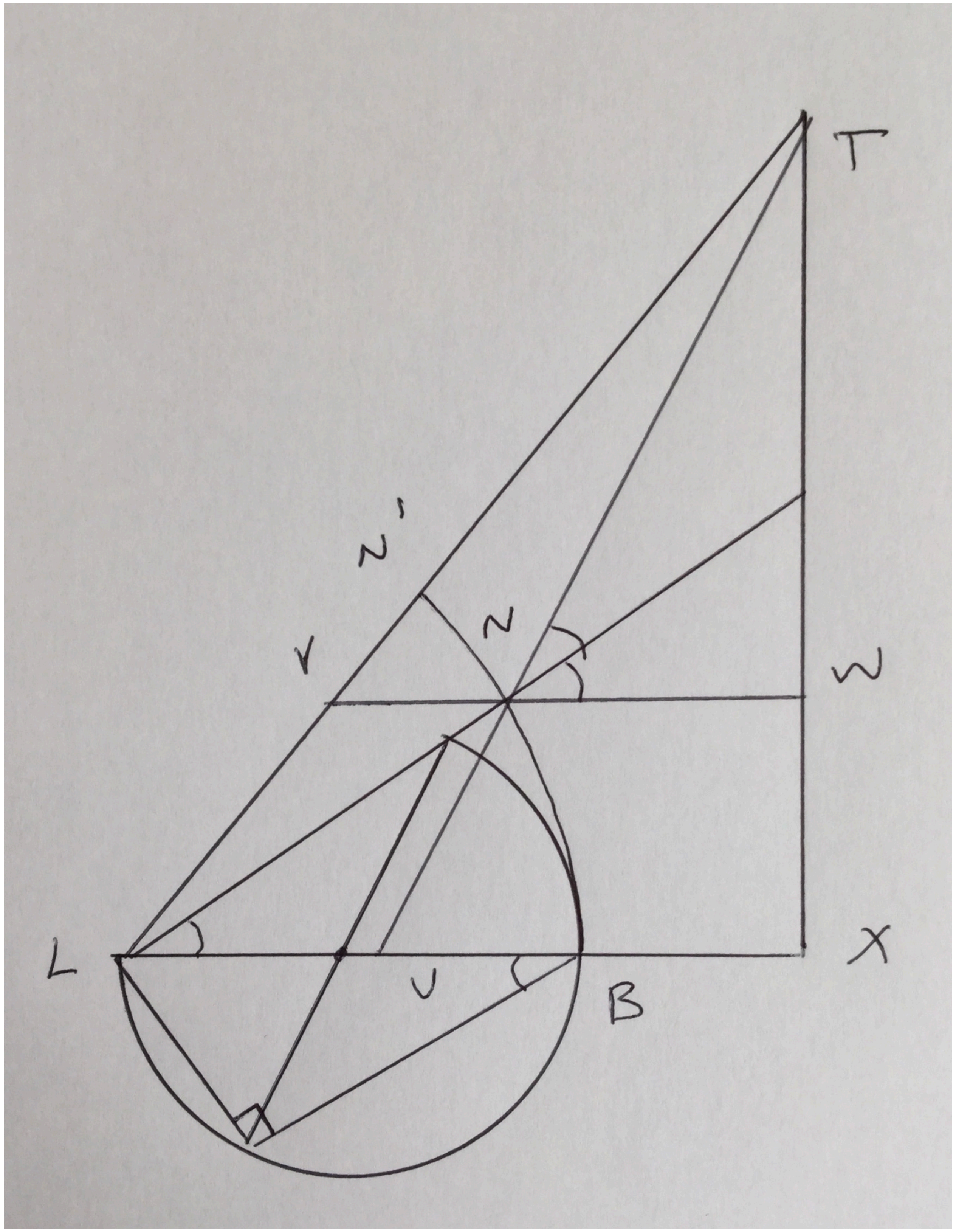
note that the condition producing a virtual image at H:



$$\frac{1}{BF} = \frac{\mathbb{R}}{BE} + \frac{\mathbb{R}}{BH}$$

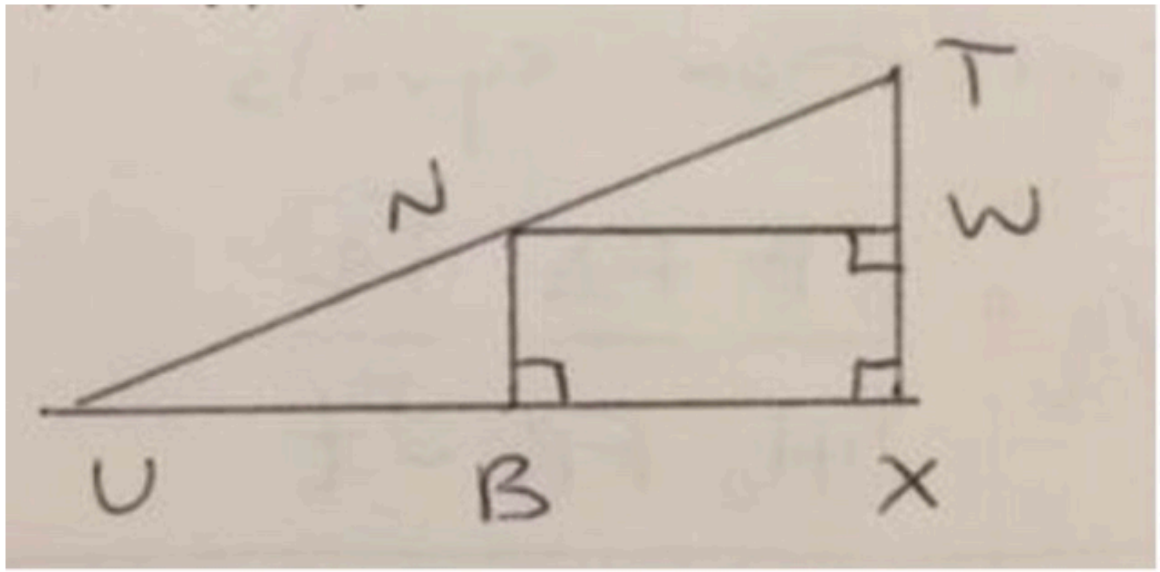
is meaningless when considering the focused axial image size magnification BH/BH_o when the standard image is real.

Figure 47:



BL Is found
by changing BX
to clearly focus
the reflected image V
of light source T

Figure 48:



make T $\Rightarrow$ X
so that 2BU $\Rightarrow$ BL
and $\angle$NBU $\Rightarrow$ $\frac{\pi}{2}$

so that:

$$\frac{XT}{XW} \rightarrow \frac{UX}{UB} \rightarrow \frac{2UX}{BL} \leftarrow \frac{2VW}{BL}$$

with a very small XT
measure XW and VW
to approximate BL

only the corneal component $\boldsymbol{K}$ of $\dfrac{\mathbb{R}}{\text{BE}}$ can be approximated with BL from the reflection off B

when its deviation from the standard 42 is assumed to equal the deviation of the total $\dfrac{\mathbb{R}}{\text{BE}}$

from its standard of 60:

$$\boldsymbol{K} + (42 - \boldsymbol{K}) = 42$$

$$\frac{\mathbb{R}}{\text{BE}} + (42 - \boldsymbol{K}) = 60$$

$$\frac{\mathbb{R}}{\text{BE}} = \boldsymbol{K} + 18$$

and since:

$$\boldsymbol{M} = \frac{\mathbb{R}}{BH_{\circ}} \quad \frac{BH}{\mathbb{R}}$$

$$\boldsymbol{M} = \frac{60}{\frac{\mathbb{R}}{BE} \pm \frac{1}{BF}}$$

(Note that the traditional sign convention when considering the distance correction 1/BF allows for the +/- sign to be replaced by simply a + sign).

When the retinal image size magnification of two real eyes are compared, retinal image size magnification loses its arbitrary nature resulting from its presumed standard. However, that does not address the arbitrary assumption in this calculation that magnification differences between two eyes result solely from their front surfaces. This calculation is only as correct as that assumption.

Section 7

Axial Magnification of Distance Correction

Figure 49:

Standard emmetropic eye:

Non-standard emmetropic eye:

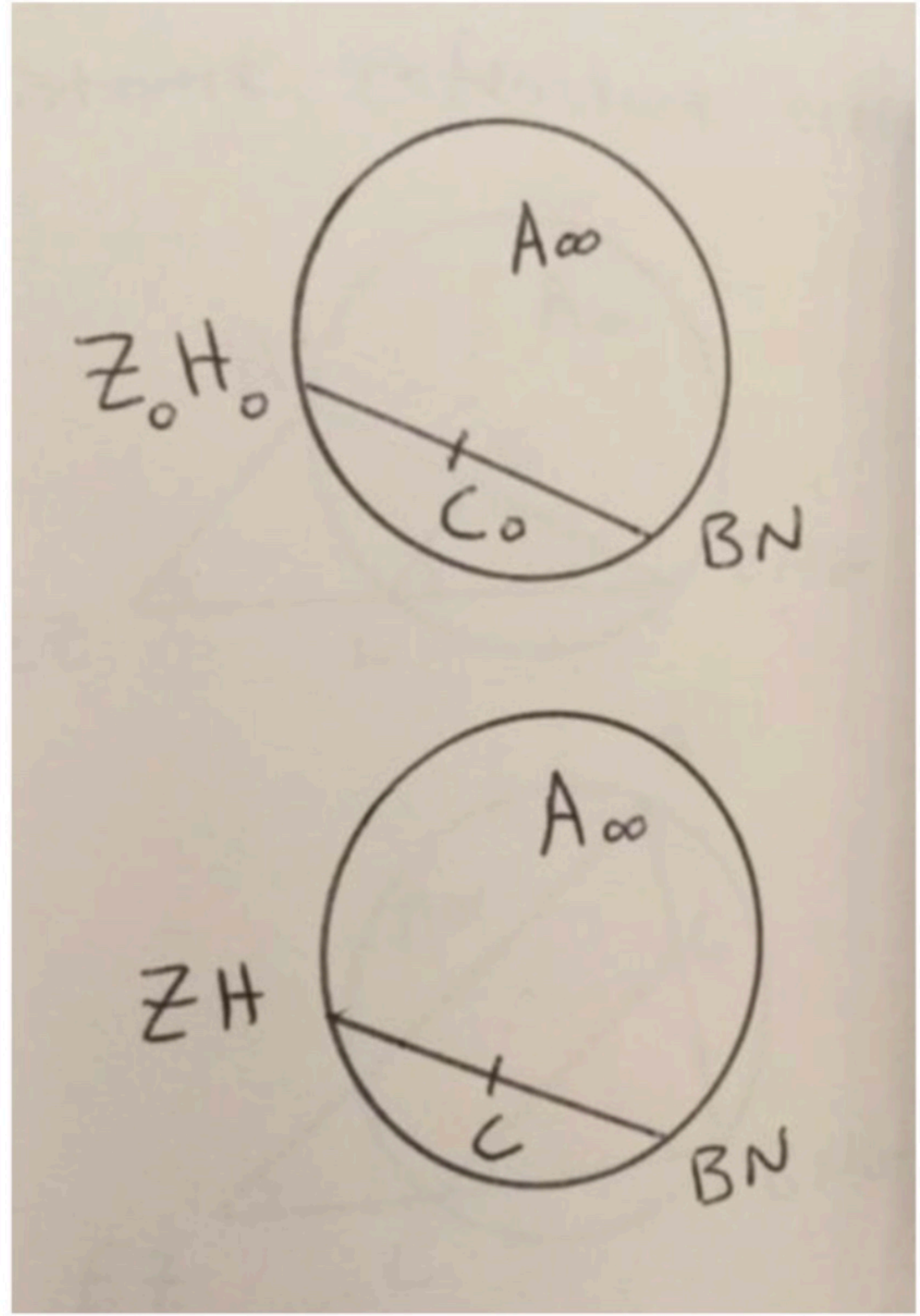


Figure 50:

Additional refraction at G (at B) creates distance refractive error with combined curvature of radius BL.

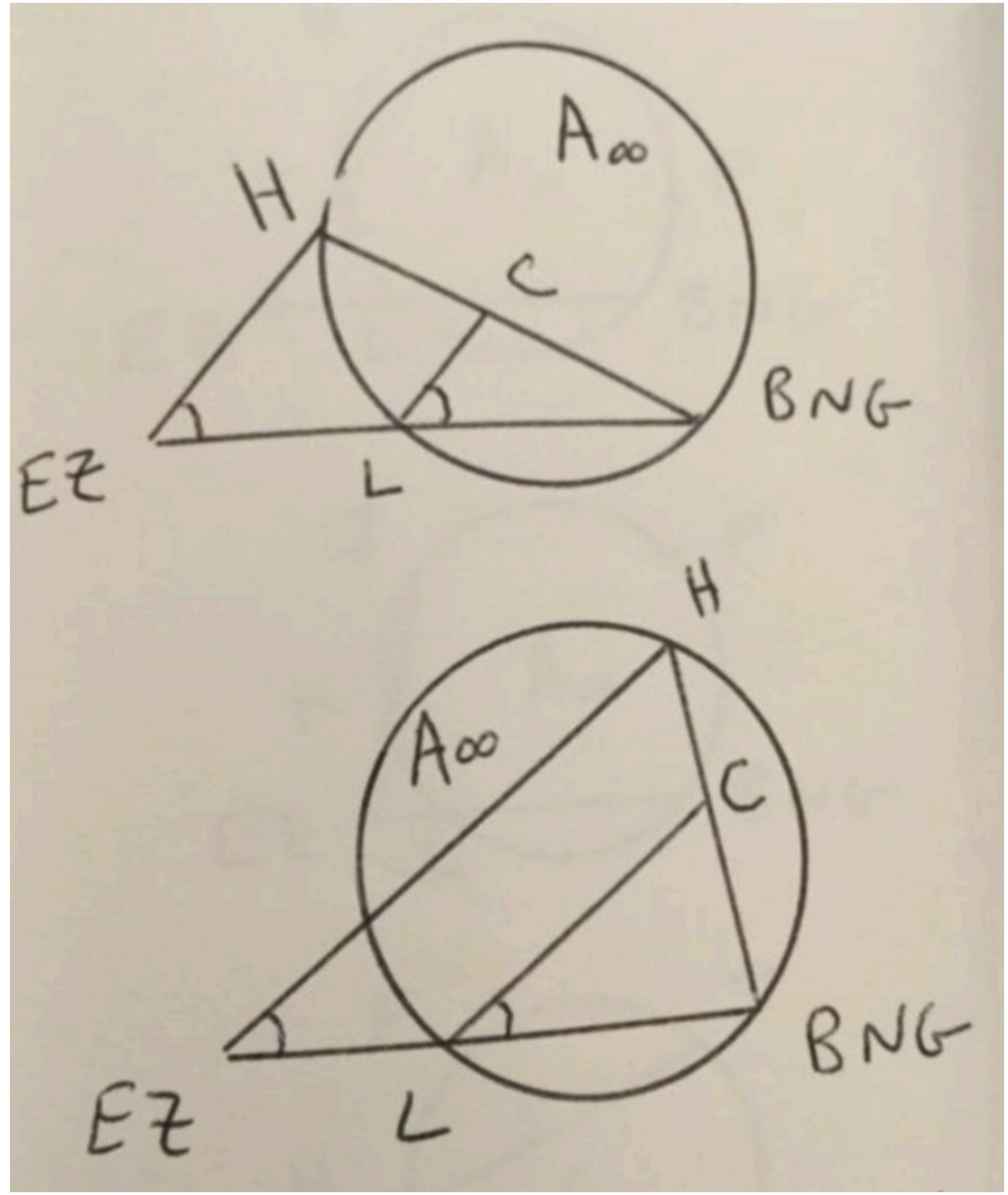


Figure 51:

The distance correction must focus infinity (A) at F so that:

JF ll BE

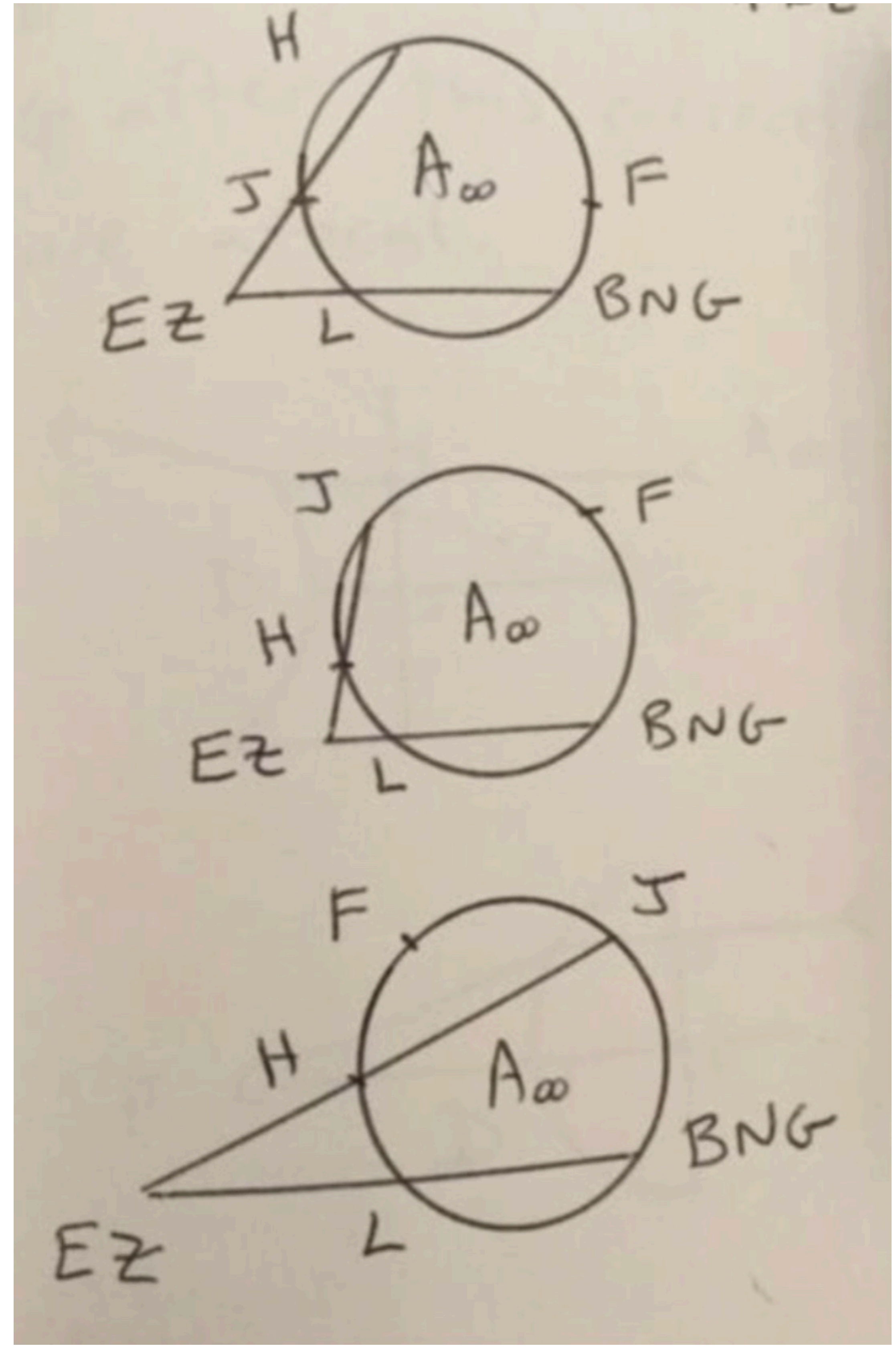
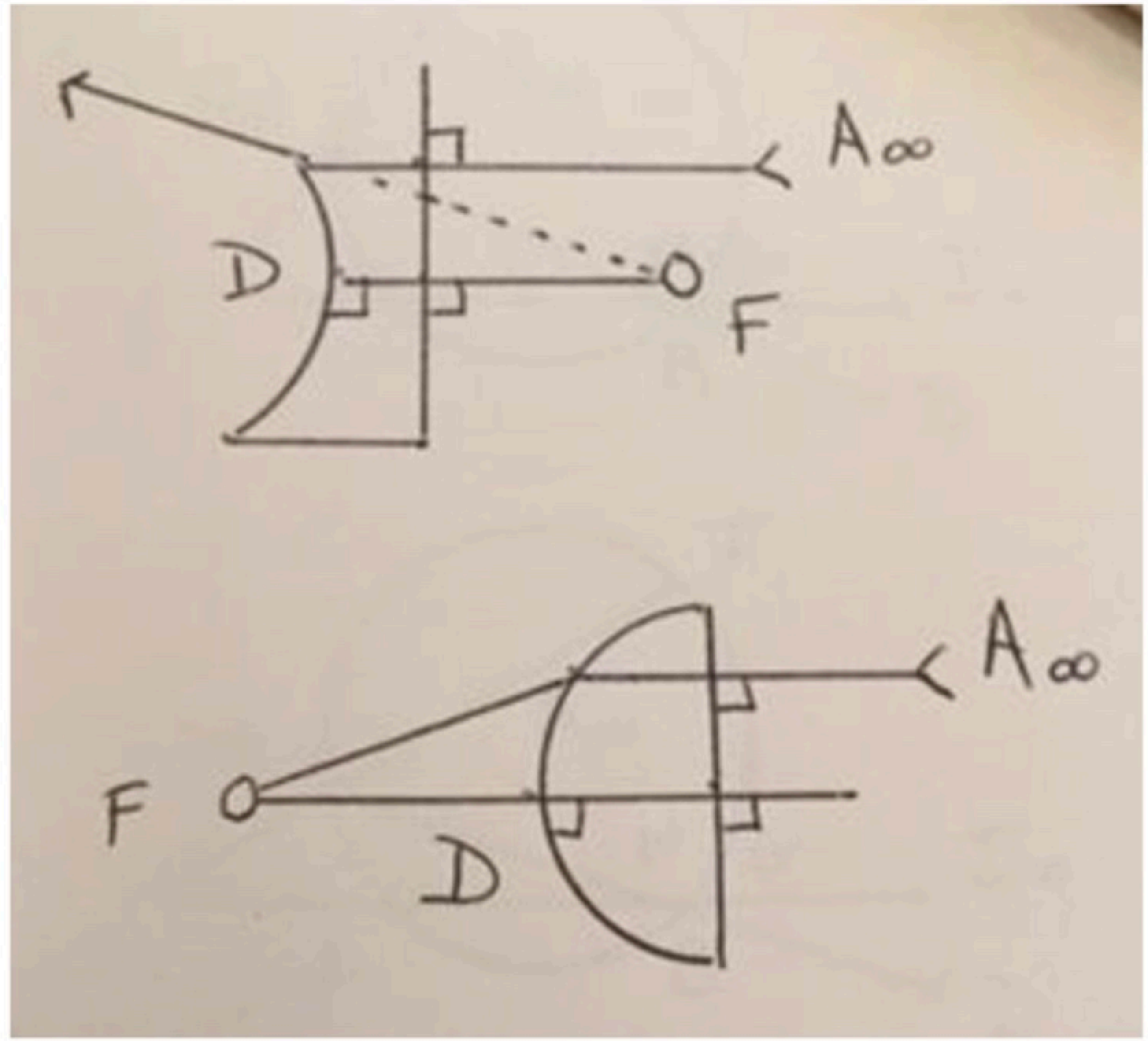
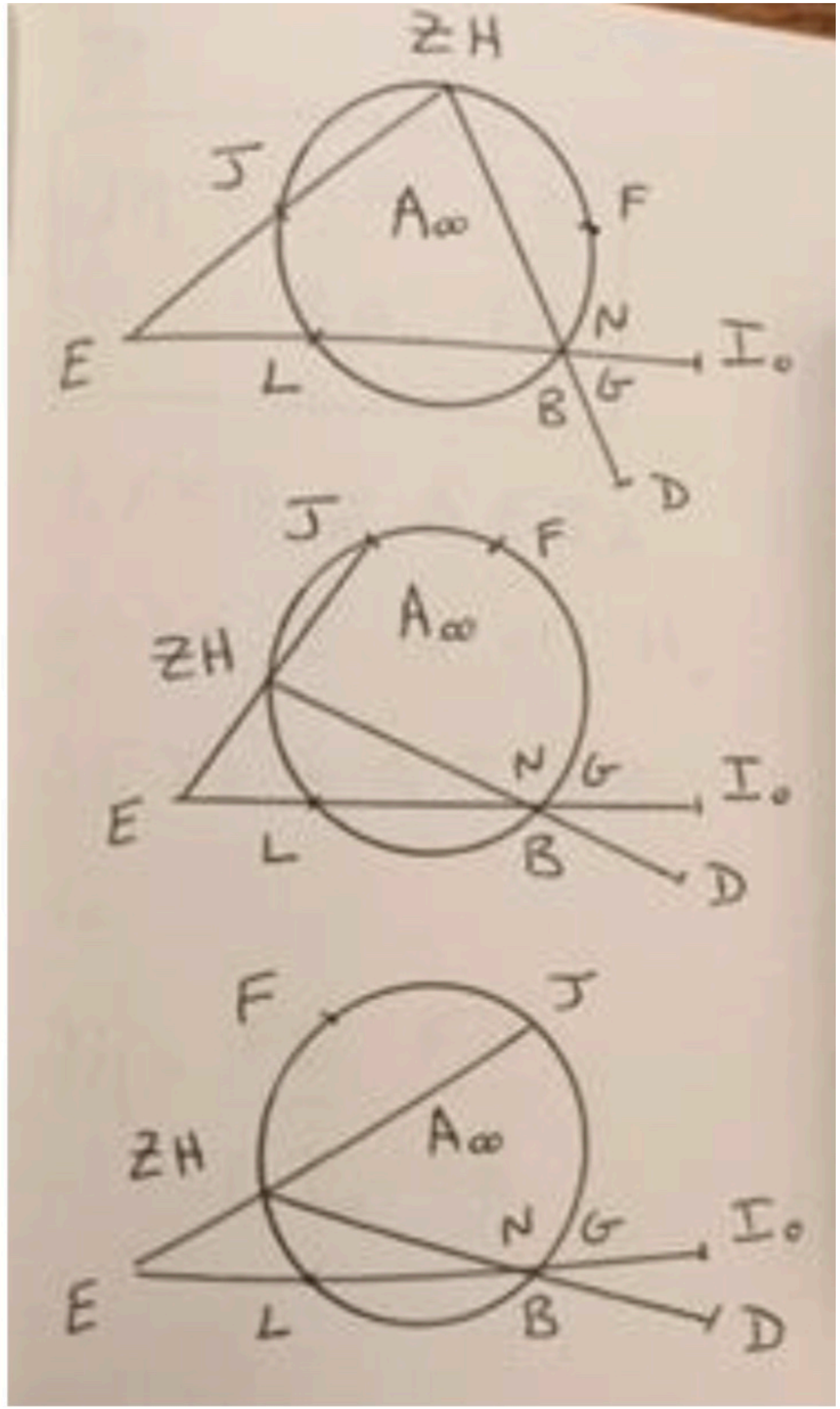


Figure 52:



since the distance correction
at D moves Z to H
rays leaving G after this correction are afocal

Figure 53:



$$\boldsymbol{M} = \frac{BH}{BH_{\circ}} \frac{FD}{FB}$$

$\Delta EBH \cong \Delta EJL$

when E is at $H_{\circ}$:

$\Delta EJL = \Delta I_{\circ}FB$ so:

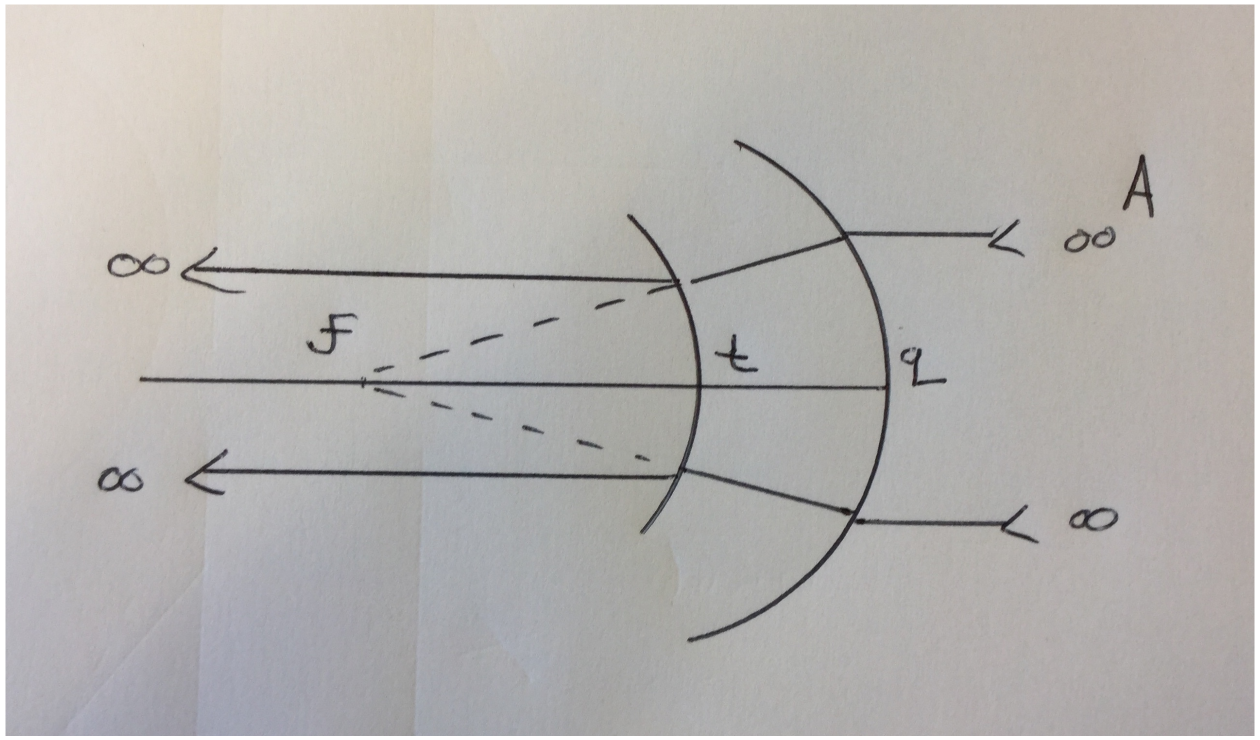
$$\boldsymbol{M} = \frac{FB}{FI_{\circ}} \frac{FD}{FB}$$

Note that when all the refractive error is due to the retina H lying at a position other than the standard, in other words, all the error is “axial” in nature, which occurs

when E is at H_o:

The magnification equals one when the distance correction at D lies at the standard eye’s front focal point.

Figure 54:



placing t at D:

$$\boldsymbol{M} = \frac{BH}{BH_o}\;\frac{FD}{FB}\;\frac{fq}{ft}$$

when the front surface of a spectacle lens that
corrects distance refractive error is not flat
it is convex
and produces additional
axial afocal angular magnification

In summary:

axial magnification of distance correction equals:

$$\boldsymbol{M} = \frac{BH}{BH_{\circ}} \frac{FD}{FB} \frac{fq}{ft}$$

where:

$\frac{BH}{BH_{\circ}}$ = axial corrected image size magnification

and:

$\frac{FD}{FB} \frac{fq}{ft}$ = axial afocal angular magnification of distance correction

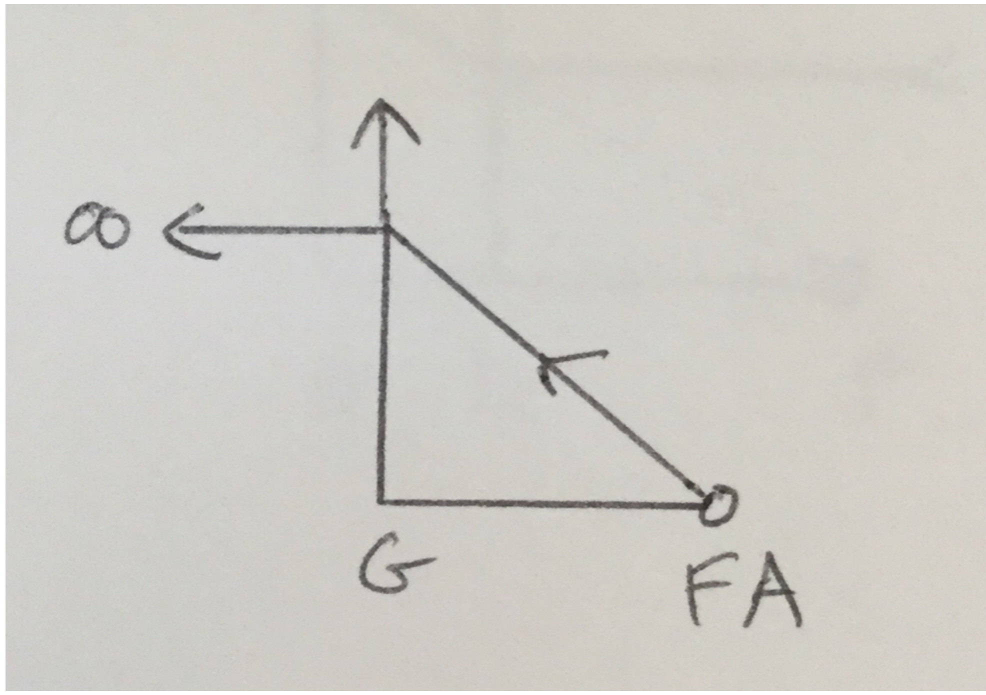
$\frac{FD}{FB}$ = "power factor"

$\frac{fq}{ft}$ = "shape factor"

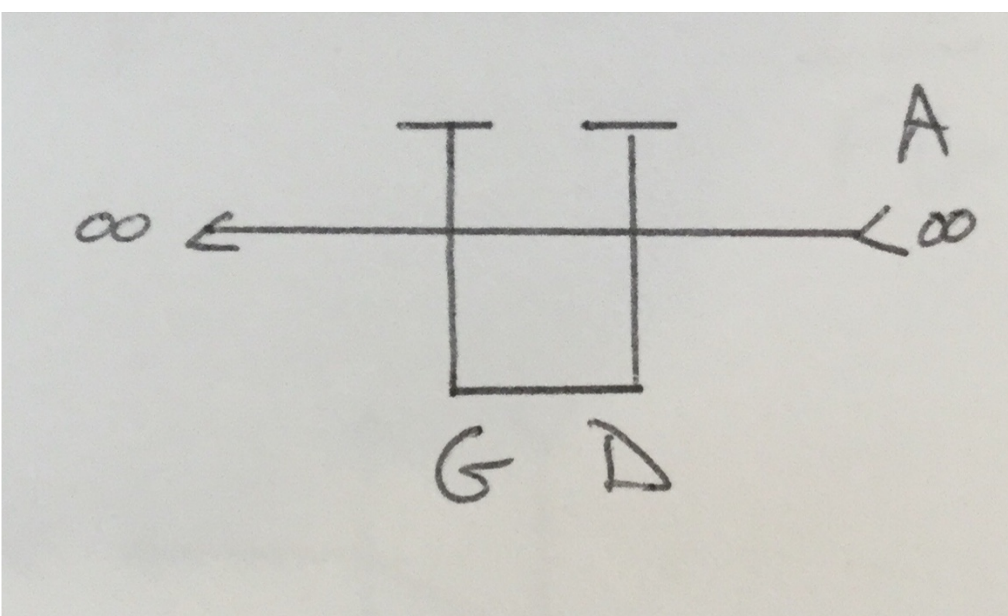
Section 8

Axial Magnification of Near Correction

Figure 55:

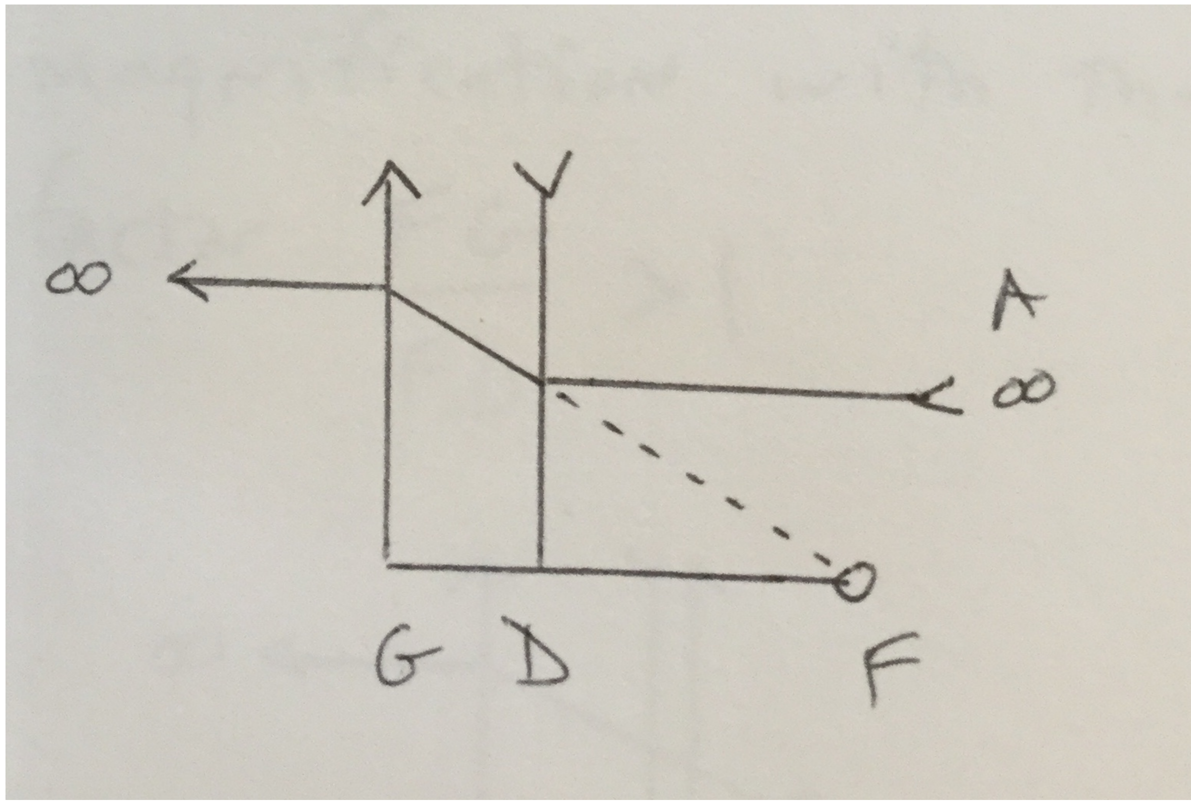


There is no afocal angular magnification when object A is at the front focal point of a myopic eye,



or at distance with an emmetropic eye.

Figure 56:

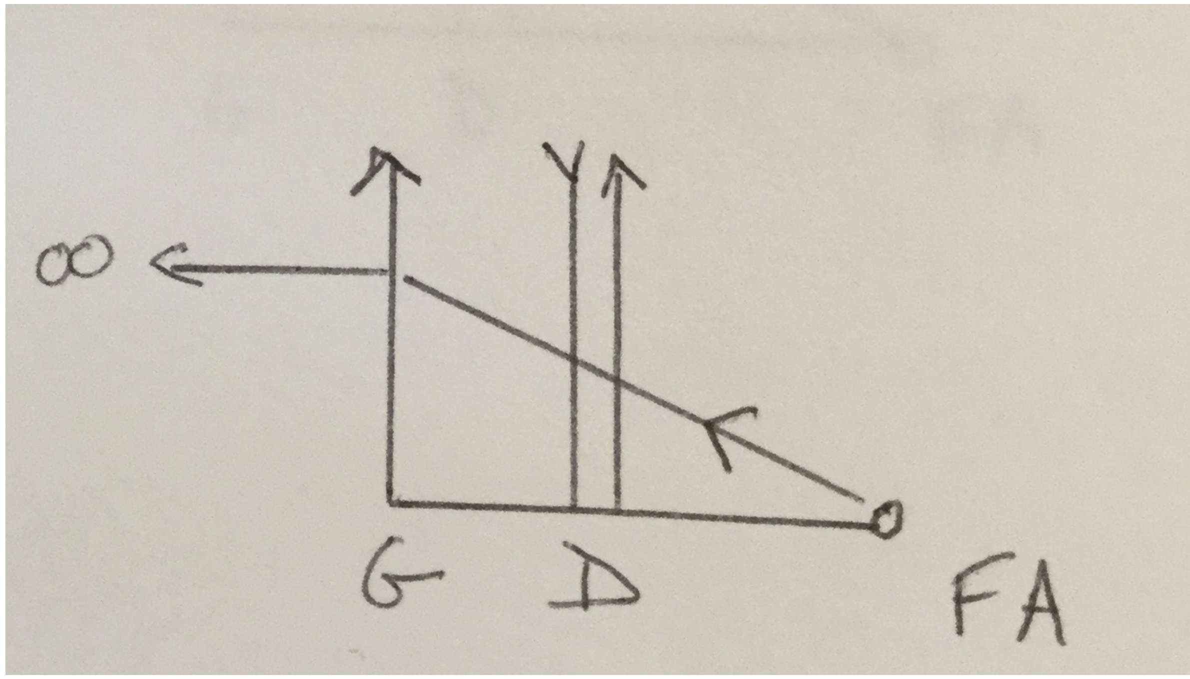


However, a distance myopic correction at D creates afocal angular magnification:

$$\frac{FD}{FG} < 1$$

and this is relative to both the myopic eye with object A at the myopic eye's front focal point F, as well as the emetropic eye with object A at distance.

Figure 57:

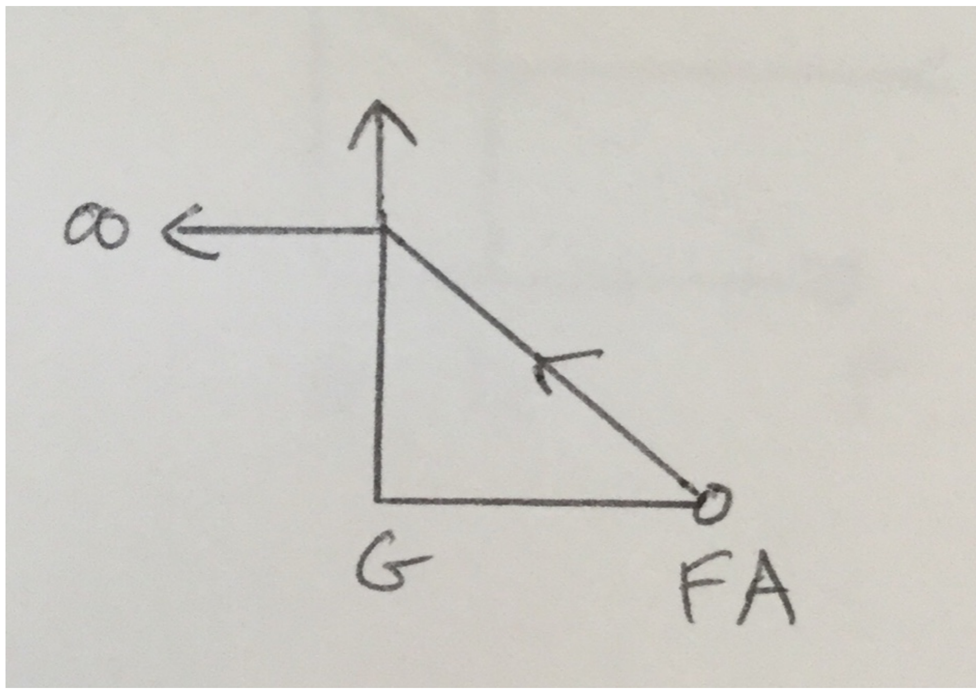


Removing the myopic distance correction at D with a converging lens at D removes this afocal angular magnification with the factor:

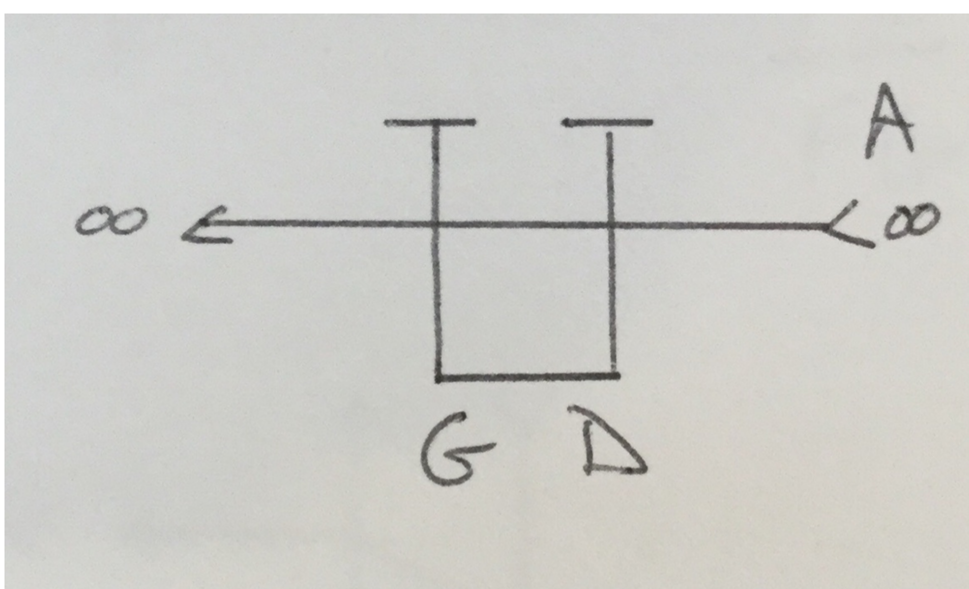
$$\frac{FG}{FD} > 1$$

and this magnification of near correction is relative to the distance corrected myope.

(Figure 55):



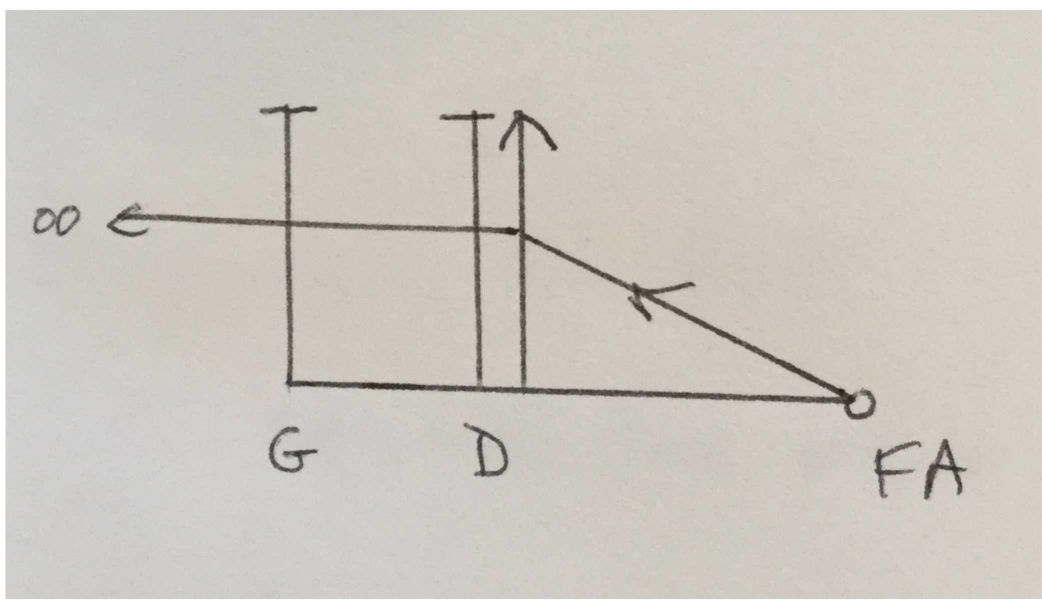
It is not relative to either the myope,



or an emmetrope.

Figure 58:

If additional converging power is added to the converging lens so that the near focal point is in focus for an emetropic eye, rather than the myopic eye, the afocal angular magnification removed with the factor:



$$\frac{FG}{FD} > 1$$

remains the same, and the reference eye is emetropic.

Figure 59:

When the converging lens at D is split into two converging lenses:

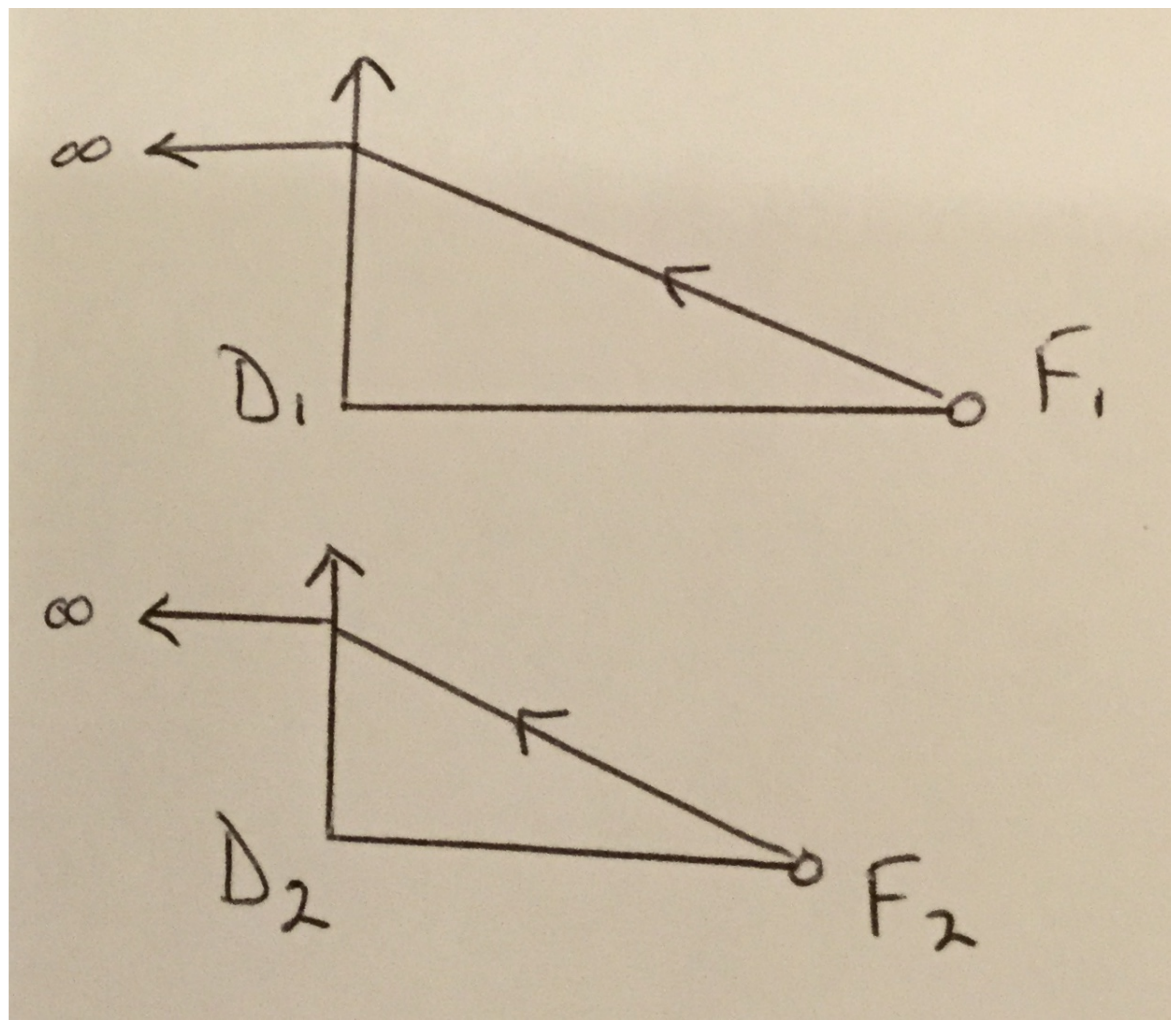


Figure 60:

With the same combined focus F:

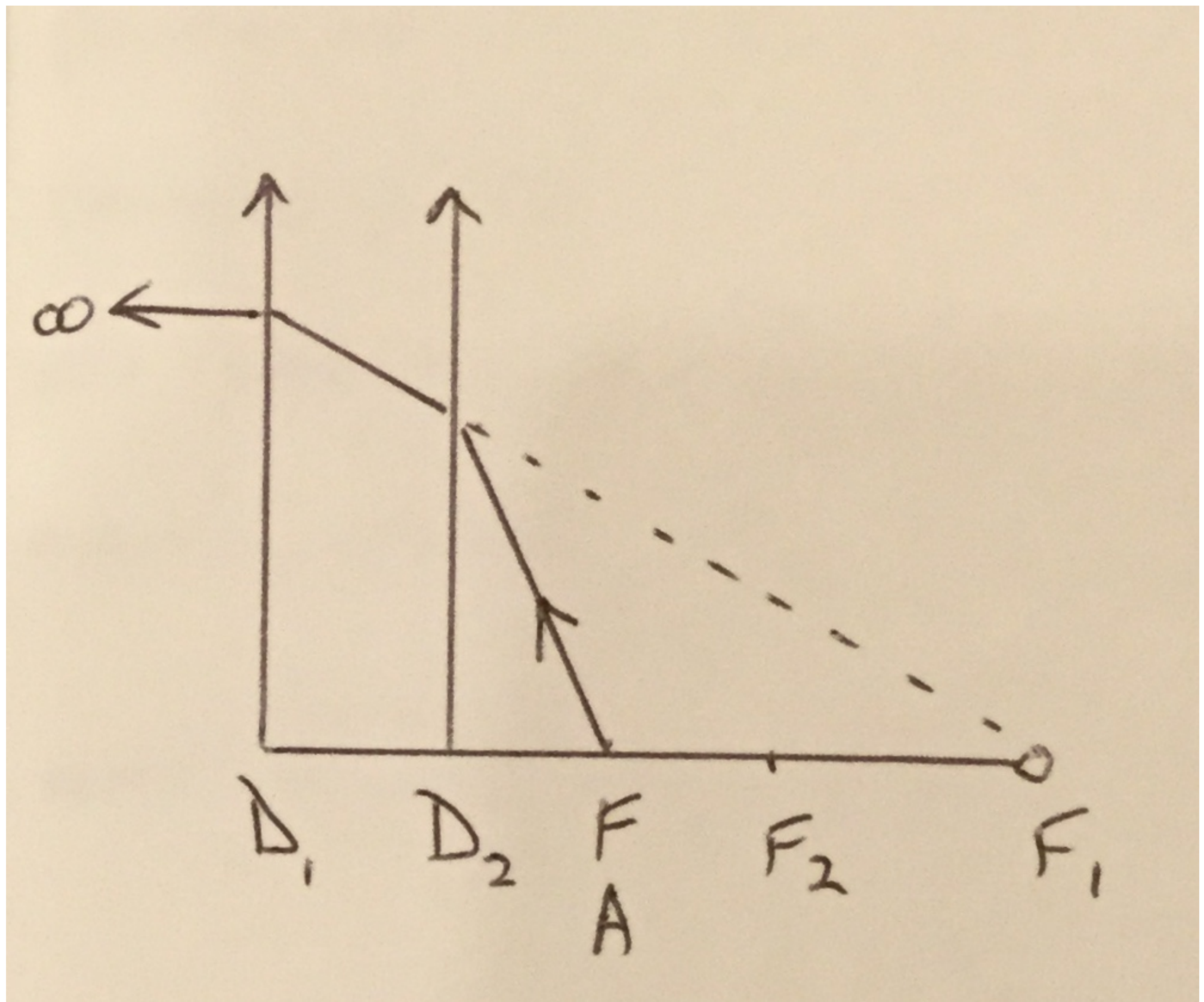
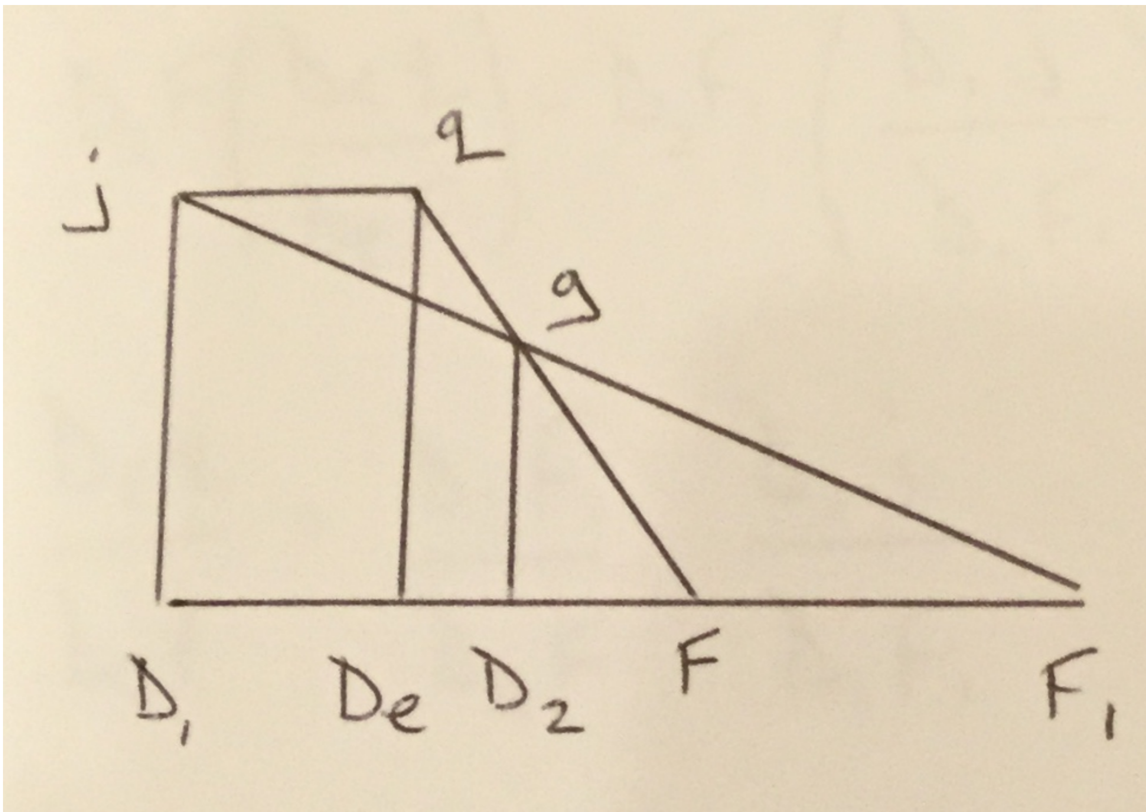


Figure 61:



The axial magnification of near correction can be specified as that produced

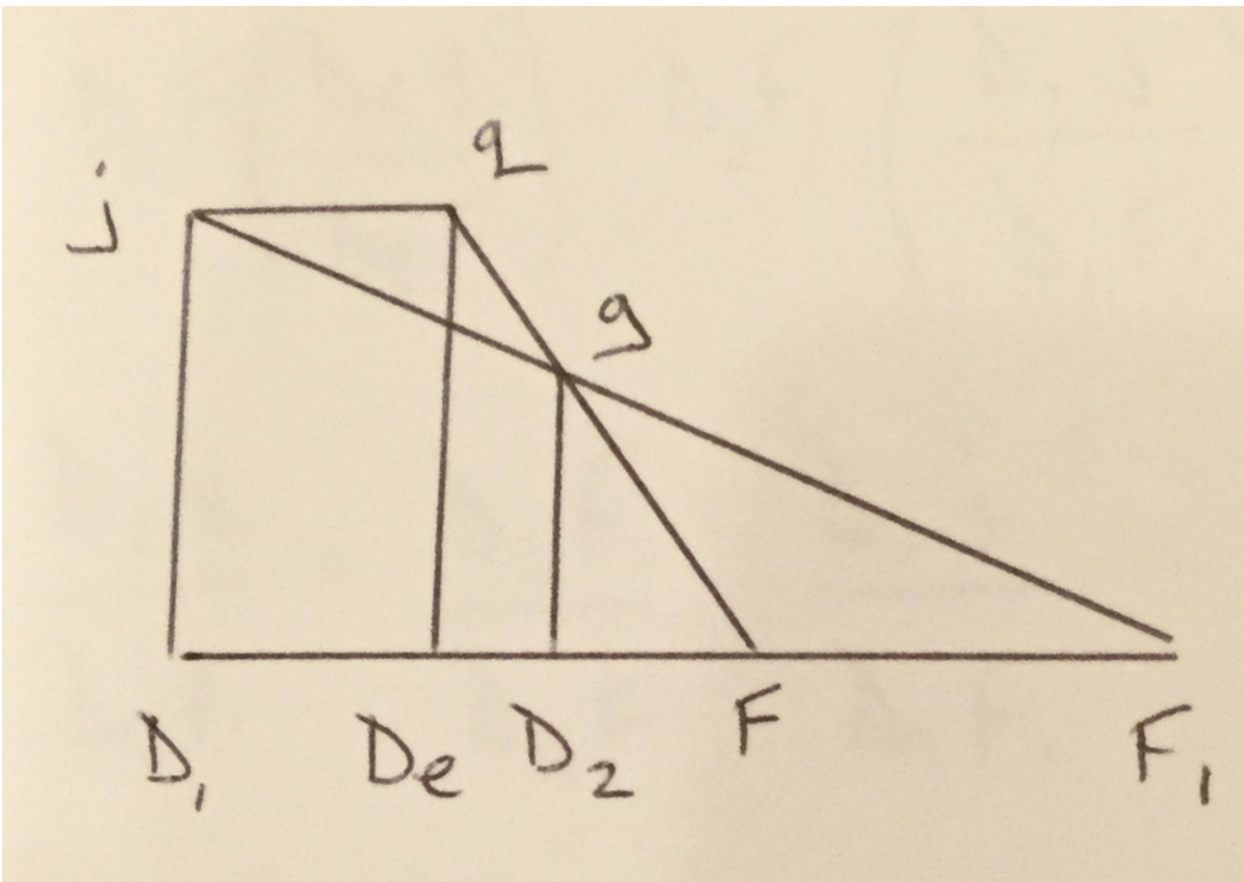
as if

all convergence occurs at a single unknown axial point De

and equals:

$$\frac{FG}{FDe} = \frac{FB}{FDe}$$

Figure 62:

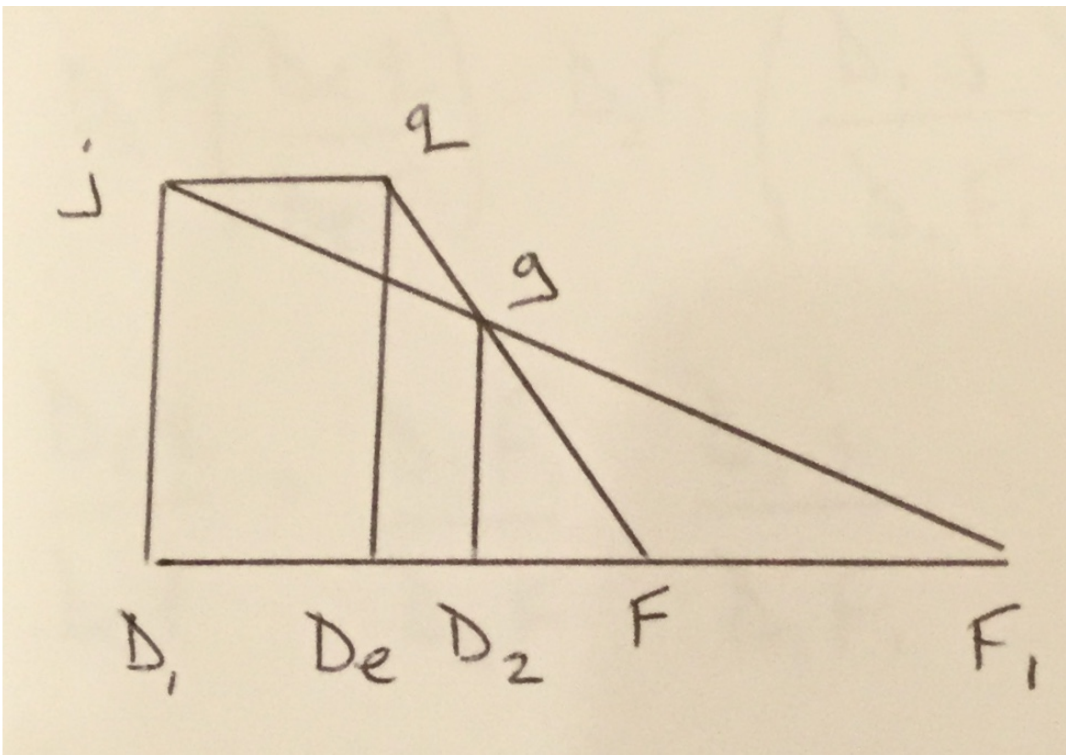


De can be located using triangles:

$$\frac{D_2g}{D_2F} = \frac{Deq}{DeF}$$

$$\frac{D_2g}{D_2F_1} = \frac{D_1j}{D_1F_1}$$

Figure 63:



$$D_2F \; \frac{Deq}{DeF} = D_2F_1 \; \frac{D_1j}{D_1F_1}$$

$$\frac{Deq}{DeF} = \frac{D_2F_1}{D_2F} \; \frac{D_1j}{D_1F_1}$$

$$\frac{1}{DeF} = \frac{D_2F_1}{D_2F} \; \frac{1}{D_1F_1}$$

$$\frac{FB}{FDe} = \frac{D_2F_1}{D_2F} \; \frac{FB}{D_1F_1}$$

Figure 64:

When an object at a standard distance Fs is moved to F:

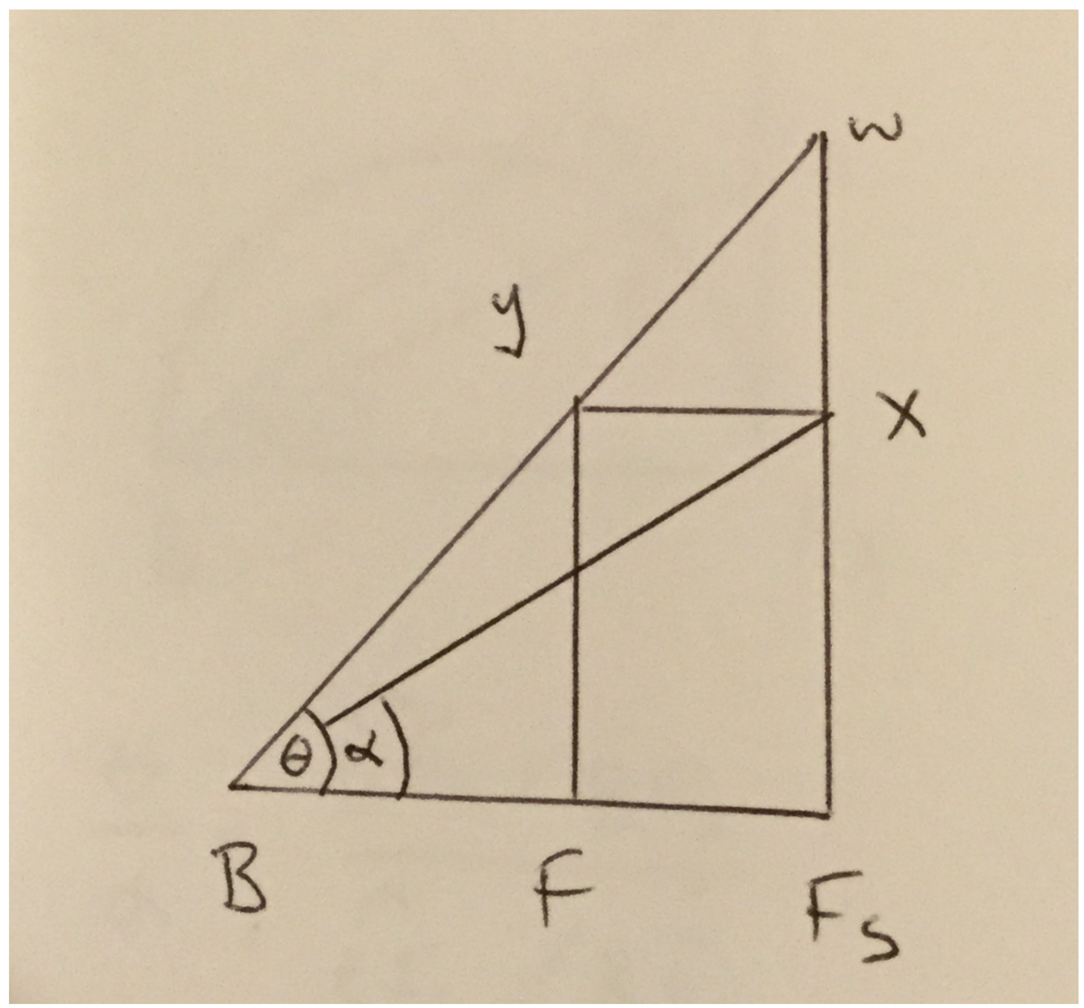
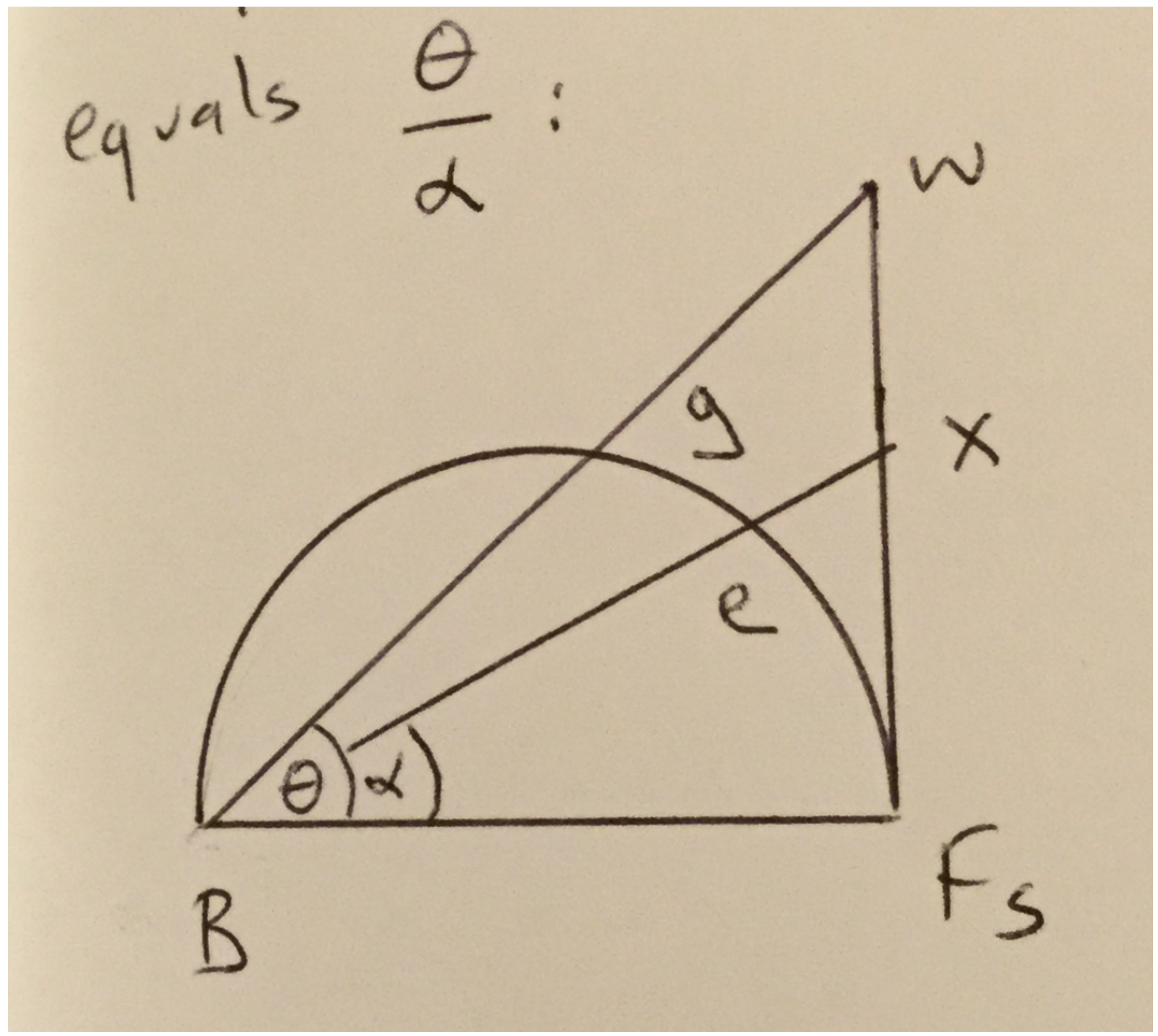


Figure 65:

The near object angular subtense magnification



$$\theta/\alpha \;=\; \frac{\sim gFs/BFs}{\sim eFs/BFs}$$

$$\text{as } yF \;=\; xFs \;\Rightarrow\; 0$$

$$\theta/\alpha \;\Rightarrow\; \frac{wFs}{xFs} \;=\; \frac{wFs}{yF} \;=\; \frac{BFs}{BF}$$

which equals the axial near object angular subtense magnification.

Multiplying the axial near subtense magnification by the axial magnification of near correction produces:

$$\frac{BFs}{FDe} = \frac{D_2F_1}{D_2F}\;\frac{BFs}{D_1F_1}$$

Since the converging lens at D_2 creates a virtual image at F_1 of an object at F, so that the enlargement of an object at F created by D_2 equals D_2F_1/D_2F; when the diagram represents a stand magnifier with lens D_2 and stand height D_2F, and the reading spectacle add is D_1, (or the ocular accommodation is D_1 at B), the magnification produced by the stand magnifier is its (constant) enlargement factor, multiplied by that produced by D_1 alone.

The ratio describing near object axial angular subtense magnification:

$$\frac{BFs}{BF}$$

when combined with the ratio describing near magnification due to a single converging lens producing parallel light for an emmetropic eye:

$$\frac{FB}{FD}$$

produces a ratio product which factors out the object's actual distance to the eye, confirming that when a converging lens is used with its front focal point at the near object, (and therefore parallel light leaves the converging lens from the object), the image size is the same regardless of the object-to-eye distance.