Images Seen Through Water

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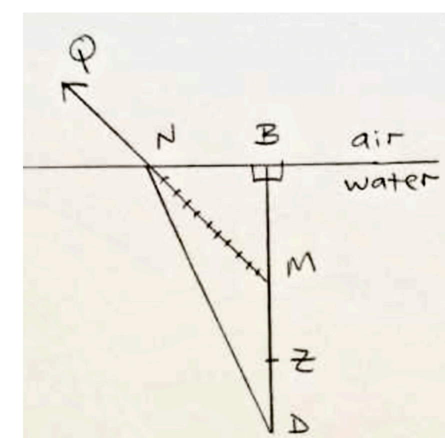
Reference:
Isaac Barrow's Optical Lectures 1667
Translated by H.C. Fay
Edited by A.G. Bennett and D.F. Edgar
Published by "The Worshipful Co. of Spectacle Makers" 1987
(Lectures 4 & 5)

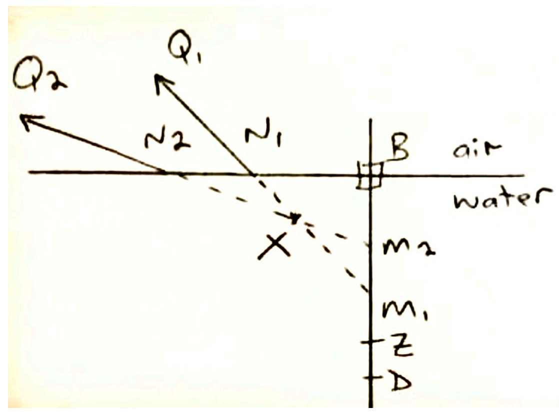
Object in Water; Image Seen From Air

If underwater object D is a perpendicular distance DB from the plane of the water surface in all radial directions, the image of object D along that perpendicular, when seen from directly above in air, is at Z, and BD/BZ = 4/3.



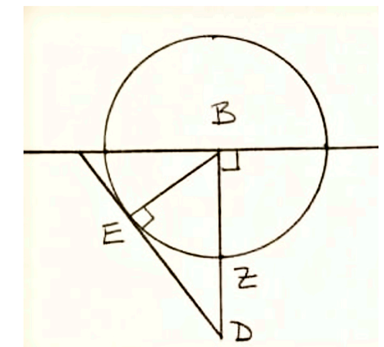
Isaac Barrow showed that the image of object D, when seen from Q *obliquely* along image ray MNQ, also lies above the object, but towards the observer relative to DB.

Isaac Barrow described a way to find all oblique image rays MNQ through a designated point X, without knowing their points of refraction (N) along the surface of the water, or their intersections (M) with the perpendicular DB.

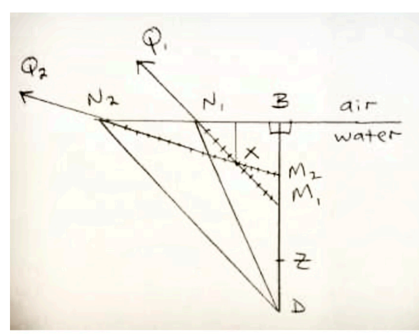


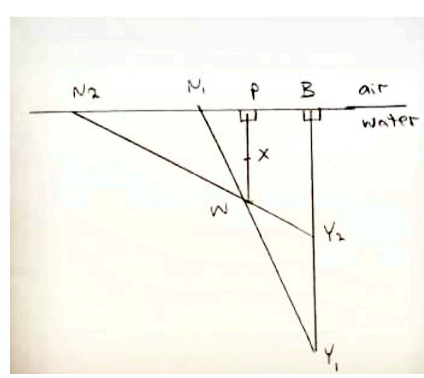
He first drew a reference right triangle created by drawing BE = BZ as shown, which created the following constant ratios for air/water refraction:

$BD/BZ = BD/BE = 4/3$
$DB/DE = 4/\sqrt{(16-9)} = 1.5$
$ED/EB = \sqrt{(16-9)}/3 = 0.87$



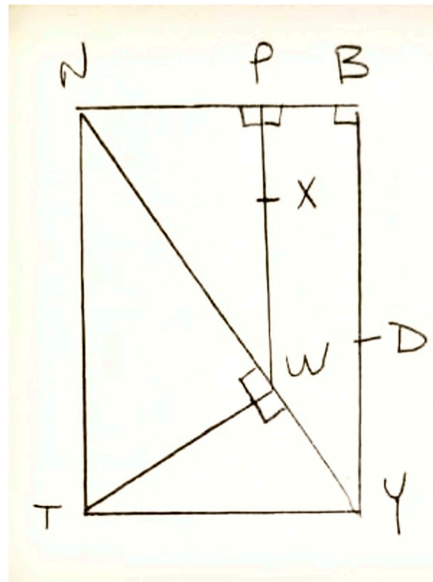
He showed that, given DB and the designated point X, if we draw:
$PW/PX = DB/DE = 1.5$
then all image rays through X, (MXNQ) are found using:
$DB/YN = ED/EB = 0.87$
by drawing all possible reference lines of length $YN = DB/0.87$ through W, in order to locate the required positions of N.





This means that for any given DB, there can be a maximum of two image rays through the designated point X, since only two reference line segments within the right angle ∠(Y)B(N), and equaling his calculated constant YN, can fit through point W.

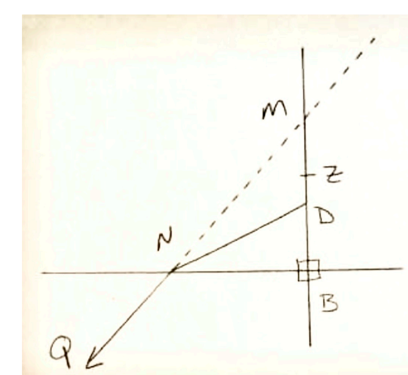
Isaac Barrow showed that YN can be drawn as the shortest segment through W bounded by the right angle ∠(Y)B(N) when right triangles ΔYBN, ΔNWT, and ΔTWY are all drawn as similar.



As length YN = DB/0.87 through W changes, so must DB, or the position of D. Since PW must remain unchanged, so must PX = PW/1.5. Therefore, when the object is in water, Isaac Barrow's method finds the image ray XMNQ for a designated clear image X, and an undesignated object D.

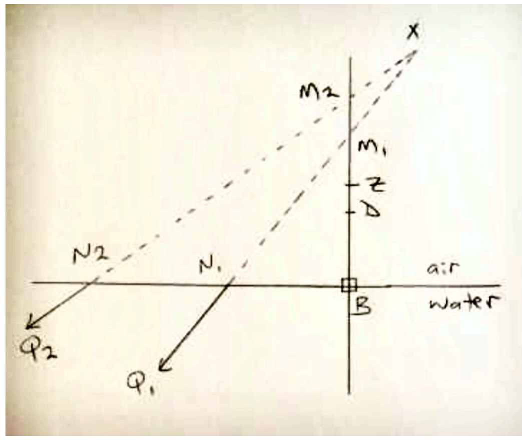
Object in Air; Image Seen From Underwater

If object D is in air, and at a perpendicular distance DB from the surface of water in all radial directions, the image of the object along that perpendicular when seen from underwater is at Z, and BZ/BD = 4/3.



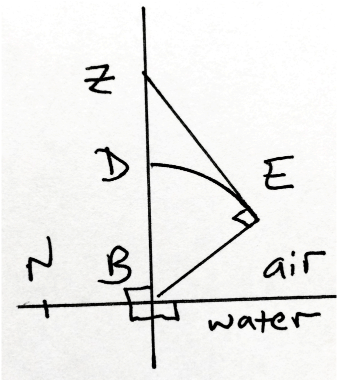
Isaac Barrow showed that the image of object D, when seen from Q *obliquely* along image ray MNQ, also lies above the object, but away from the observer relative to DB.

Isaac Barrow described a way to find all oblique image rays MNQ through a point X, without knowing their points of refraction (N) along the surface of the water, or their intersections (M) with the perpendicular DB.

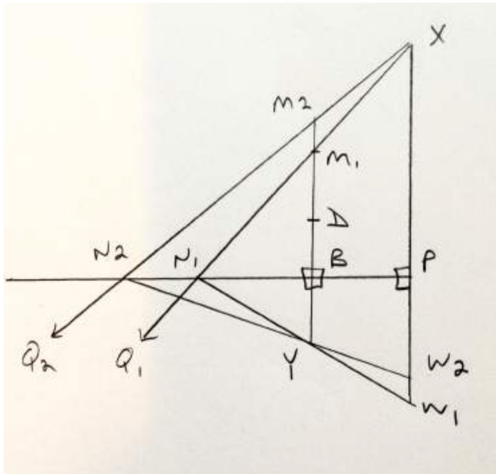


He first drew a reference right triangle created by drawing BE = BD as shown, which created the following constant ratios for air/water refraction:

BZ/BD = BZ/BE = 4/3
ZB/ZE = 4/√(16-9) = 1.5
EZ/EB = √(16-9)/3 = 0.87

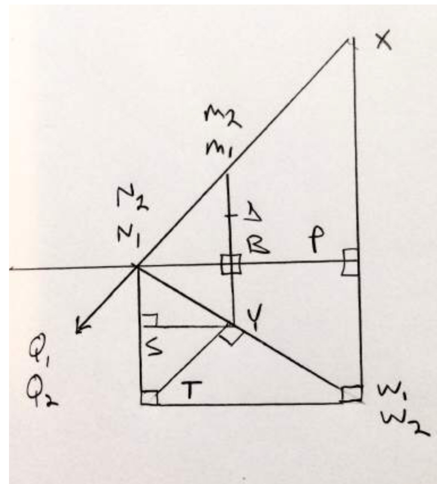


He showed that, given DB and the designated point X, if we draw BY/BD = ZB/ZE = 1.5 then all image rays through X, (XMNQ) are found using: XP/WN = MB/YN = EZ/EB = 0.87 by drawing all possible reference lines of length WN = XP/0.87 through Y, in order to locate the required positions of N.



This means that for any given DB, there can be a maximum of two image rays through the designated point X, since only two reference line segments within the right angle ∠(W)P(N), and equaling his calculated constant WN, can fit through point Y.

Isaac Barrow showed that WN can be drawn as the shortest segment through Y bounded by the right angle ∠(W)P(N) when right triangles ΔWPN, ΔNYT, and ΔWYT are all drawn as similar.



As length WN = XP/0.87 through Y changes, so must XP, or the position of X. Since BY must remain unchanged, so must DB = BY/1.5. Therefore, when the object is in air, Isaac Barrow's method finds the image ray XMNQ for a designated object D, and an undesignated clear image X.