Plano Refraction

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Continuity of Conic Sections/ Plane Geometry

Characteristics of Conic Sections/ Synthetic Geometry

Plano Refraction

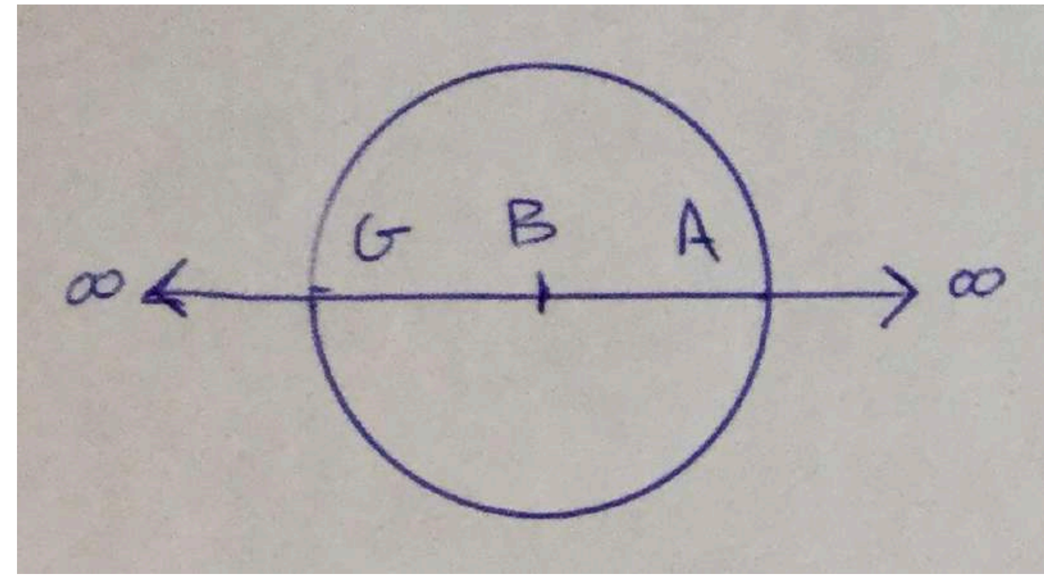
- Image rays from plano refraction
- Locating the plano image ray through an off-axis point
- Locating the clear off-axis image from plano refraction

Continuity of Conic Sections /Plane Geometry

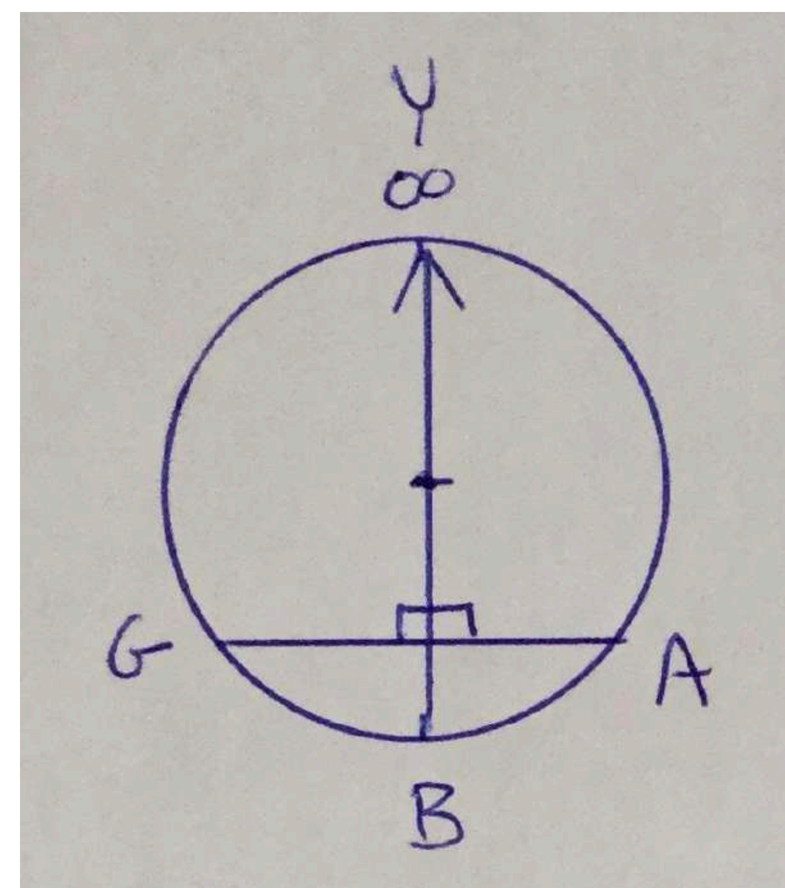
Although solid geometry can be used to visualize the continuity of conic sections, this can be also visualized with plane geometry.

Circle

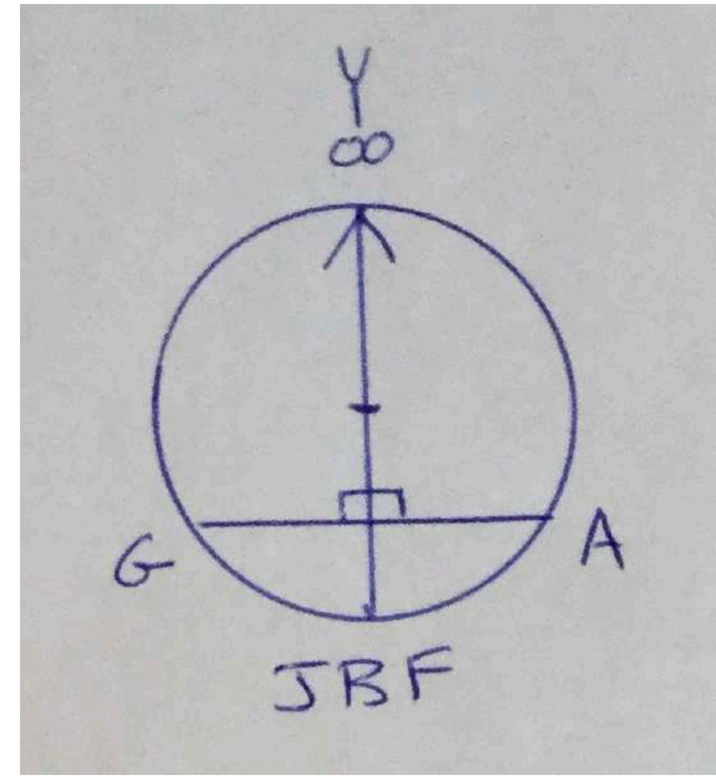
If we consider a circle with center B and diameter GBA with an “axis” infinitely long through GBA:



We can represent GBA along a circle of infinite diameter with BY, and draw BG = BA. This infinitely large reference circle is equally divided along ray BY, with Y at infinity.

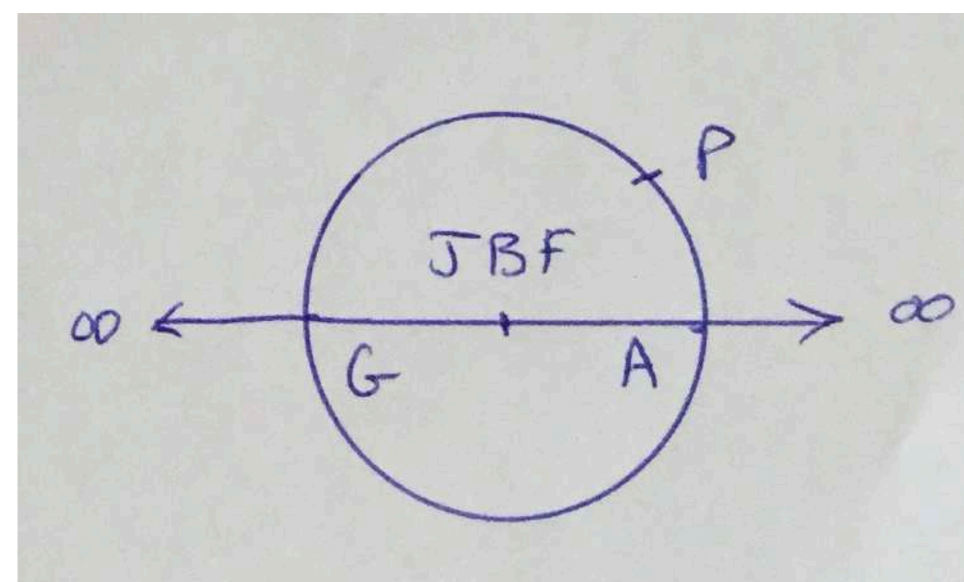


If we call points J & F, (both of which in this case lie at B), the "focal points" of the finite circle, we can consider the shape of the finite circle with diameter GBA to equal its "eccentricity" = e = BF/BA = 0.

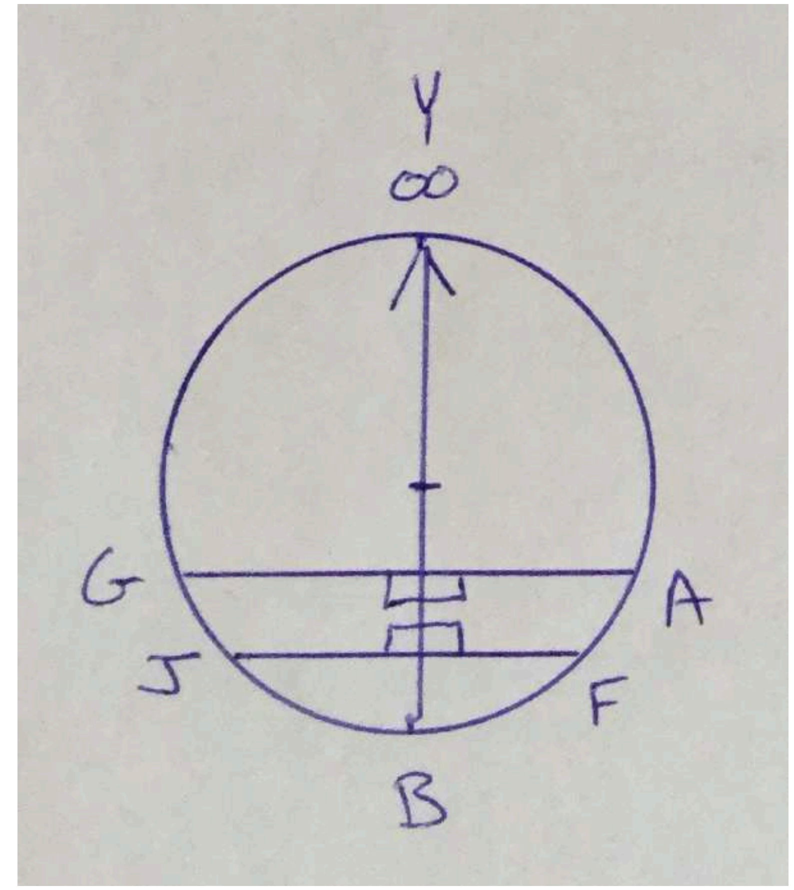


We will have drawn a defined circle where AJ + AF = AG along its diameter GJBFA, if it is also true that:

PJ + PF = AG

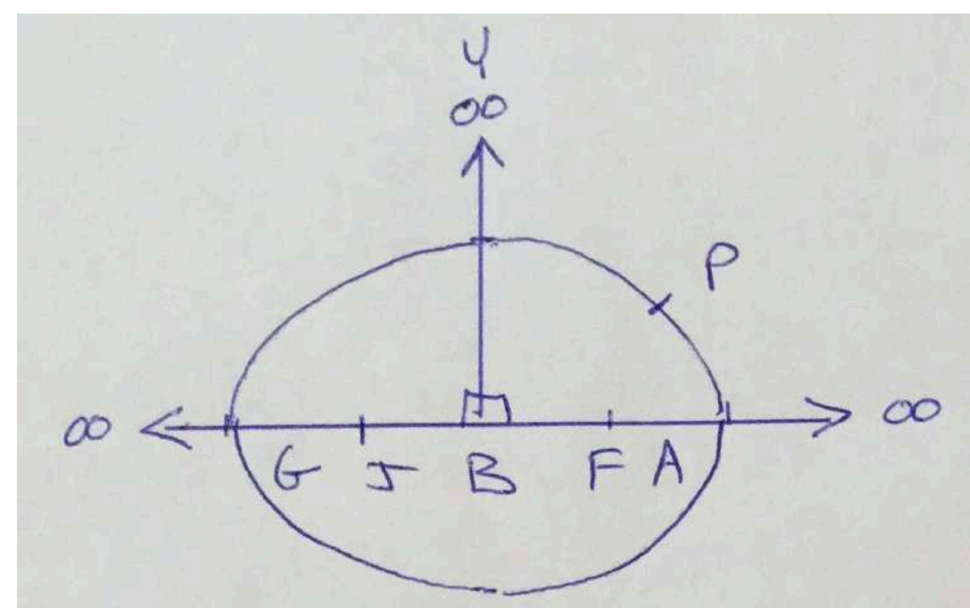


Ellipse



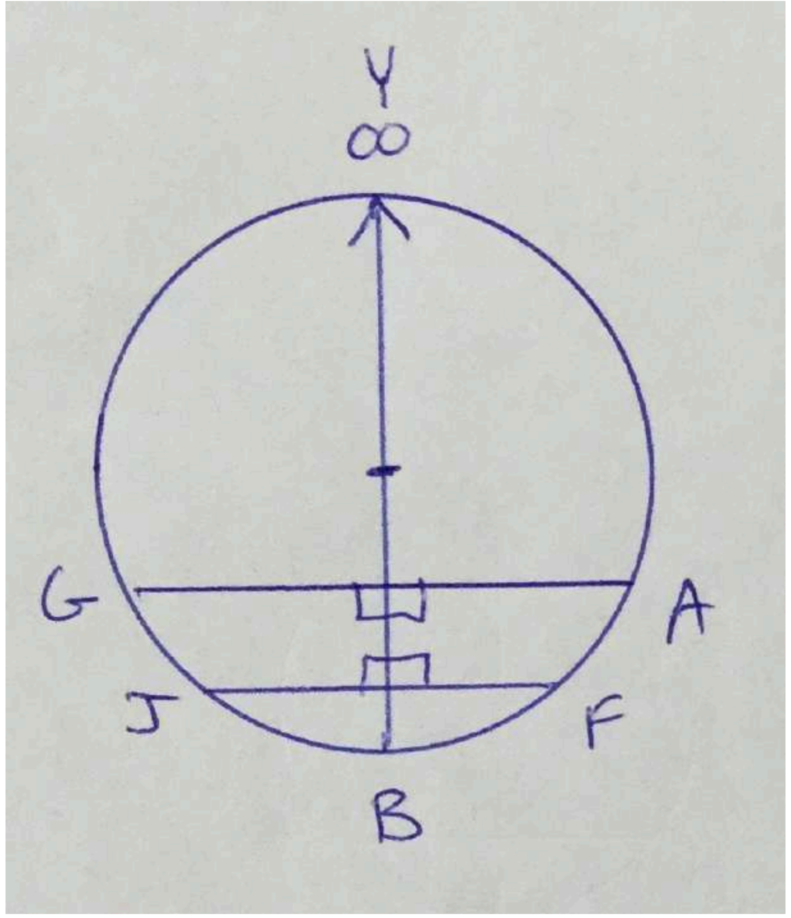
Draw: $0 < BF = BJ < \infty$
so that: $0 < e = BF/BA < 1$

We will have drawn a defined ellipse where AJ + AF = AG along its "major axis" GJBFA, if it is also true that PJ + PF = AG.

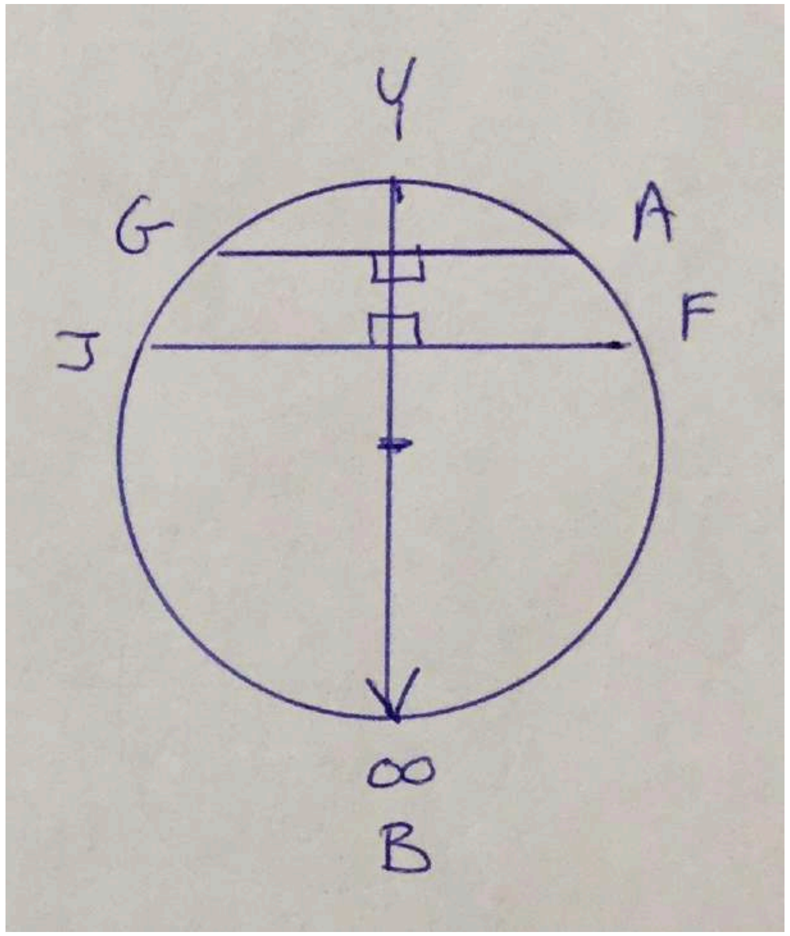


Hyperbola

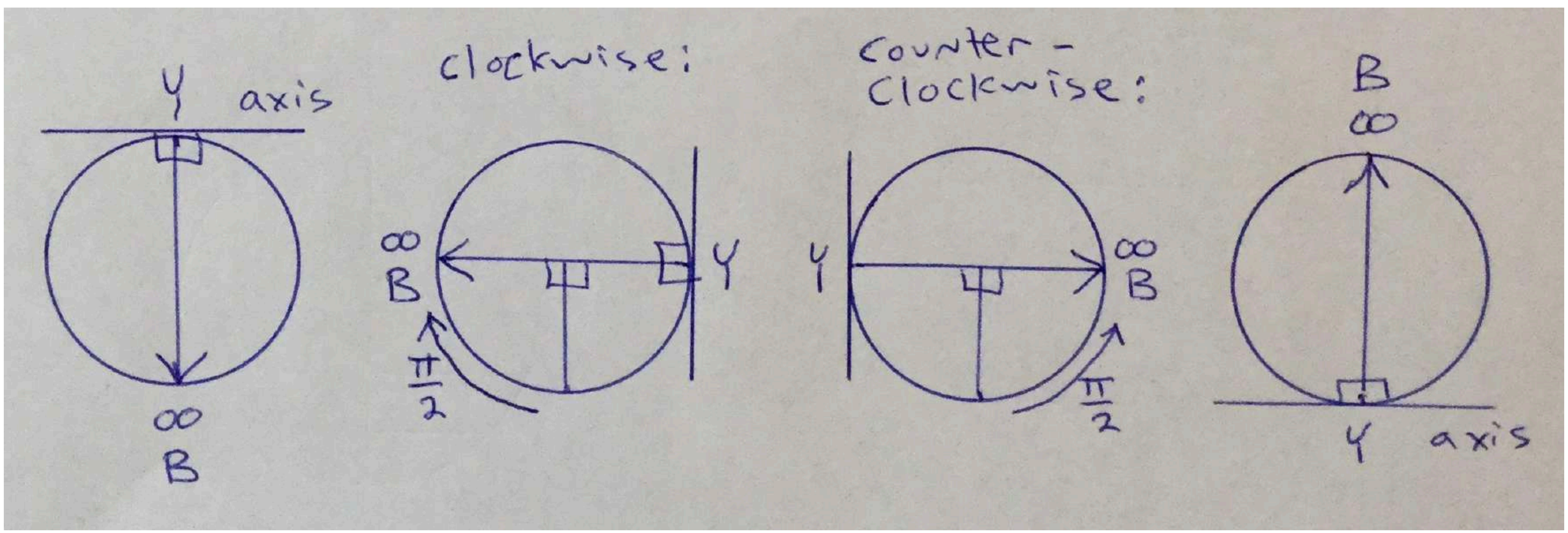
As B $\Rightarrow \infty$



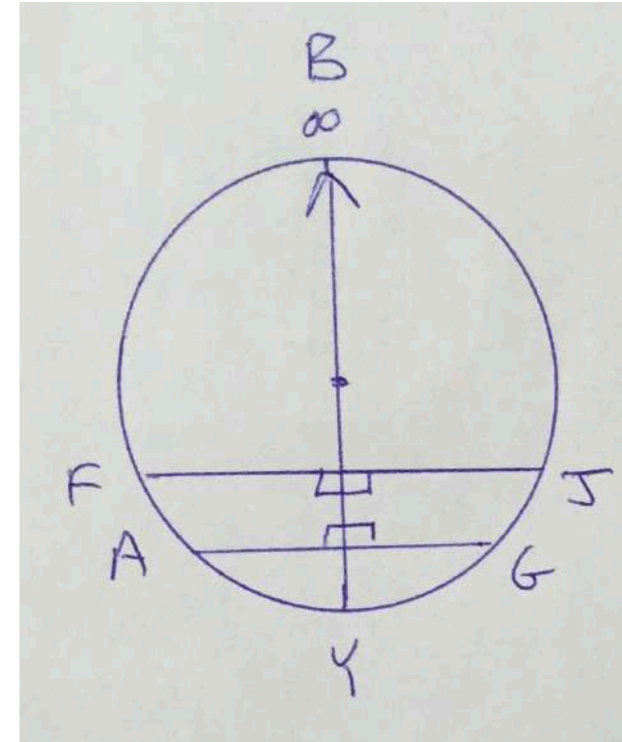
becomes:



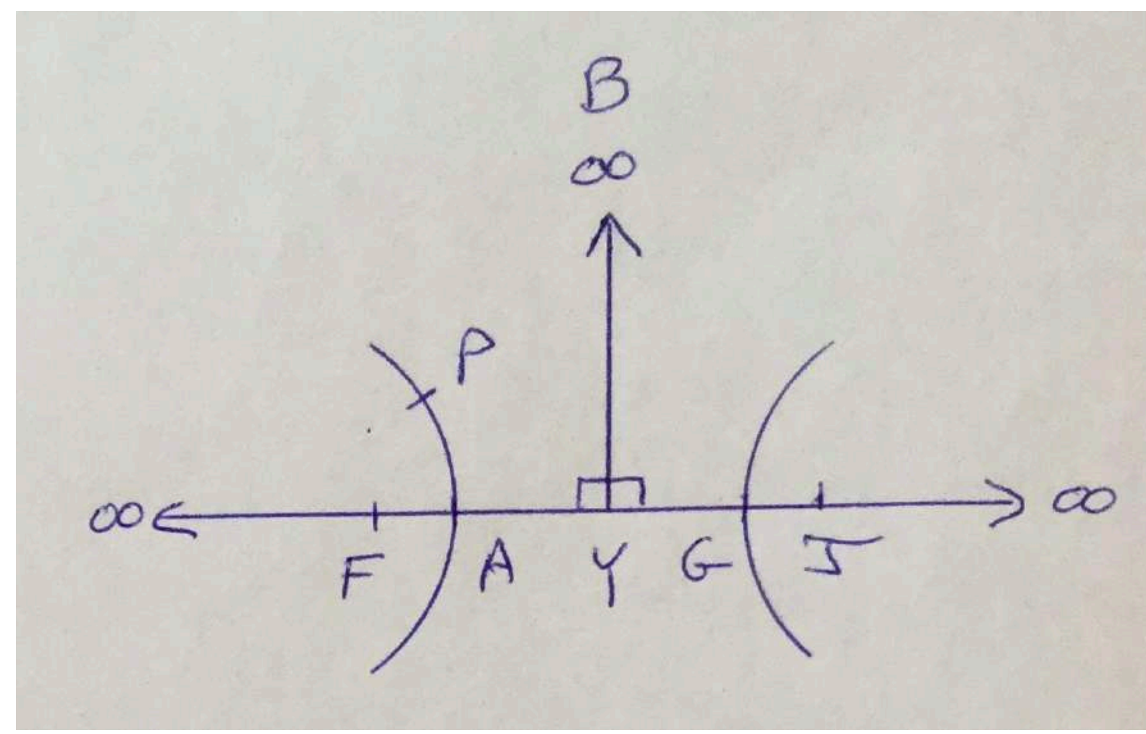
However, the infinitely large reference circle must then rotate by π radians in either direction to remain a circle equally divided by an infinitely long upward ray with its base on the axis; and it is this common perspective that allows for an appreciation of the continuity of these curves without using solid geometry.



Draw: $0 < YF = YJ < \infty$
so that: $0 < e = YF/YA > 1$



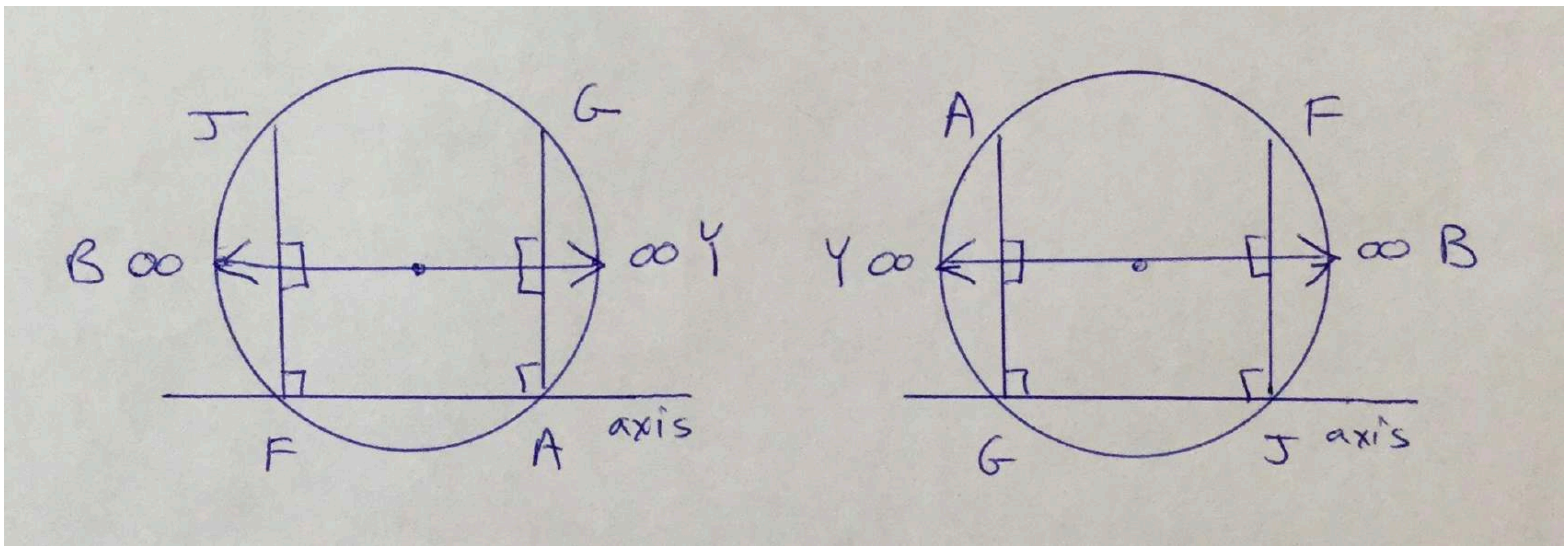
We will have drawn a defined hyperbola where AJ - AF = AG along its "transverse axis" FAYGJ, if it is also true that PJ - PF = AG.



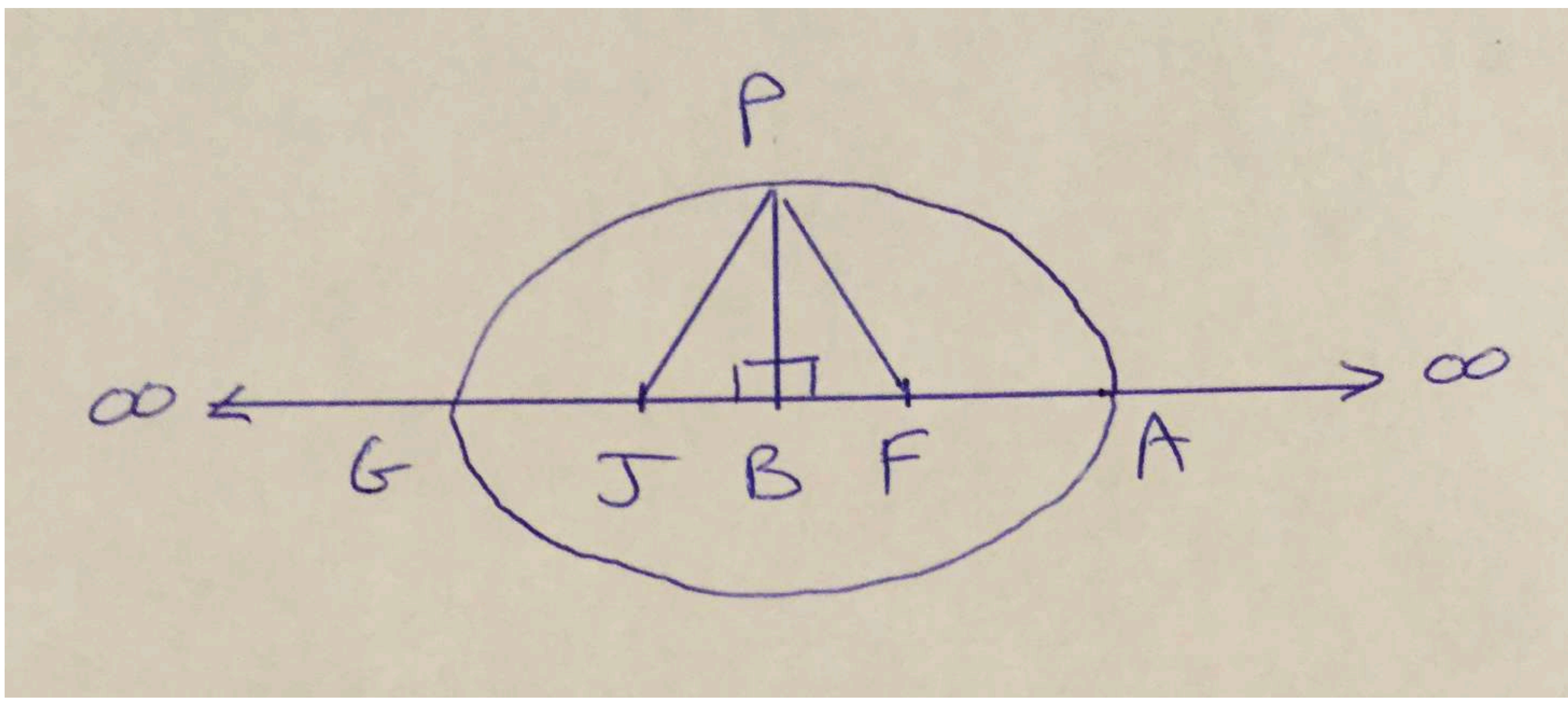
Parabola

When the infinitely large reference circle only rotates by π/2 radians in either direction, it no longer remains a circle equally divided by an infinitely long upward ray with its base on the axis. In fact, reference points B and Y are both infinitely far from the resulting curves they reference, and can be no longer used to specify the curves, or their eccentricity. However, due to the halfway rotation of the reference circle, we can assume either of the resulting curves would have an eccentricity halfway between that of an ellipse ($e < 1$), and that of an hyperbola ($e > 1$).

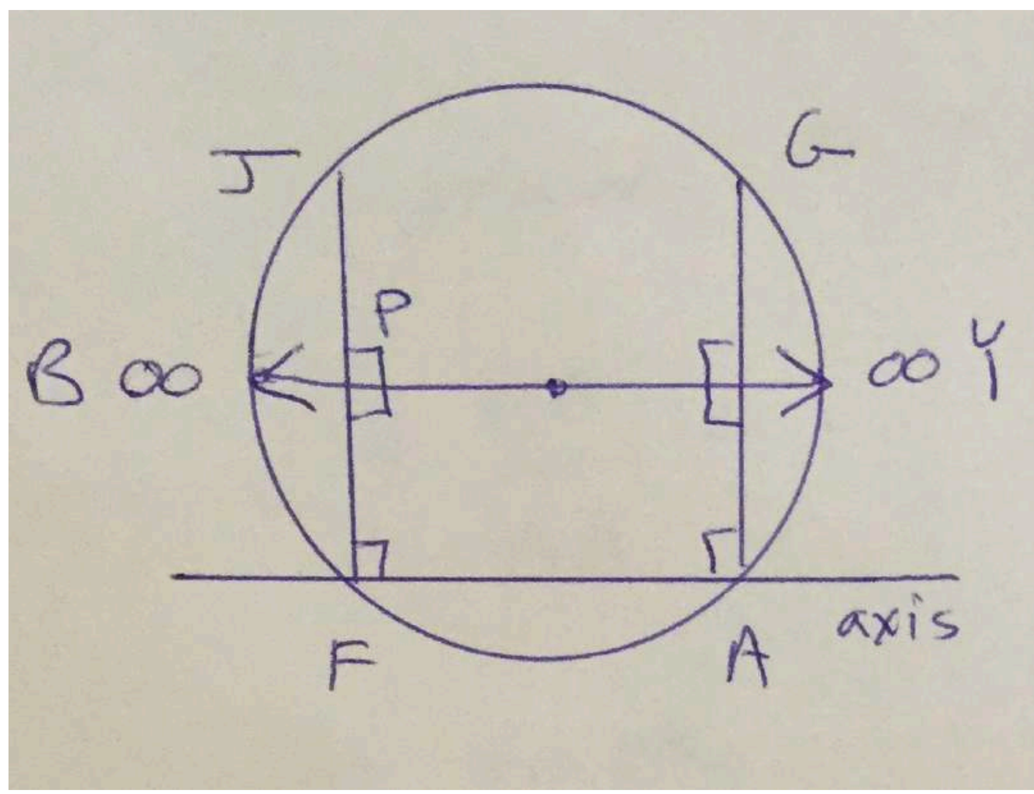
These resulting curves are defined as a parabolas (e = 1), and like the circle (e = 0), they represents a special case with a singular shape, or eccentricity.



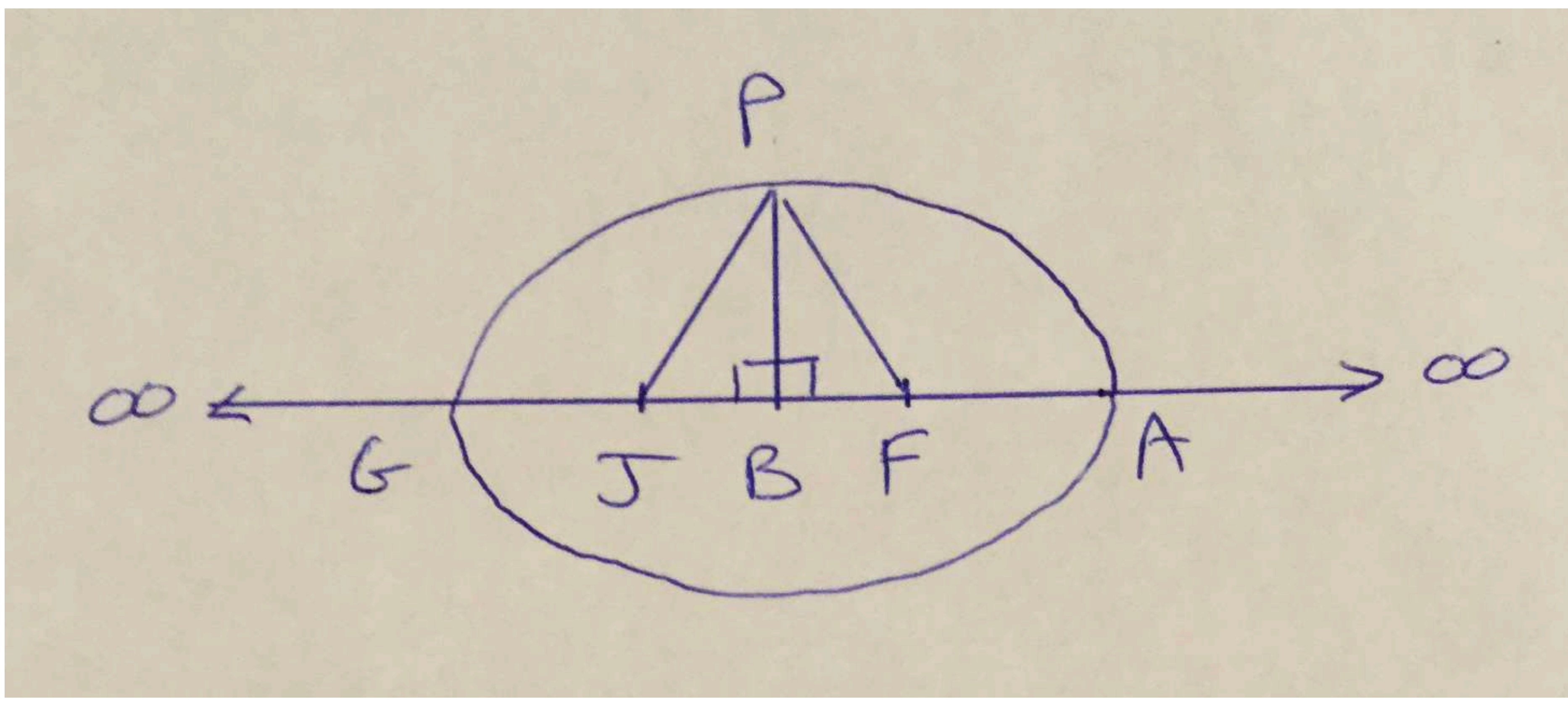
The path of an ellipse opens towards G from A only so long as PF and PJ become more equal. The ellipse remains open towards G as a parabola only as $PF = PJ \Rightarrow \infty$.



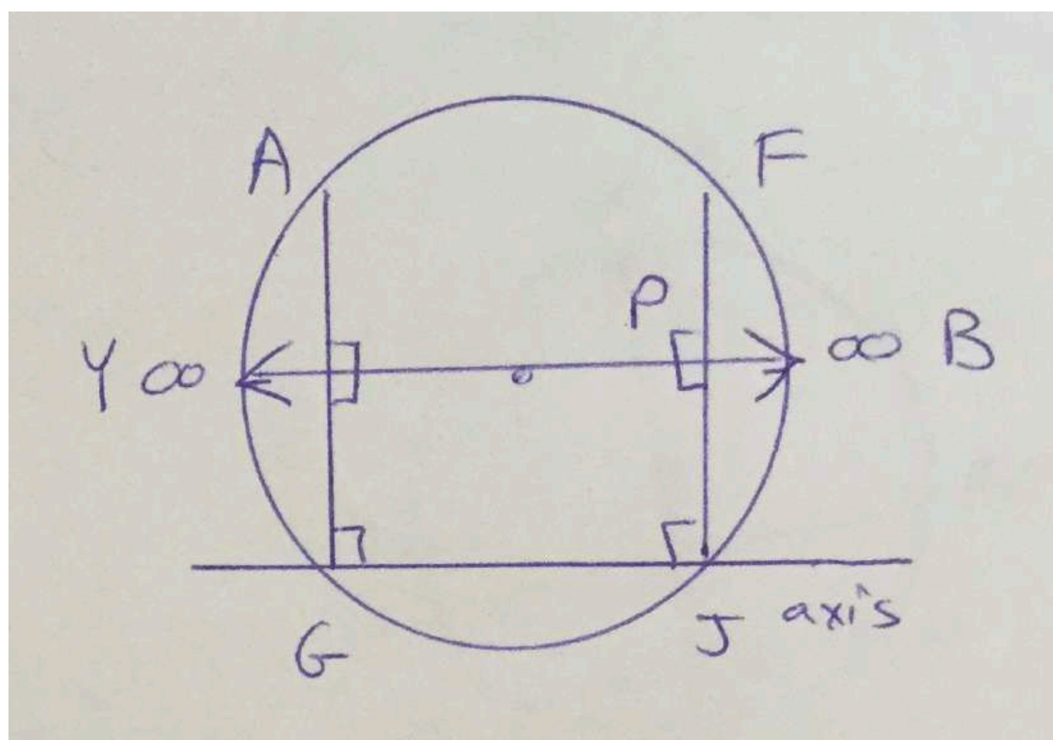
When P then moves from ∞ back toward A, this parabolic shape can be maintained using this knowledge by locating P on the following diagram halfway between F and J.



The path of an ellipse opens towards A from G only so long as PF and PJ become more equal. The ellipse remains open towards A as a parabola only as $PF = PJ \Rightarrow \infty$.

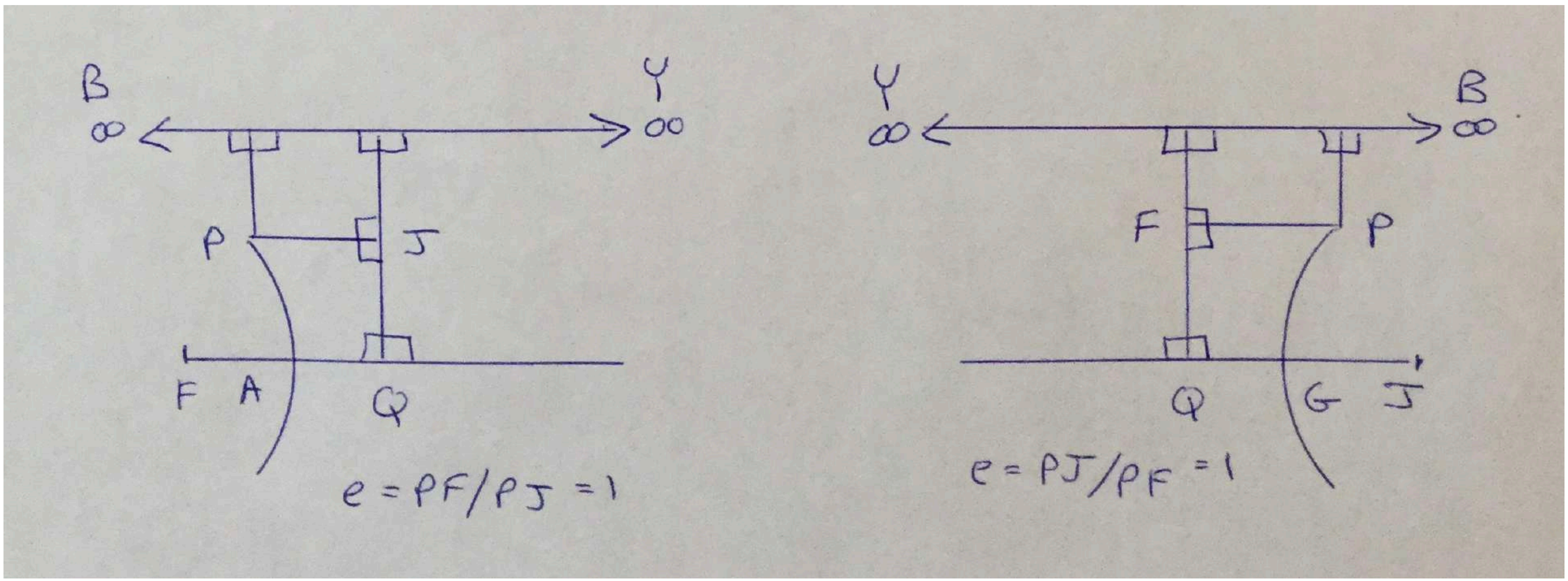


When P then moves from ∞ back toward G, this parabolic shape can be maintained using this knowledge by locating P on the following diagram halfway between F and J.



We have seen that as $PF = PJ$, and as $P \Rightarrow \infty$ from either A or G, an ellipse can transform into either of two parabolas opening toward each other with vertices at A or G, but this can not happen simultaneously. However, if we start with a hyperbola, and let $P \Rightarrow \infty$ from A, so that $PF \Rightarrow PJ$, we create two simultaneous parabolas facing away from each other.

QJ and QF can simply be chosen so that PF = PJ, and for these curves to be *simultaneously* true, Q must lie halfway between A and G. It is also obvious that FA must equal AQ, and that JG must equal GQ.



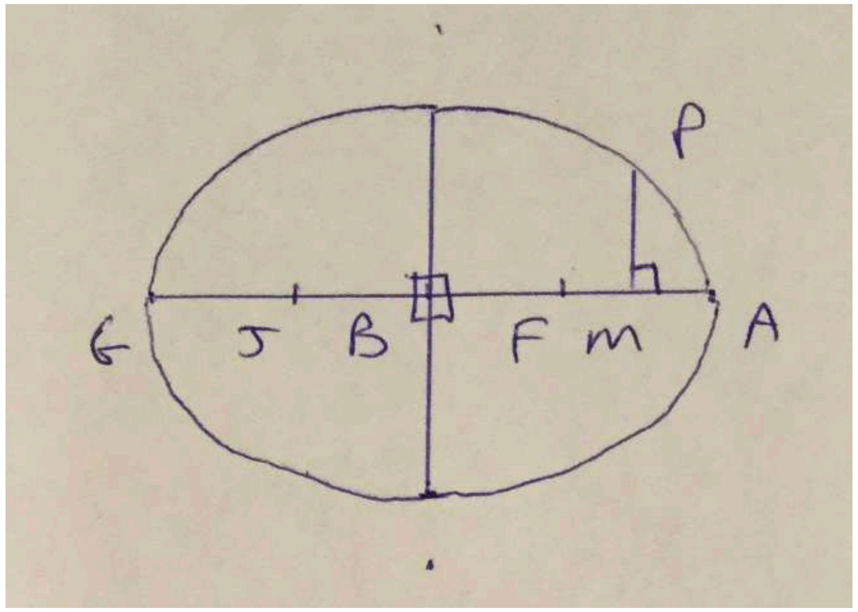
Characteristics of Conic Sections /Synthetic Geometry

The following discussions reference the 1899 revised edition of “Plane and Solid Geometry,” by G. A. Wentworth. (Later editions of this text do not discuss these curves). This text is particularly useful when studying Isaac Barrow’s 1667 Optical Lectures*,* because it also uses the language of *synthetic* geometry, rather than analytical geometry.

Ellipse

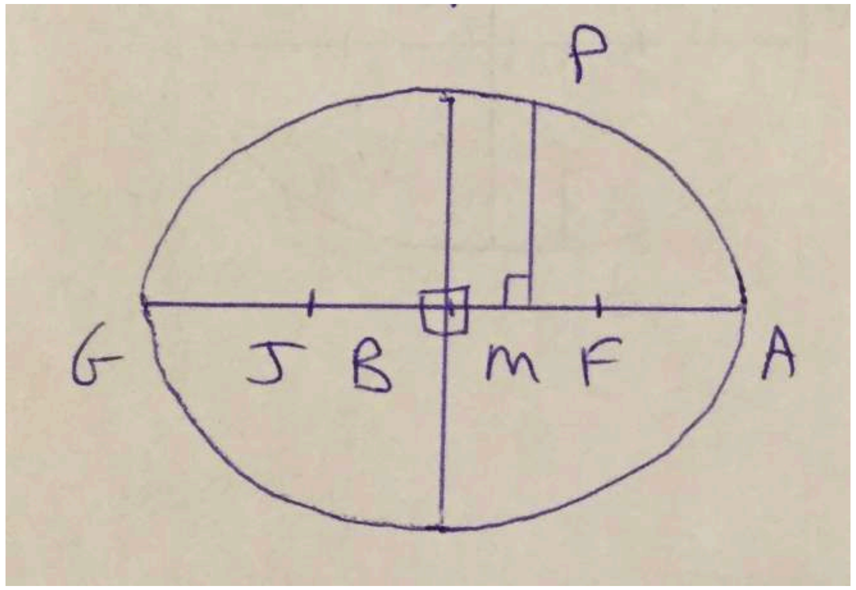
2(BF) = MJ - MF

2(BM) = MJ + MF



2(BF) = MJ + MF

2(BM) = MJ - MF



$$PJ^2 - FP^2 = (MJ^2 + MP^2) - (MF^2 + MP^2)$$

$$(PJ + FP)(PJ - FP) = (MJ + MF)(MJ - MF)$$

$$AG(PJ - FP) = 2(BM)\,2(BF)$$

$$PJ - FP = [2(BM)\,2(BF)]/2(BA)$$

$$\text{eccentricity} = e = BF/BA$$

$$PJ - FP = 2(BM)e$$

Since:

$$FP + PJ = AG = 2(BA)$$

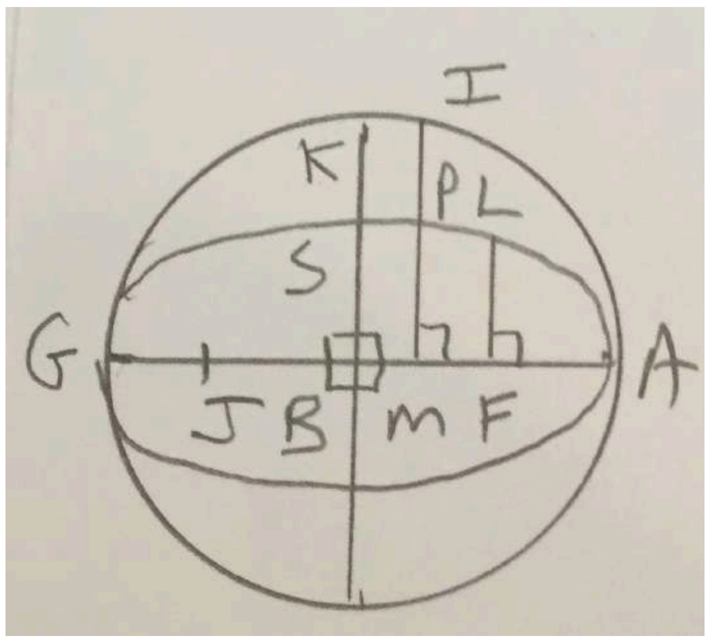
$$(FP + PJ) + (PJ - FP) = 2(PJ) = 2(BA) + 2(BM)e$$

$$(FP + PJ) - (PJ - FP) = 2(FP) = 2(BA) - 2(BM)e$$

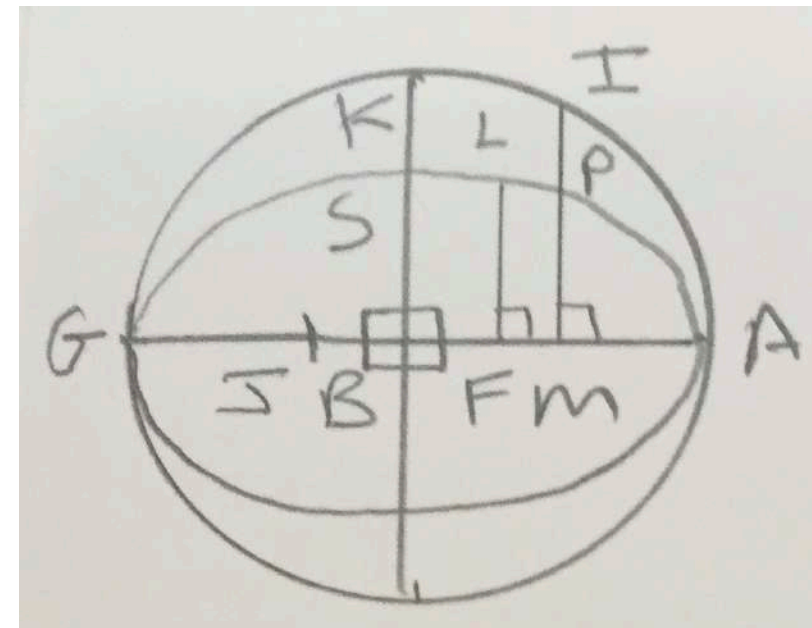
$$\mathbf{PJ = BA + (BM)e}$$

$$\mathbf{PF = BA - (BM)e}$$

$FM = BF - BM$



$FM = BM - BF$



$$FM^2 = BF^2 + BM^2 - 2(BF)BM$$

$$e = BF/BA = FB/FS$$

$$BA^2 = BF^2 + BS^2$$

$PF^2 = [BA - (BM)e]^2$

$PF^2 = BA^2 + (BM)^2e^2 - 2(BM)BF$

$PM^2 = PF^2 - FM^2$

$PM^2 = [BA^2 + (BM)^2e^2 - 2(BM)BF]$

$- [BF^2 + BM^2 - 2(BF)BM]$

$PM^2 = BS^2 + BM^2(e^2 - 1)$

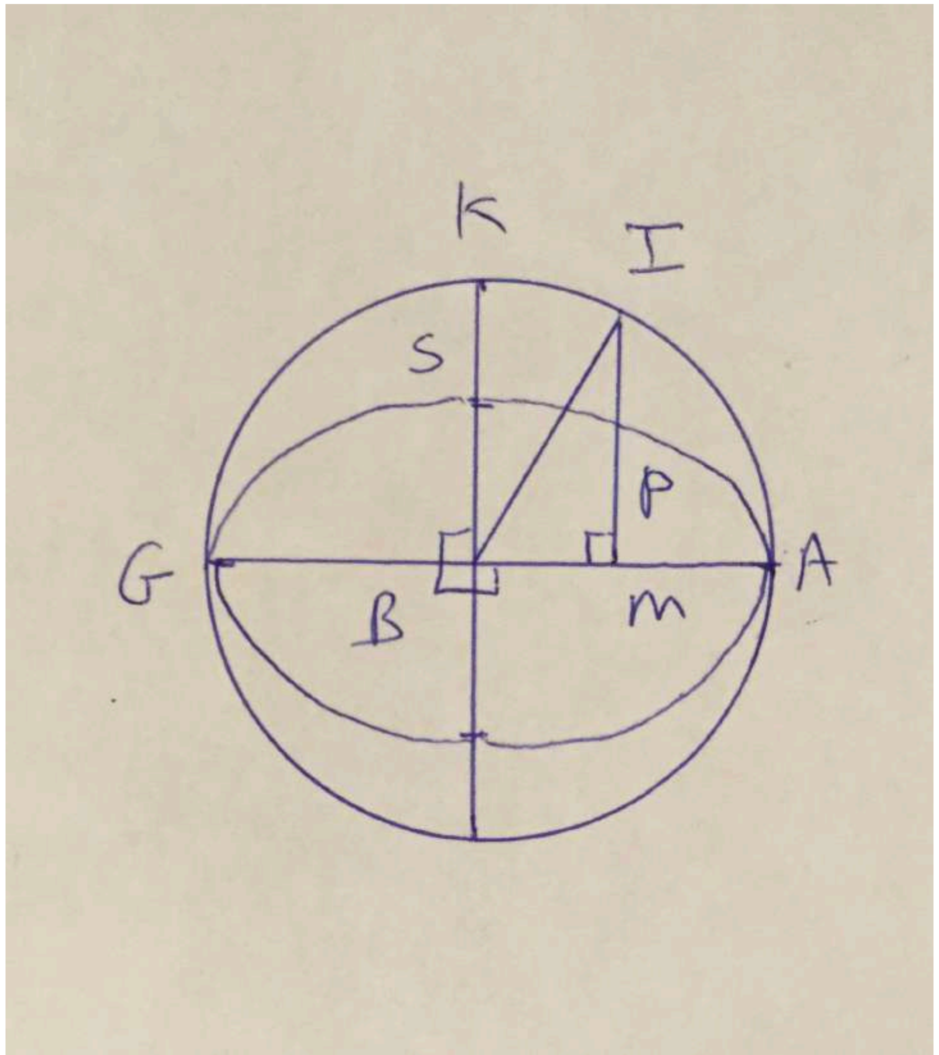
$PM^2 = BS^2 - BM^2(1-e^2)$

$(PM)^2BA^2 = (BS)^2BA^2 - BM^2[BA^2 - BF^2]$

$(PM)^2BA^2 = BS^2[BA^2 - BM^2]$

$(MP/MI)^2 = (BS/BA)^2$

$MP/MI = BS/BK$



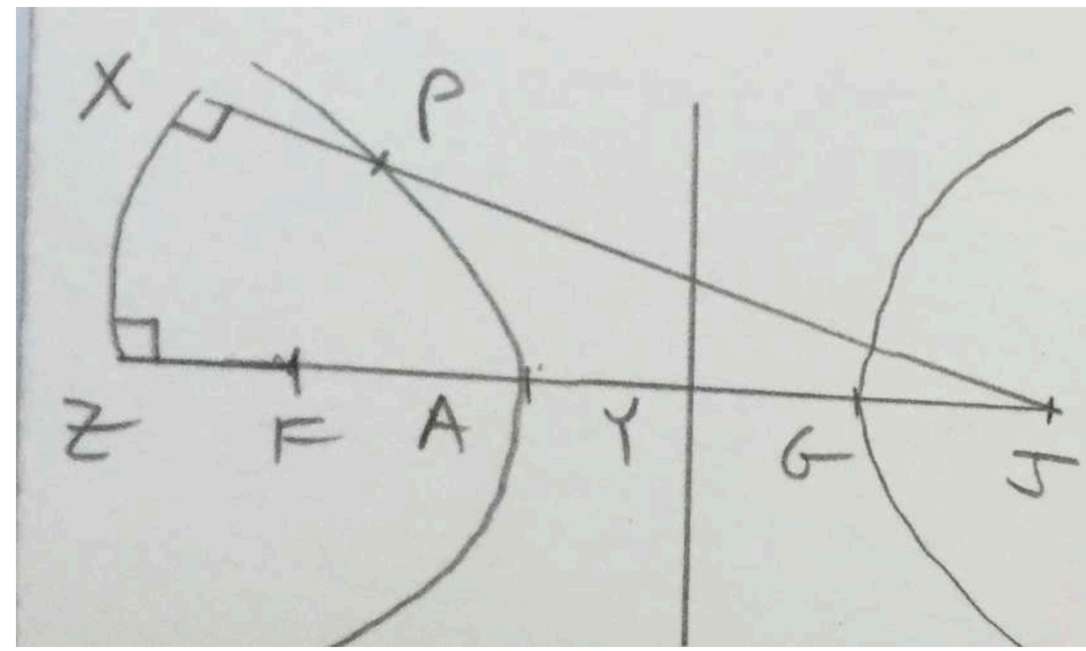
Hyperbola

Draw hyperbola arm AP:

Make: ZJ - AG = XP + FP

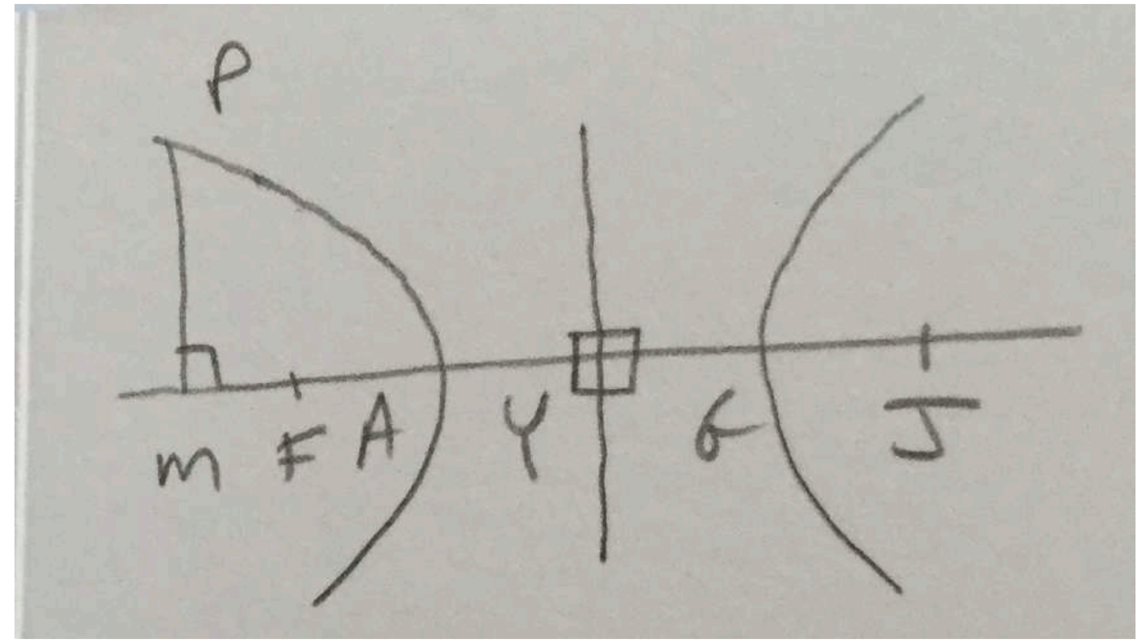
So: XJ - XP = FP + AG

and PJ - FP = AG



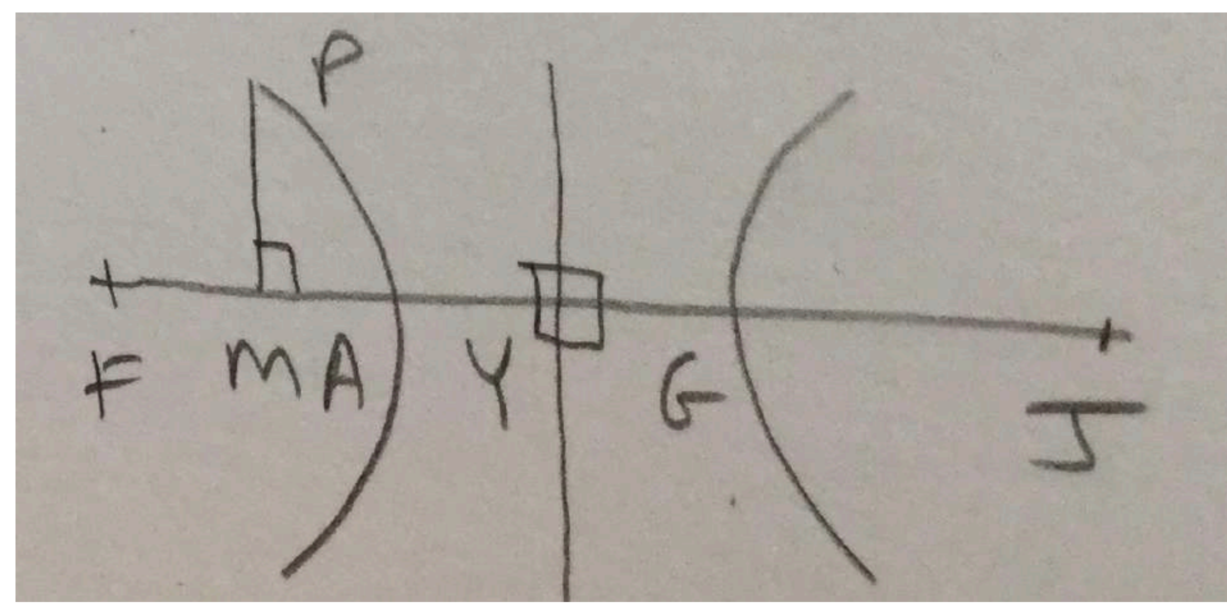
MJ - MF = 2(YF)

MJ + MF = 2(YM)



MJ - MF = 2(YM)

MJ + MF = 2(YF)



$$PJ^2 - FP^2 = (MP^2 + MJ^2) - (MP^2 + MF^2)$$

$$(PJ + FP)(PJ - FP) = (MJ + MF)(MJ - MF)$$

$$(PJ + FP)AG = 2(YM)\,2(YF)$$

$$PJ + PF = [2(YM)\,2(YF)]/2(YA)$$

eccentricity = $e = YF/YA$

$$PJ + PF = 2(YM)e$$

Since: $PJ - PF = AG = 2(YA)$

$(PJ + PF) + (PJ - PF) = 2(PJ) = 2(YM)e + 2(YA)$

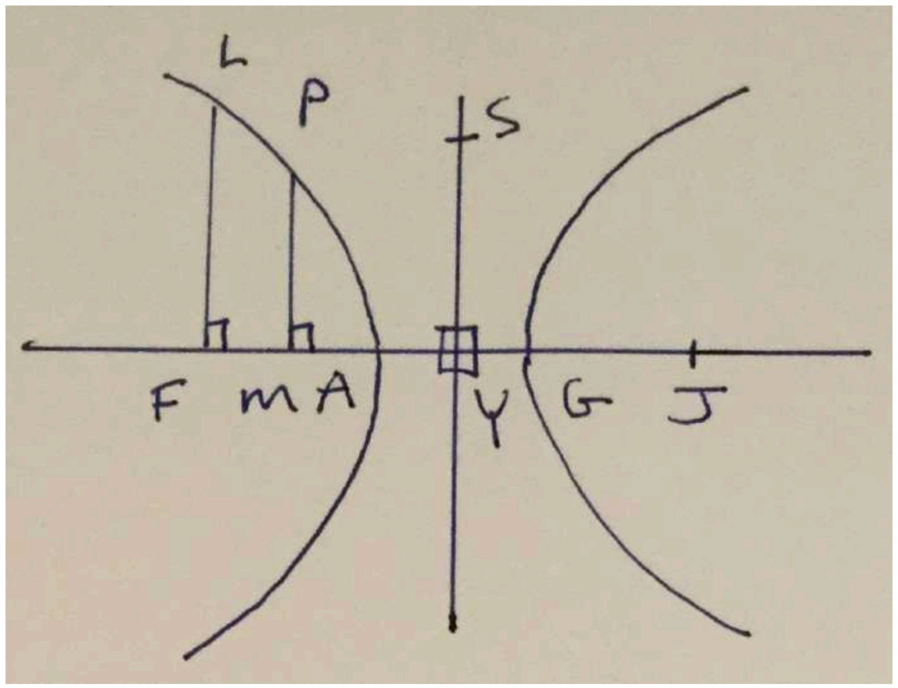
$(PJ + PF) - (PJ - PF) = 2(PF) = 2(YM)e - 2(YA)$

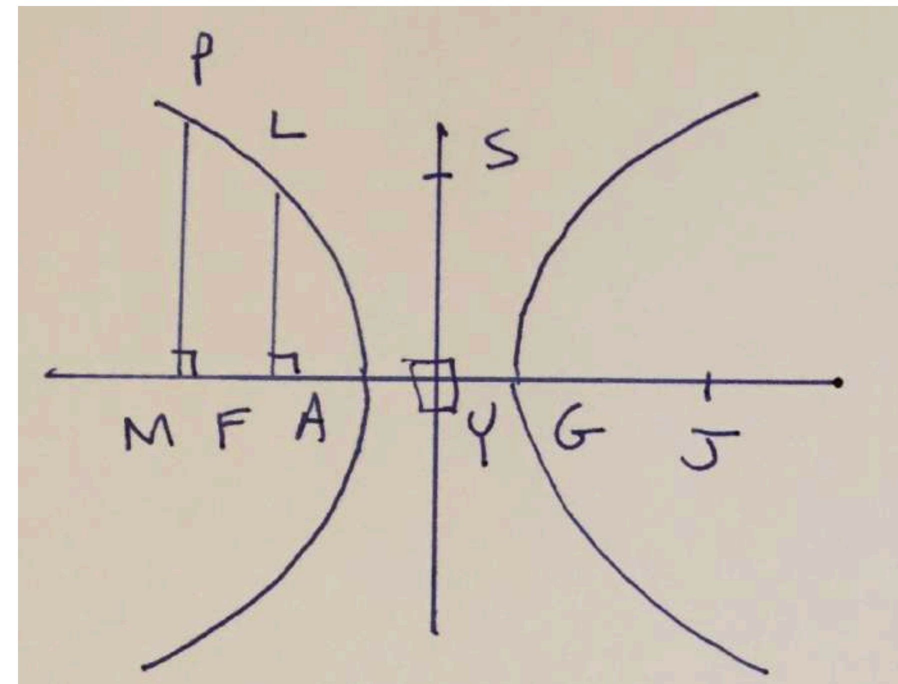
$PJ = (YM)e + YA$

$PF = (YM)e - YA$

FM = YF - YM

FM = YM - YF





$$FM^2 = YF^2 + YM^2 - 2(YF)YM$$

$$e = YF/YA = AS/AY$$

$$YF^2 = YA^2 + YS^2$$

$$PF^2 = [(YM)e - YA]^2$$

$$PF^2 = YM^2e^2 + YA^2 - 2(YM)YF$$

$$PM^2 = PF^2 - FM^2$$

$$PM^2 = [YM^2e^2 + YA^2 - 2(YM)YF]$$

$$- [YF^2 + YM^2 - 2(YF)YM]$$

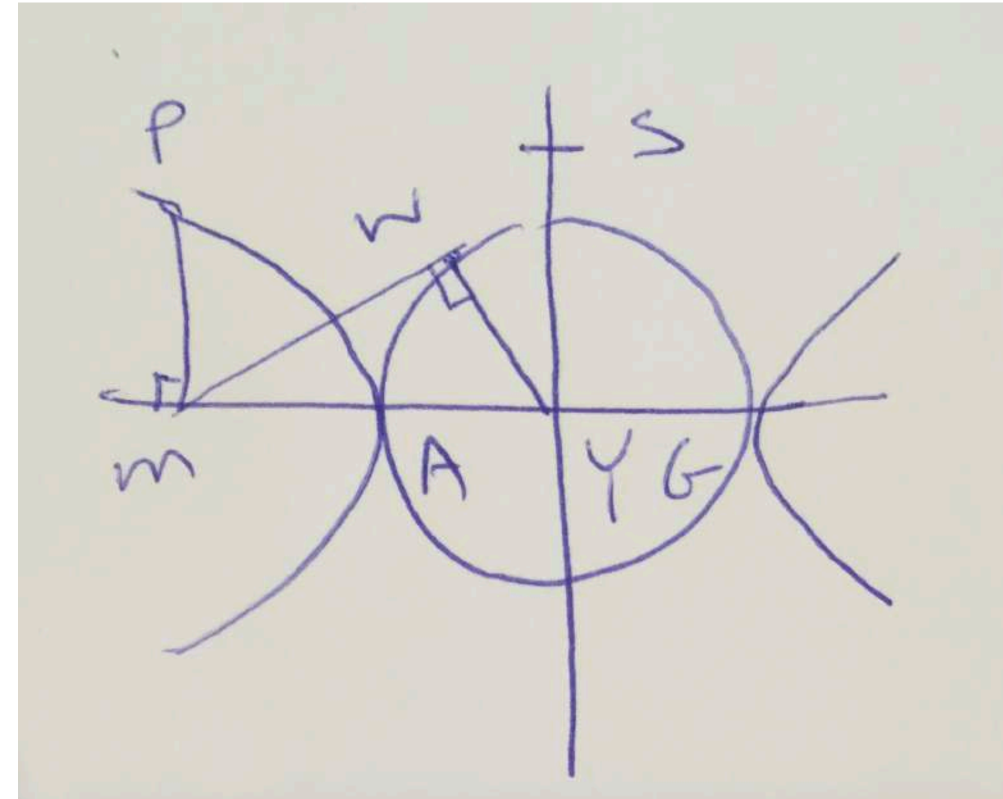
$$PM^2 = YM^2(e^2 - 1) - YS^2$$

$$PM^2 \, YA^2 = YM^2[YF^2 - YA^2] - YS^2 \, YA^2$$

$$PM^2 \, YA^2 = YS^2(YM^2 - YA^2)$$

$$(MP/MW)^2 = (YS/YA)^2$$

$$MP/MW = YS/YA$$



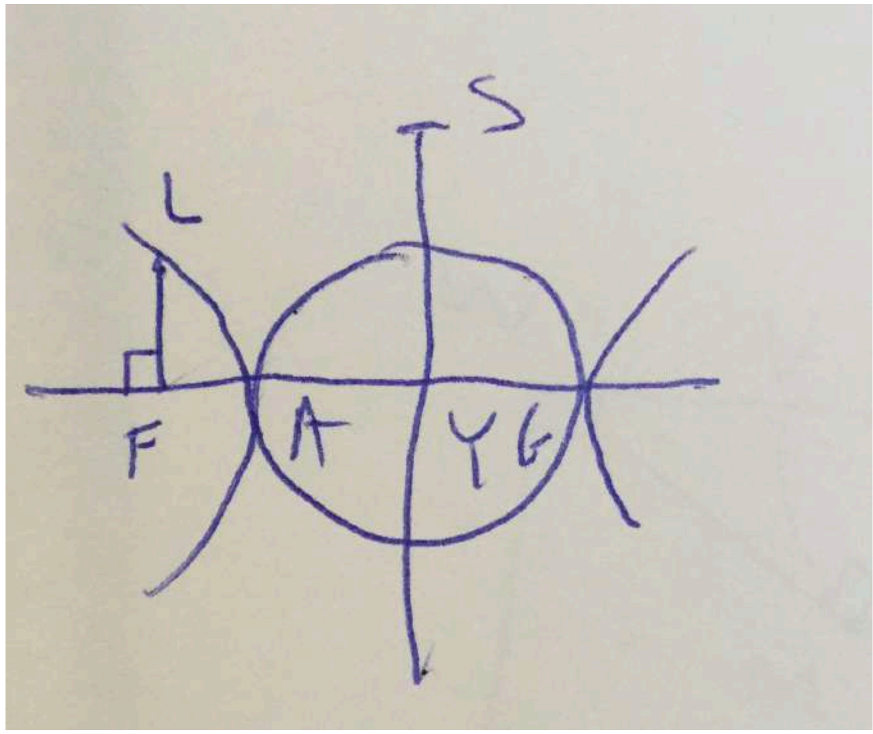
$$MW^2 = (MA)MG$$

$$MP^2/(MA)MG = (YS/YA)^2 = FL^2/(FA)FG$$

$$(FA)FG = (YF - YA)(YF + YA)$$

$$(FA)FG = YF^2 - YA^2 = YS^2$$

$$FL/YS = YS/YA$$



Plano Refraction

The following discussion references H. C. Fay's 1987 English translation of *Isaac Barrow's Optical Lectures 1667,* edited by A. G. Bennett and D. F. Edgar; and published in 1987 by the "Worshipful Company of Spectacle Makers."

Image rays from plano refraction

Locating the plano image ray through an off-axis point

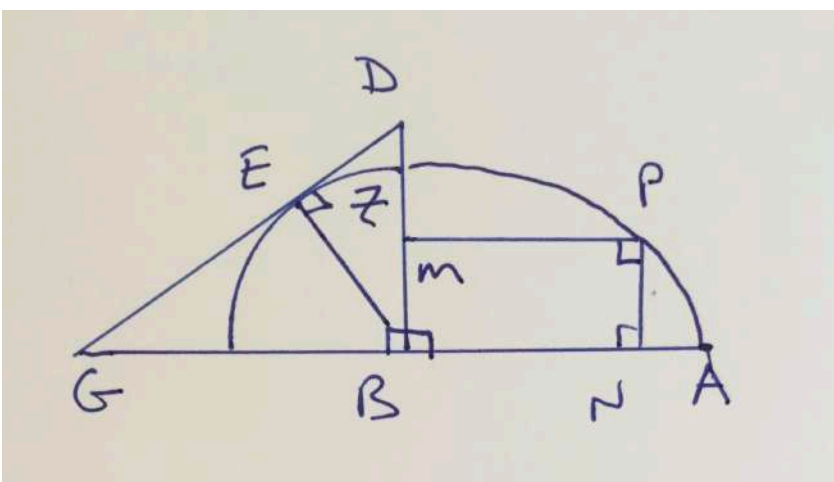
Locating the clear off-axis image from plano refraction

Image rays from plano refraction

The remainder of the discussion will be presented in two columns for clarity. The column on the right concerns when the object lies in air, and the column on the left when the object lies in glass. Each condition will be examined with the other in tandem, so that a quick glance across the columns will allow for a comparison, and reduce confusion.

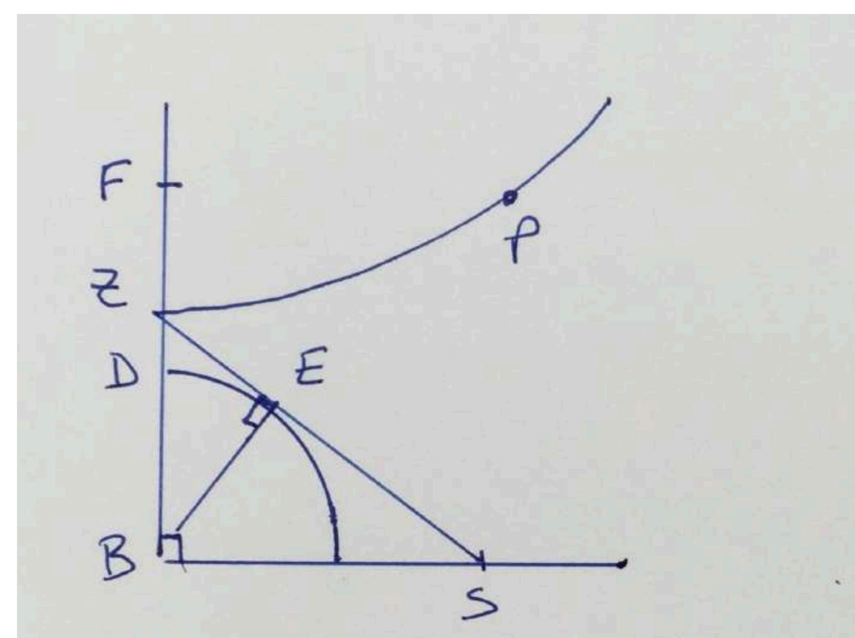
Object D in glass:

Given axial object distance DB, and axial image distance ZB:

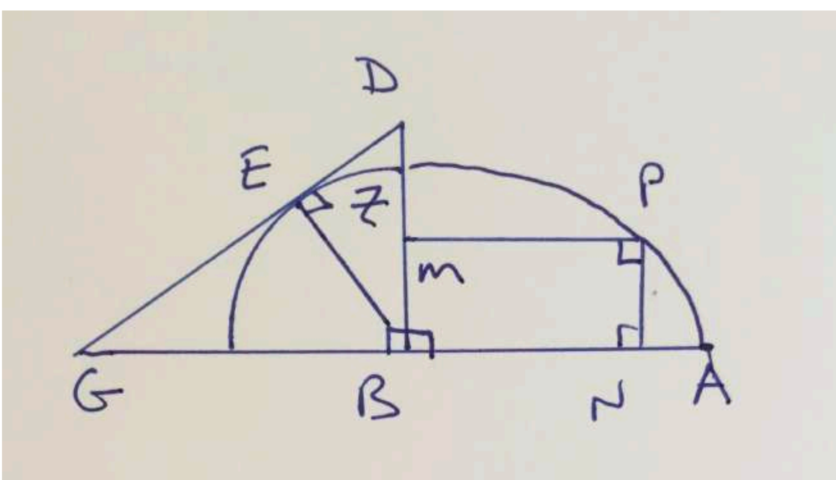


Object D in air:

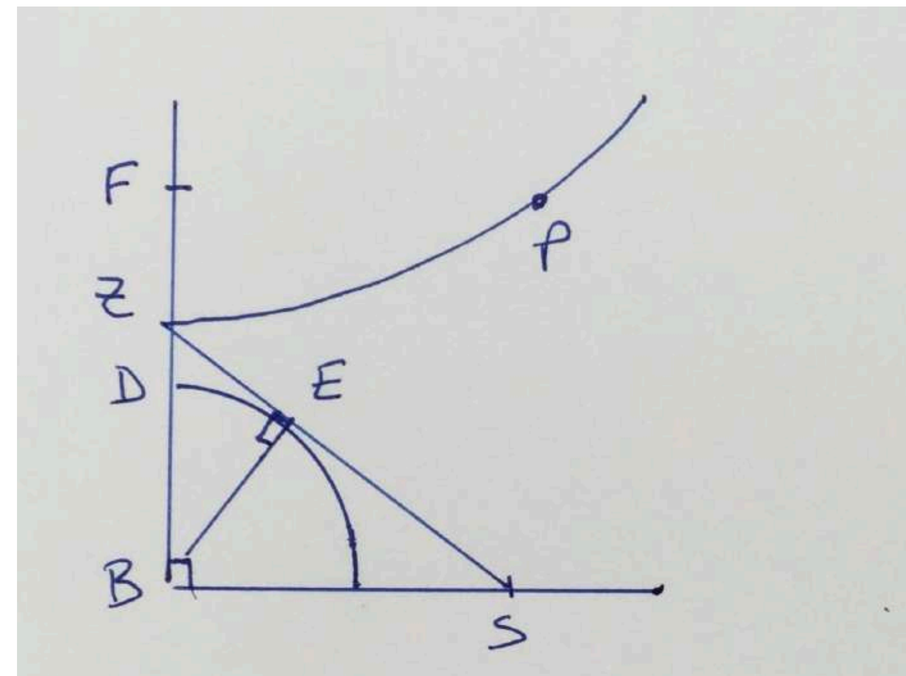
For ease of comparison, the axial point of refraction has been designated as B, rather than Y, even though the reference curve is a hyperbola rather than an ellipse.



we can find any off-axis refracted image ray (MN) using the reference semi-ellipse GZPA,

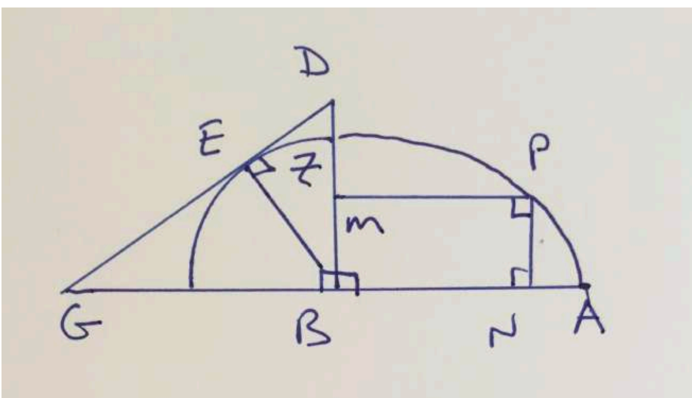


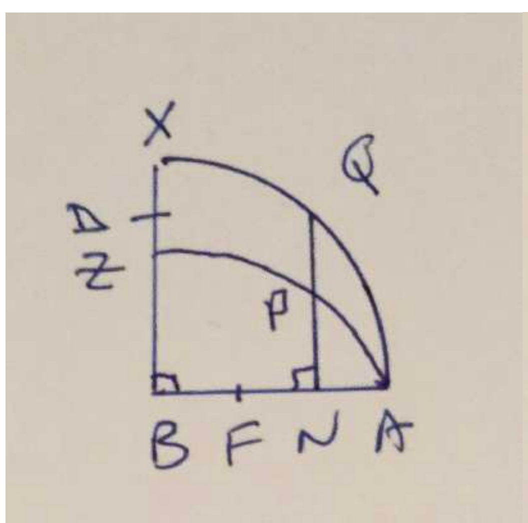
Given the axial object distance DB, and axial image distance ZB:

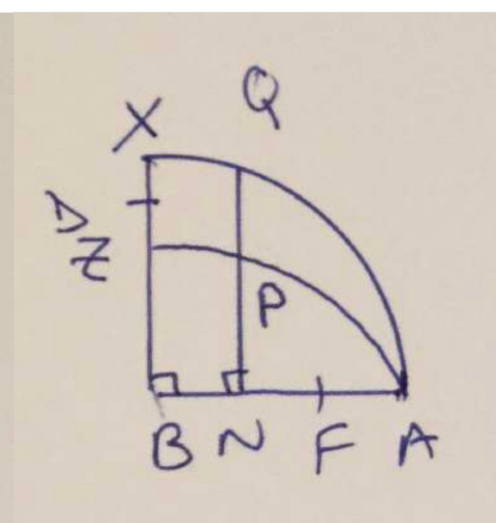


we can find any off-axis refracted image ray using the reference hyperbola arm ZP, If:

e = BF/BA = FB/FZ

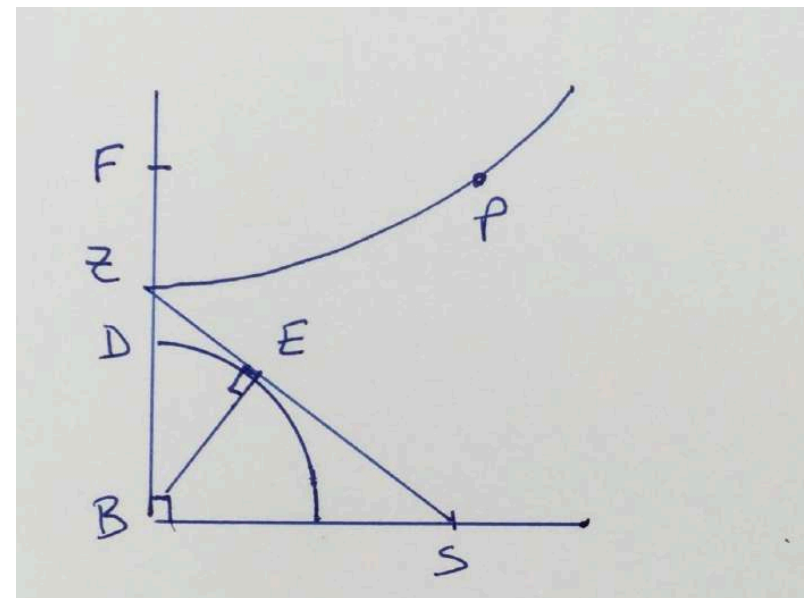






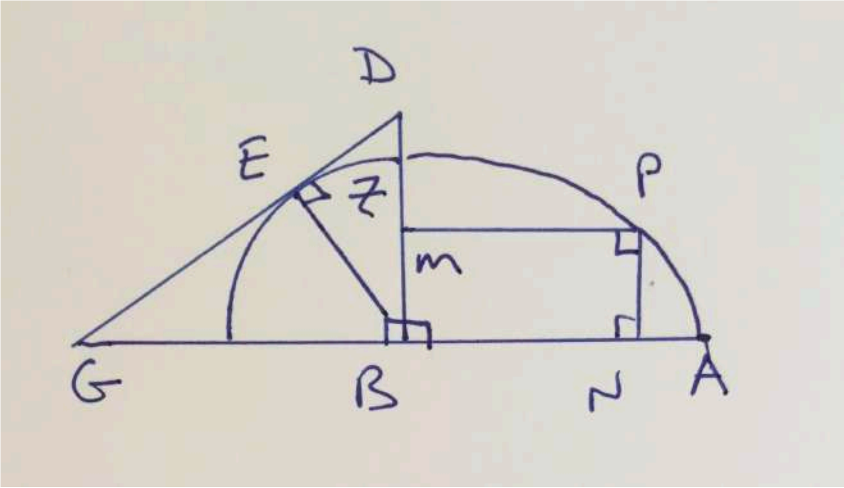
If:

e = BF/BZ = ZS/ZB
(e = ZB/ZE)



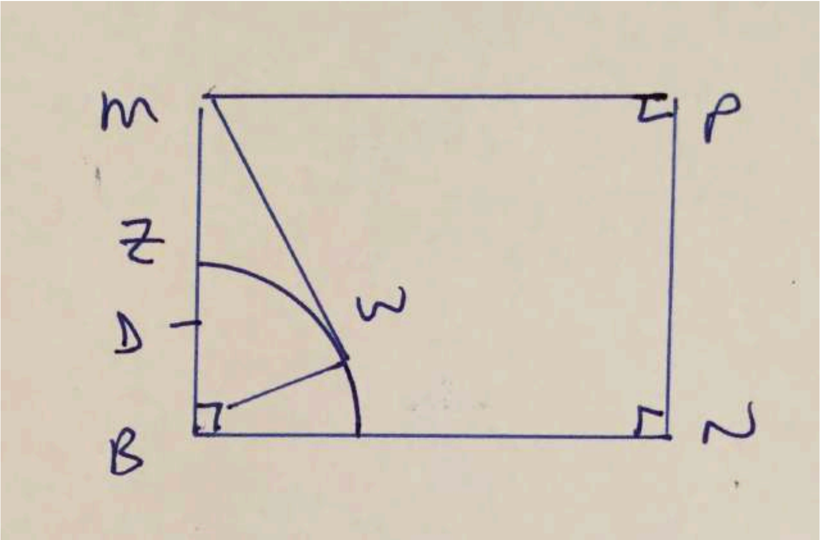
because an off-axis image ray MN is determined by:

MN/DN = BZ/BD



because an off-axis image ray MN is determined by:

MN/DN = BZ/BD



and for the ellipse:

$NQ/NP = BX/BZ$

$BZ^2/NP^2 = BA^2/(BA^2 - BN^2)$

$(BZ^2 - NP^2)/NP^2 = BN^2/(BA^2 - BN^2)$

$(BZ^2 - NP^2)/BN^2 = NP^2/(BA^2 - BN^2)$

$(BZ^2 - NP^2)/BN^2 = NP^2/NQ^2$
$= BZ^2/BG^2$

$BZ^2/BG^2 = BE^2/BG^2 = ED^2/BD^2$
$= (BD^2 - BZ^2)/BD^2$

and for the hyperbola:

$MW/MP = BZ/BS$

$MW^2/MP^2 = (MB^2 - ZB^2)/BN^2$

$BZ^2/BS^2 = EZ^2/EB^2$
$= (ZB^2 - DB^2)/DB^2$

$(MB^2 - ZB^2)/BN^2$
$= (ZB^2 - DB^2)/DB^2$

Since BD > BZ > BM

$(NP^2 - BZ^2)/BN^2$
$= (BZ^2 - BD^2)/BD^2$

$(MN^2 - BZ^2)/BN^2 = BZ^2/BD^2$

$(MN^2 - BZ^2)/BZ^2 = BN^2/BD^2$

$MN^2/BZ^2 = (BN^2 + BD^2)/BD^2$

$MN^2/DN^2 = BZ^2/BD^2$

$MN/DN = BZ/BD$

$(MB^2 - ZB^2 + BN^2)/BN^2$
$= BZ^2/BD^2$

$(MN^2 - BZ^2)/BZ^2$
$= BN^2/BD^2$

$MN^2/ZB^2 = DN^2/DB^2$

$MN^2/DN^2 = BZ^2/BD^2$

$MN/DN = BZ/BD$

Locating the plano image ray through an off-axis point

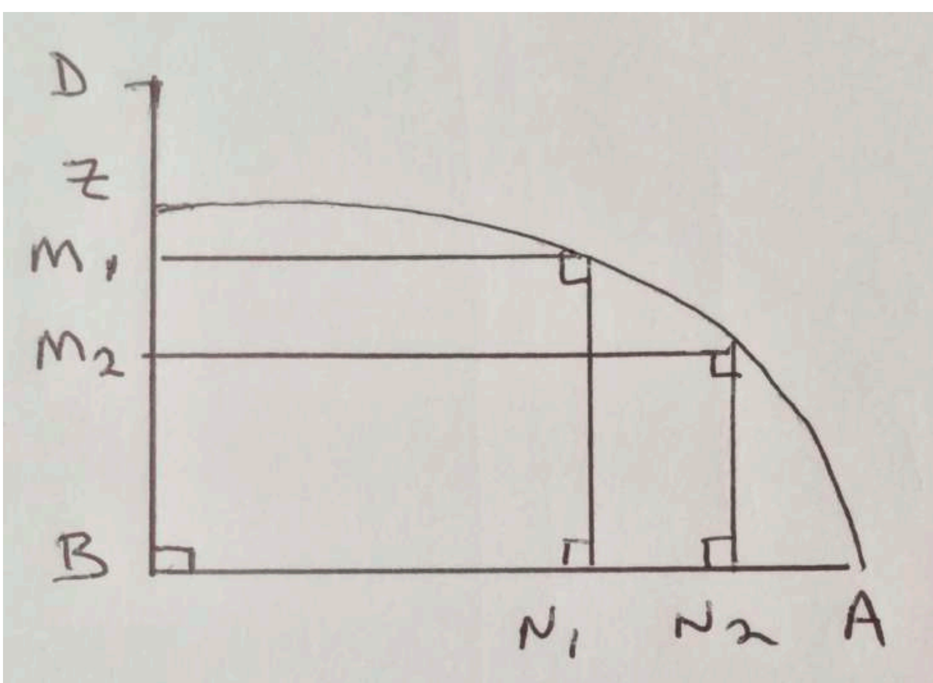
The following section examines how Isaac Barrow used a reference hyperbola to locate the plano image ray through an off-axis point.

Given object D and axial image Z:

$\mathbb{R} = BD/BZ$

$\mathbb{R} = N_1D/N_1M_1$

$\mathbb{R} = N_2D/N_2M_2$

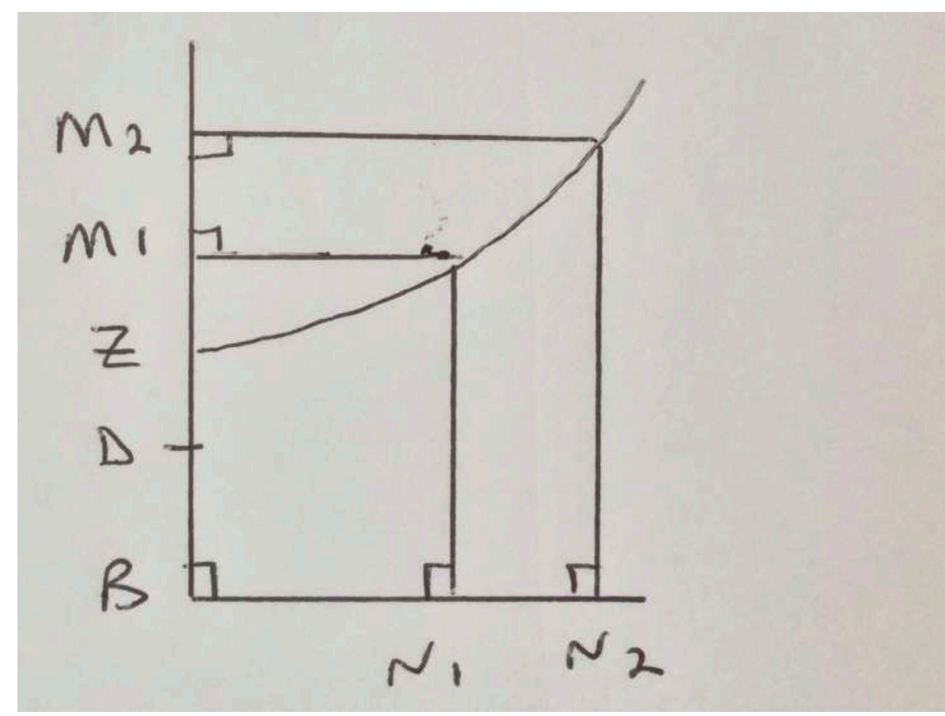


Given object D and axial image Z:

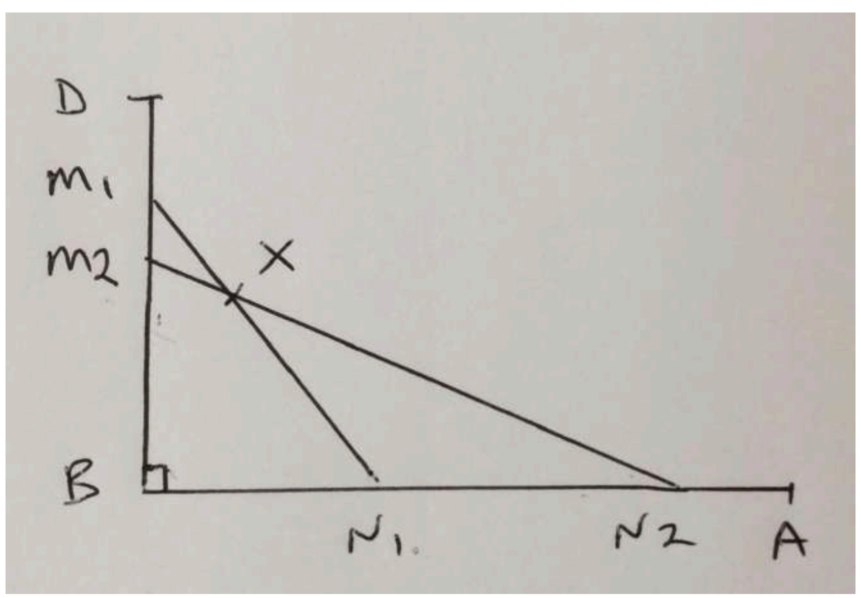
$\mathbb{R} = BZ/BD$

$\mathbb{R} = N_1M_1/N_1D$

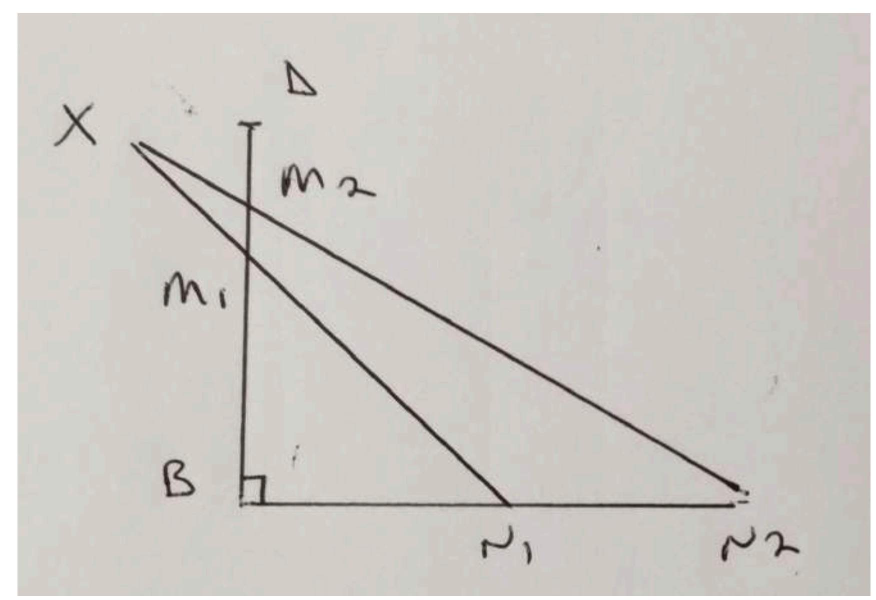
$\mathbb{R} = N_2M_2/N_2D$



Because the refraction is described by a reference ellipse through Z and A,
$BM_1 > BM_2$
and N_1M_1 crosses N_2M_2 at X within the right angle ∠DBA.

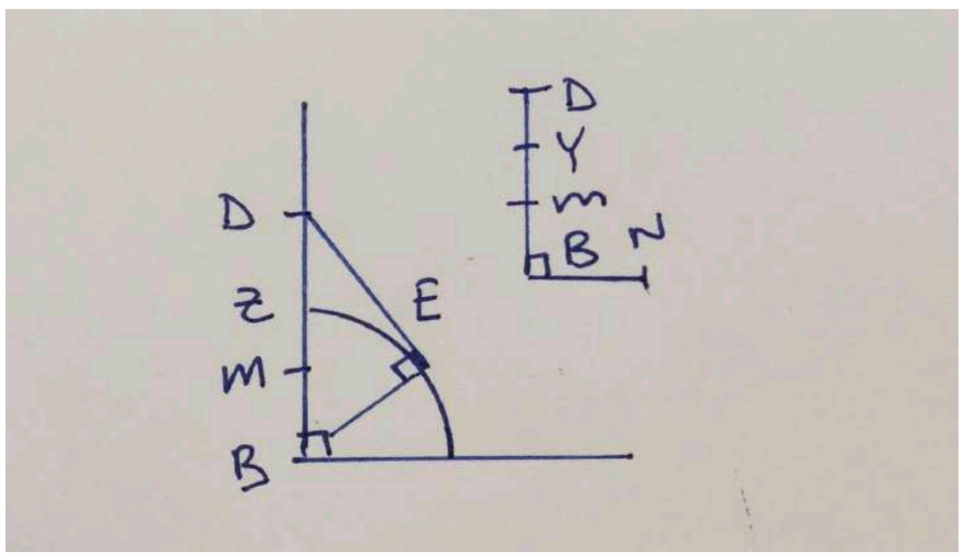


Because the refraction is described by a reference hyperbola through Z,
$BM_2 > BM_1$
and N_1M_1 crosses N_2M_2 at X outside the right angle ∠DBN_2.



Given object D and axial image Z:

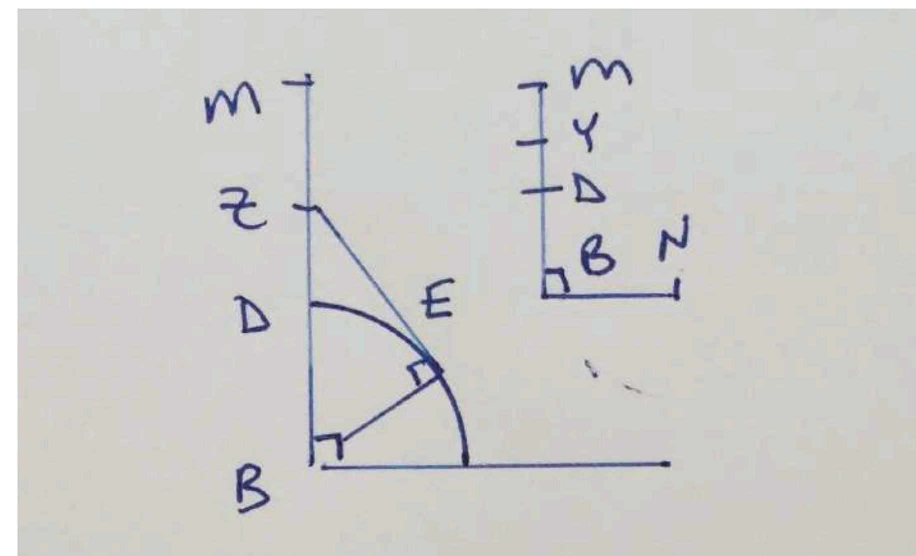
$\mathbb{R}$ = BD/BZ = ND/NM



We need to find a way to locate point N.

Given object D and axial image Z:

$\mathbb{R}$ = BZ/BD = NM/ND



We need to find a way to locate point N.

Of course, we can use the reference ellipse previously described, but we will need a different method when given a point on the image ray X, but not (M).

Of course, we can use the reference hyperbola previously described, but we will need a different method when given a point on the image ray X, but not (M).

To accomplish that, the following ratio manipulations prove that if:

BY/MB = DB/DE

then:

DB/YN = ED/EB

To accomplish that, the following ratio manipulations prove that if:

BY/DB = BZ/EZ

then:

MB/YN = EZ/EB

$$MB^2 = MN^2 - BN^2$$

$$MB^2 = MN^2 - YN^2 + BY^2$$

$$BY^2/(MN^2 - YN^2 + BY^2)$$
$$= DB^2/(DB^2 - BZ^2)$$
$$= DN^2/(DN^2 - MN^2)$$

$$BY^2/(YN^2 - MN^2) = DN^2/MN^2$$

$$BY^2 = YN^2 - BN^2$$

$$BY^2/DB^2 = BZ^2/(BZ^2 - EB^2)$$

$$BY^2/(BY^2 - DB^2) = BZ^2/DB^2$$

$$BZ^2/DB^2 = MN^2/DN^2$$

$$BY^2/MN^2 = (BY^2 - DB^2)/DN^2$$

$$(BY^2 + MN^2)/MN^2$$
$$= (BY^2 - DB^2 + DN^2)/DN^2$$

$$BY^2 = YN^2 - DN^2 + DB^2$$

$$(YN^2 - DN^2 + DB^2)/(YN^2 - MN^2)$$
$$= DN^2/MN^2$$

$$a/b = c/d;\ (a + c)/(b + d) = c/d$$

$$(YN^2 + DB^2)/YN^2 = DN^2/MN^2$$

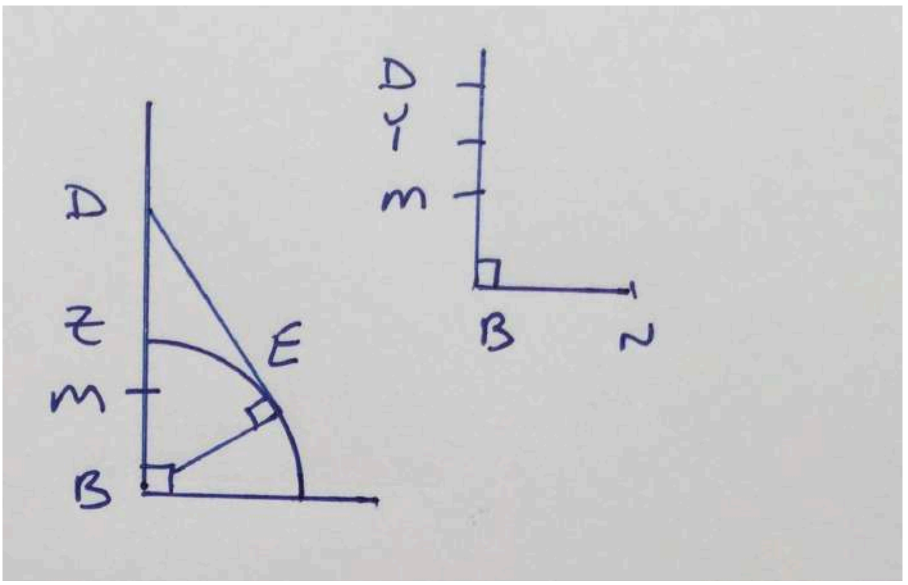
$$DB^2/YN^2 = (DN^2 - MN^2)/MN^2$$
$$= (DB^2 - BZ^2)/DB^2 = ED^2/EB^2$$

$$(BY^2 + MN^2)/(BY^2 + BN^2)$$
$$= MN^2/DN^2$$

$$(MN^2 - BN^2)/NY^2$$
$$= (MN^2 - DN^2)/DN^2$$

$$MB^2/YN^2 = (BZ^2 - DB^2)/DB^2$$
$$= EZ^2/EB^2$$

Therefore, given object D and axial image Z:



ℝ = DB/BZ = ND/NM

if: BY/MB= DB/DE
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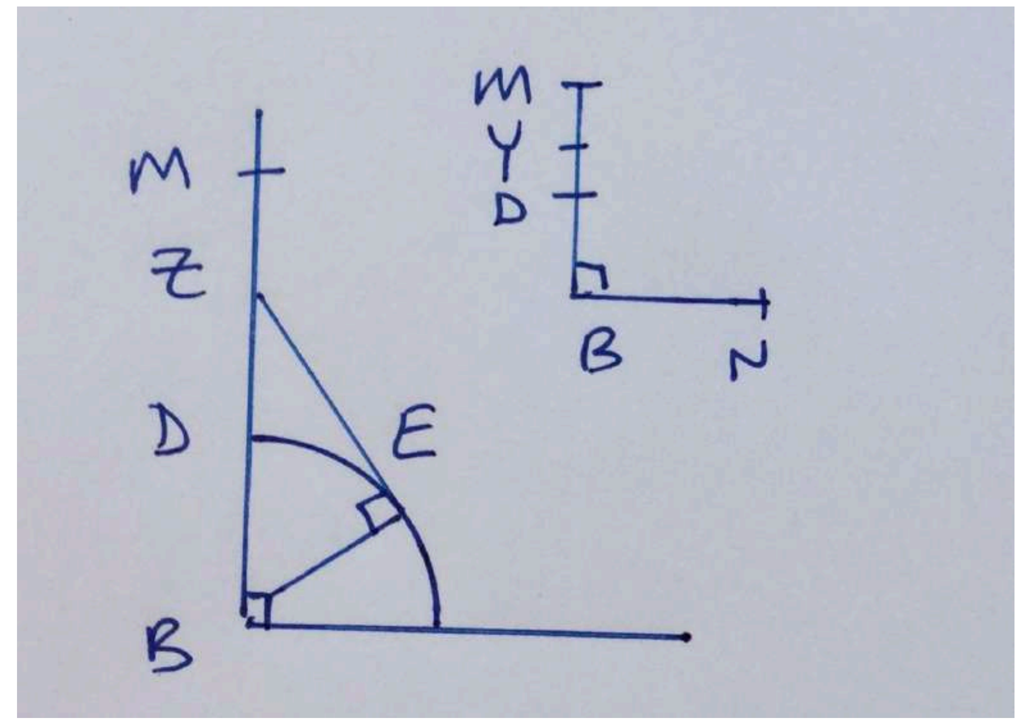
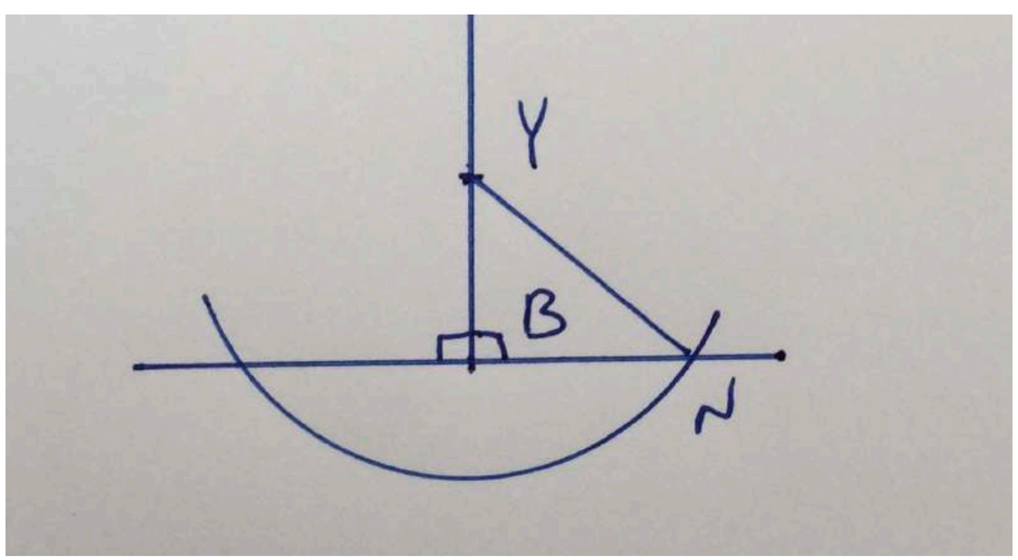


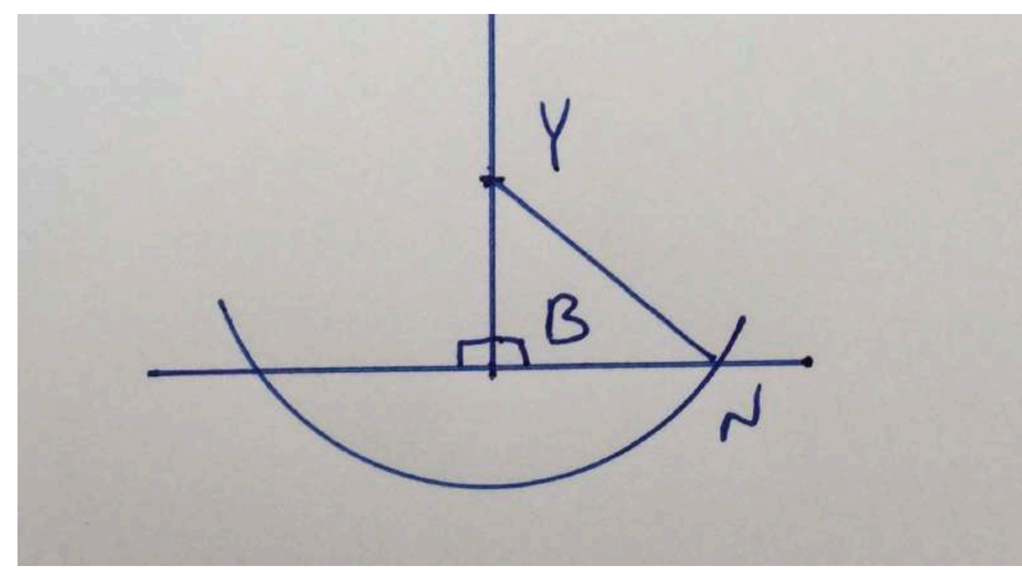
ℝ = BZ/DB= NM/ND

if: BY/DB= ZB/EZ
then: MB/YN= EZ/EB

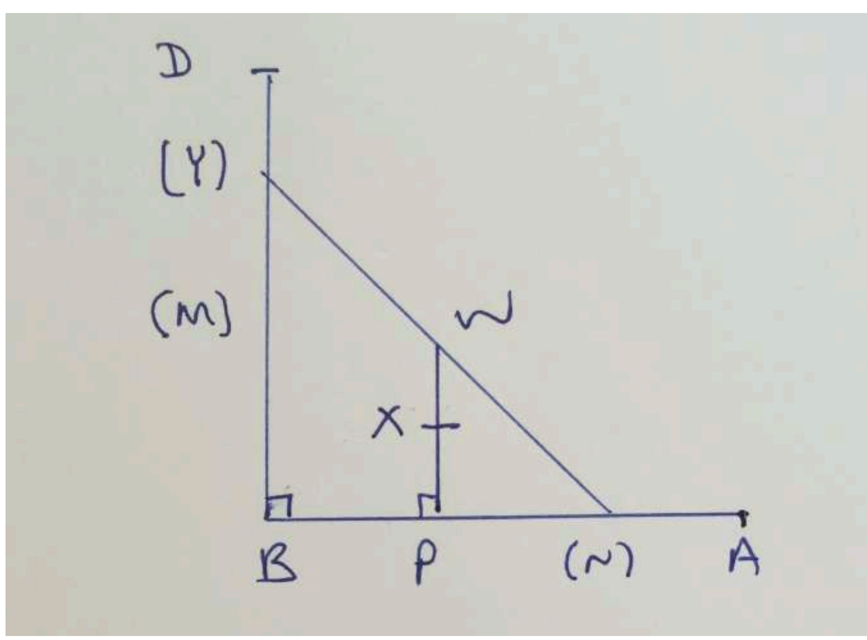
After calculating BY with known BM, (as well as known DB/DE); we can use known DB, (as well as known ED/EB), to calculate YN and use that as a radius about Y to find N:



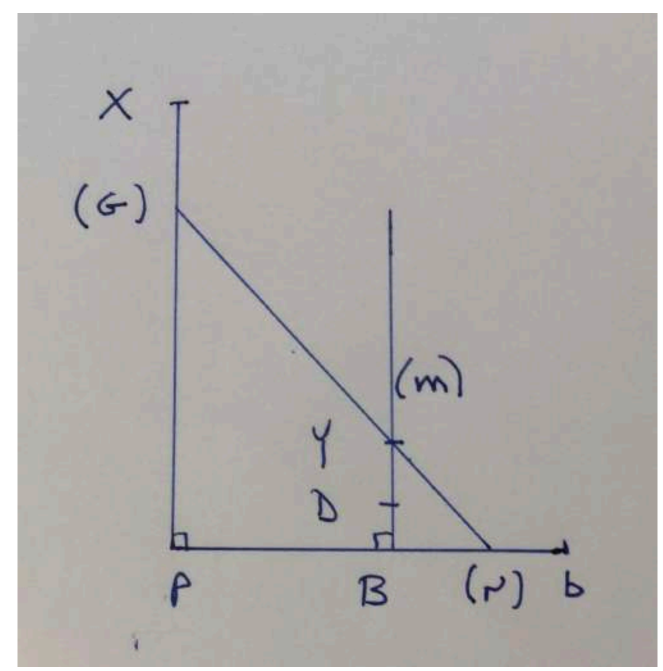
After calculating BY with known DB, (as well as known ZB/ZE); we can use known MB, (as well as known EZ/EB), to calculate YN and use that as a radius about Y to find N:



This method gives us no advantage over the previously described method. However, it will allow us to develop a way to find the line segment (M)X(N) without knowing (M), (or N).



This method gives us no advantage over the previously described method. However, it will allow us to develop a way to find the line segment X(M)(N) without knowing (M), (or N).



After calculating PW with known PX and DB/DE using:

PW/PX = (BY/MB) = DB/DE

since DB and ED/EB are also known:

DB/YWN = (WP/XN) = ED/EB

allows us to calculate the length of YWN, and we can then find (N) by inserting the calculated length YWN within the right angle $\angle$DBA through W.

After calculating BY with known DB and ZB/ZE using:

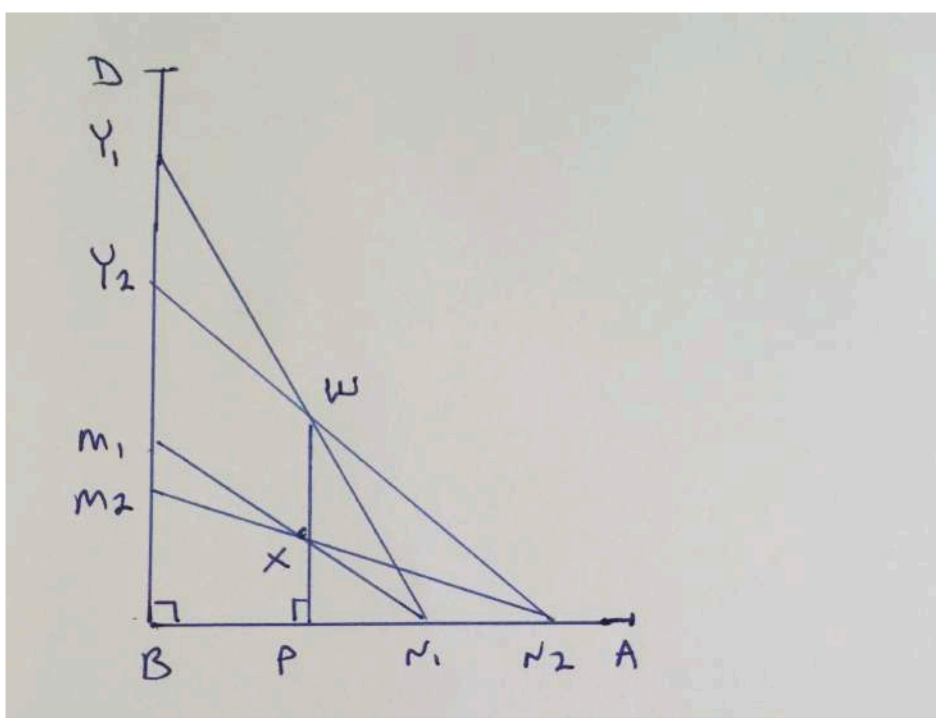
BY/DB = ZB/EZ

since PX and EZ/EB are also known:

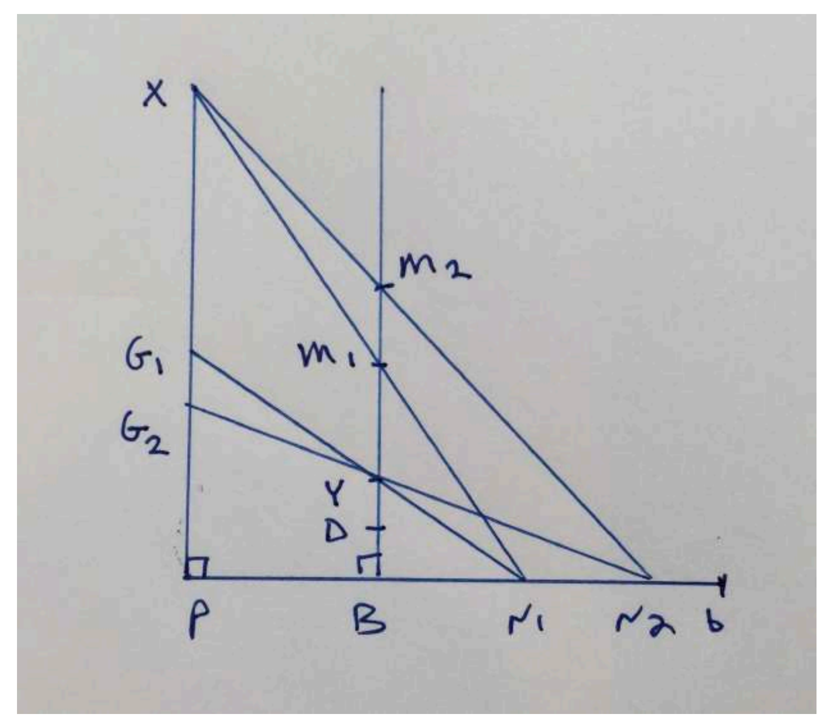
PX/GYN = (MB/YN) = EZ/EB

allows us to calculate the length of GYN, and we can then find (N) by inserting the calculated length GYN within the right angle $\angle$XPb through Y.

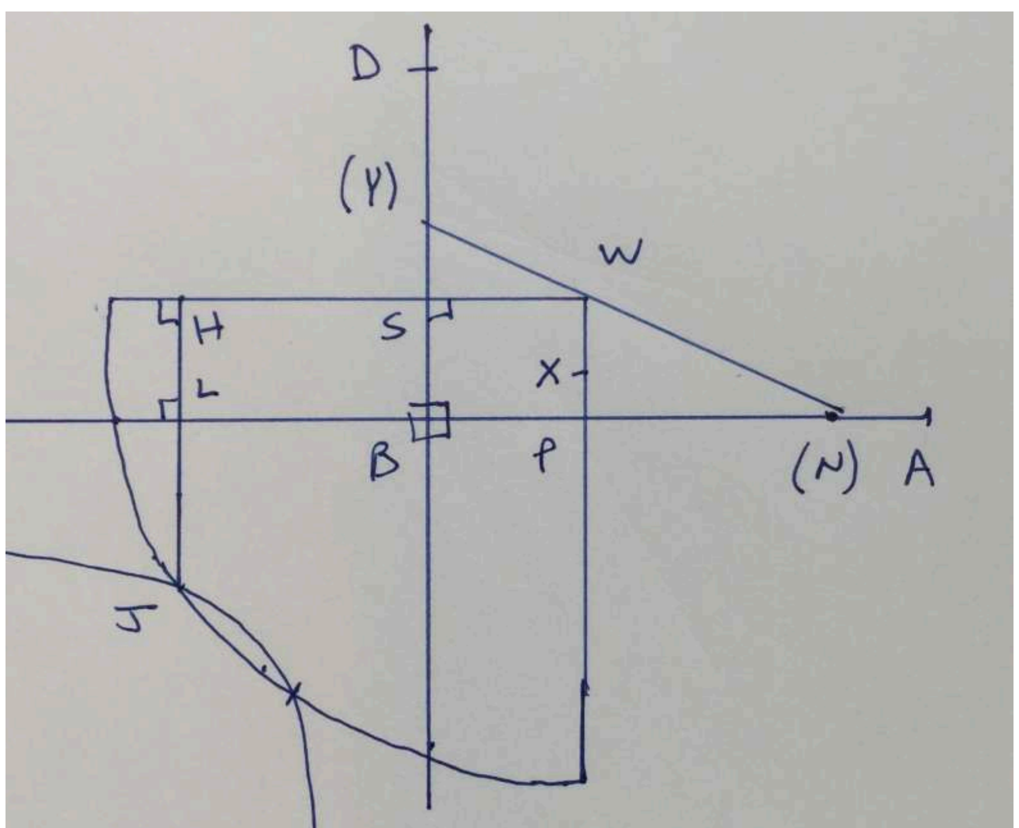
For any given calculated value of YWN, draw the maximum number of line segments, (two),
$Y_1WN_1 = Y_2WN_2$
though W, within the right angle ∠DBA to find both N_1 and N_2 for the image rays through X.



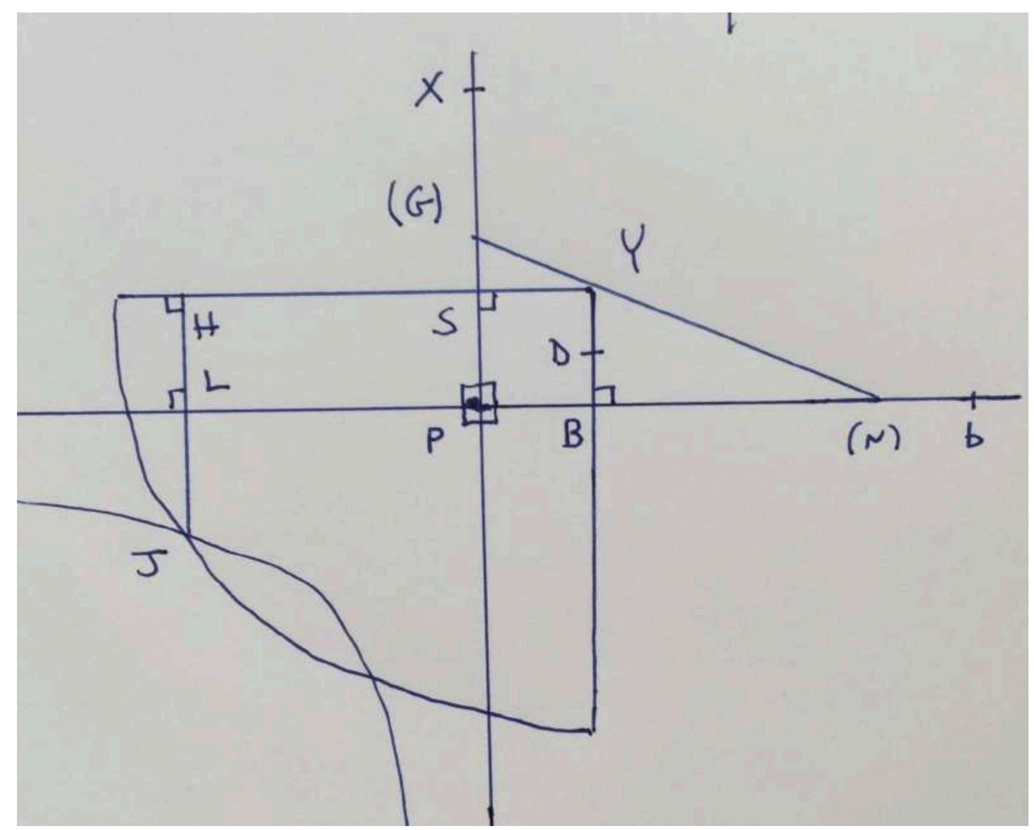
For any given calculated value of GYN, draw the maximum number of line segments, (two),
$G_1YN_1 = G_2YN_2$
though Y, within the right angle ∠XPb to find both N_1 and N_2 for the image rays through X.



To do this, consider the given point W to be on a reference hyperbola where:
(LB)LJ = (BP)PW
LB/PW= BP/LJ
and draw its opposite arm as shown:



To do this, consider the given point Y to be on a reference hyperbola where:
(LP)LJ = (BP)BY
PL/BY= BP/LJ
and draw its opposite arm as shown:



If we make radius WJ
equal the (calculated)
YWN, and construct:
P(N) = BL, then:
(Y)W(N) = WJ = YWN
because:

by construction:
P(N)/PW = BP/LJ

but:
P(N)/PW = SW/S(Y) so:
SW/S(Y) = BP/LJ.

If we make radius YJ
equal the (calculated)
GYN, and construct:
B(N) = PL, then:
(G)Y(N) = YJ = GYN
because:

by construction:
B(N)/BY = BP/LJ

but:
B(N)/BY = SY/S(G) so:
SY/S(G) = BP/LJ.

And since SW = BP:
S(Y) = LJ

S(Y) + SB = LJ + HL
B(Y) = HJ

and by construction:
B(N) = LP

since LP = HW:
B(N) = HW.

Therefore the right triangle Δ(N)B(Y) equals the right triangle ΔWHJ, so:
WJ = (Y)W(N) = YWN

And since SY = BP:
S(G) = LJ

S(G) + SP = LJ + HL
P(G) = HJ

and by construction:
P(N) = LB

since LB = HY:
P(N) = HY.

Therefore the right triangle Δ(N)P(G) equals the right triangle ΔYHJ, so:
YJ = (G)Y(N) = GYN

Since the radius WJ cuts the hyperbola at a maximum of two points J_1 and J_2, both $(Y_1)W(N_1)$ and $(Y_2)W(N_2)$ can be found using this reference hyperbola.

Since the radius YJ cuts the hyperbola at a maximum of two points J_1 and J_2, both $(G_1)Y(N_1)$ and $(G_2)Y(N_2)$ can be found using this reference hyperbola.

Locating a clear image from plano refraction

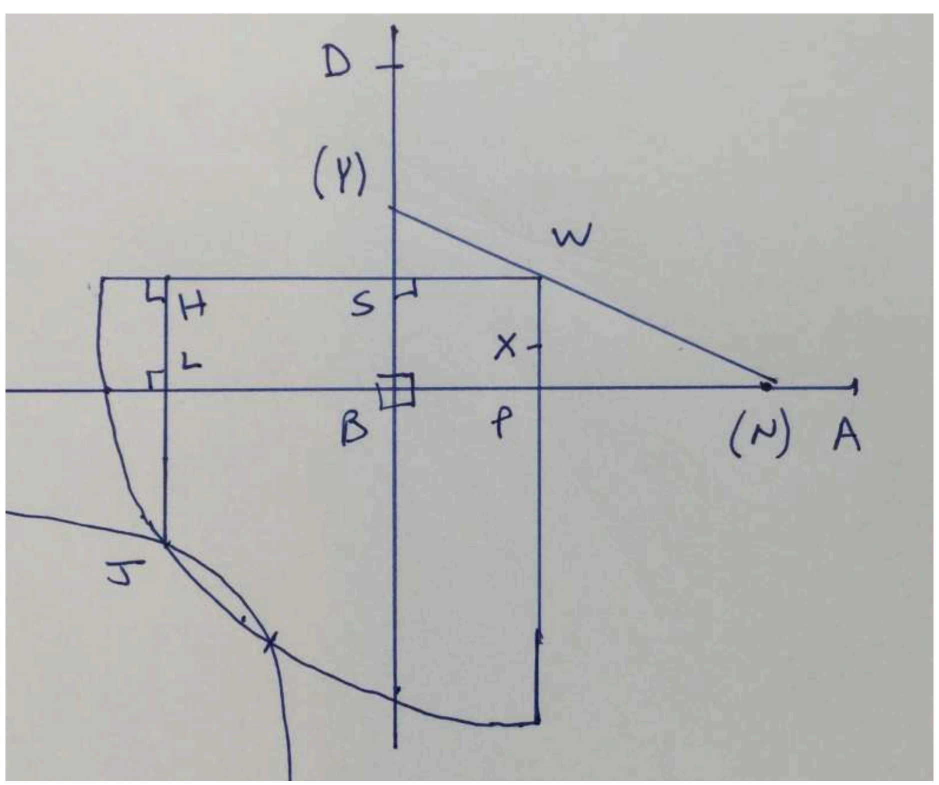
The point X_c on an image ray is the clear image of object D only when its two possible locations of N, (N_1 and N_2), overlap.

X is the clear image of object D only when N_1 and N_2 overlap, which occurs when $Y_1WN_1 = Y_2WN_2$ represents the line segment of minimum length through W within the right angle ∠DBA.

X is the clear image of object D only when N_1 and N_2 overlap, which occurs when $G_1YN_1 = G_2YN_2$ represents the line segment of minimum length through Y within the right angle ∠XPb.

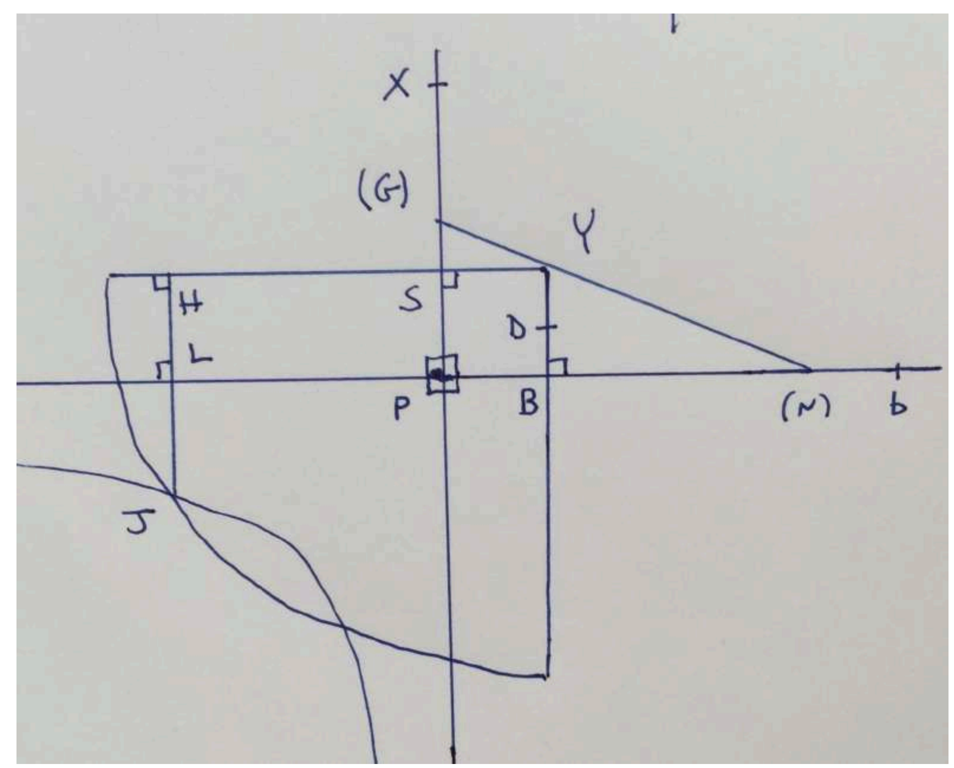
This minimum length equals the radius WJ of the reference circle when it is just tangent to the reference hyperbola. In this way, a position of $(Y_c)W(N_c)$ can be found that produces a clear image X_c, when:

PW/PX = DB/DE and:
DB/Y_cWN_c = ED/EB



This minimum length equals the radius YJ of the reference circle when it is just tangent to the reference hyperbola. In this way, a position of $(G_c)Y(N_c)$ can be found that produces a clear image X_c, when:

BY/DB = ZB/EZ and:
PX/G_cYN_c = EZ/EB



Concluding comments

This course described the continuity of conic sections and their characteristics with synthetic plane geometry, rather than analytical and/or solid geometry. This led to a presentation of Isaac Barrow’s method of locating the plano image ray through an axial point or a point of refraction, using either a reference ellipse or reference hyperbola in object space. A discussion of Isaac Barrow’s use of a reference hyperbola across from object space illustrated his method of finding an image ray through an off-axis point alone, and demonstrated why there can be a maximum of only two such image rays through any off-axis point. Finally, a method of determining the clear plano image location of an object was presented.

Review of course objectives

1). Describe conic sections and their continuity using synthetic plane geometry, rather than analytical and/or solid geometry.

2). Present Isaac Barrow's method of using a reference ellipse or reference hyperbola to find a plano image ray's direction, given its axial point or point of refraction.

3). Understand Isaac Barrow's method of locating the plano image ray through an off-axis point using a reference hyperbola when no axial point or point of refraction is known.

4). Understand why there can be a maximum of only two such rays through any specific off-axis point.

5). Understand why an off-axis point is only a clear image of its object when those two rays overlap.

6). Construct the different figures required for understanding these concepts when the object lies in air or glass.

Twenty Questions

1). Which conic section would Isaac Barrow likely use to find plano image rays with an object in air?
A) Ellipse
B) Hyperbola
C) Parabola
D) Sphere

2). Which conic section would Isaac Barrow likely use to find plano image rays with an object in glass, (or water)?
A) Ellipse
B) Hyperbola
C) Parabola
D) Sphere

3). The continuity of conic sections can be visualized with plane geometry by viewing the hyperbola transverse axis, and ellipse major axis, as a:

A) Circle of infinite diameter

B) Crossed cylinder

C) Parabolic curve

D) Inverted circle

4). The eccentricity of a circle is:

A) one

B) zero

C) >1

D) <1

5). The eccentricity of an ellipse is:

A) one

B) zero

C) >1

D) >0 and <1

6). The eccentricity of a parabola is:
A) one
B) zero
C) >1
D) >0 and <1

7). The eccentricity of a hyperbola is:
A) one
B) zero
C) >1
D) >0 and <1

8). Eccentricity allows for the easy calculation of the distance from a point on a hyperbola to either of its foci by using a corresponding distance along its:
A) transverse axis
B) major axis
C) minor axis
D) inverse curve

9). Eccentricity allows for the easy calculation of the distance from a point on an ellipse to either of its foci by using a corresponding distance along its:

A) transverse axis

B) major axis

C) minor axis

D) inverse curve

10). Eccentricity allows for the easy calculation of the distance from a point on an ellipse or hyperbola to either of the curves' respective foci by using:

A) half the distance between the foci

B) the distance between the foci minus that between the vertices

C) half the distance between the vertices

D) the distance between the foci plus that between the vertices

11). When an object is in air, it's plano refracted image rays meet:

a). in glass, in the same quadrant as the refraction
b). in glass, in the opposite quadrant as the refraction
c). in air, in the same quadrant as the refraction
d). in air, in the opposite quadrant as the refraction

12). When an object is in glass, it's plano refracted image rays meet:

a). in air, in the same quadrant as the refraction
b). in air, in the opposite quadrant as the refraction
c). in glass, in the same quadrant as the refraction
d). in glass, in the opposite quadrant as the refraction

13). Specifying the index of refraction as the hypotenuse of a right triangle with one leg equaling the unit measure, allows for:

a). locating the plano image rays through an off-axis point
b). showing that there are a maximum of only two such rays
c). locating a clear off-axis plano image
d). all of the above

14). When an object is in glass, a reference ellipse is most useful for determining the direction of the plano refracted image ray in air when:

a). only an off-axis point on the image ray is known
b). the image ray point of refraction is known, and lies along the major axis of the reference ellipse
c). a right triangle determined by the index of refraction is used
d). the point of incidence lies off the major axis of the reference ellipse

15). When an object is in air, and at least one on-axis point of the plano image ray is known, which is used for determining the direction of that image ray?

a). a reference ellipse
b). a reference hyperbola
c). a reference circle
d). a right triangle determined by the index of refraction

16). When an object is in air, and only an off-axis point of the plano image ray is known, which is used for determining the direction of that image ray?

a). a reference ellipse
b). a reference hyperbola arm in air
d). a reference parabola
d). a reference hyperbola arm in glass

17). How many plano image rays pass through any given off-axis point?

a). a maximum of one
b). a maximum of two
c). a minimum of one
d). a minimum of two

18). When an object is in glass, and a reference ellipse is used for determining the direction of a plano refracted image ray in air when given its point of incidence, if that point reaches the edge of the reference ellipse along its major axis:

a). the reference ellipse becomes a hyperbola
b). the reference ellipse can simply be made larger
c). it is refracted along that major axis, which lies along the refracting plane, and all more obliquely incident rays are therefore internally reflected
d). the incident ray is reflected back onto itself

19). When an object is in air, and a reference hyperbola is used for determining the direction of a plano refracted image ray in glass when given the point of incidence, because the point of refraction along the plane never reaches the edge of the reference hyperbola:

a). it is refracted along the transverse axis, which lies along the refracting plane
b). it is never internally reflected
c). internal reflection occurs when a reference circle is tangent to the reference hyperbola
d). the reference hyperbola can not describe refraction along the entire surface of the plane

20). When an object is in air, and a reference hyperbola is used for determining the direction of a plano refracted image ray in glass when no point of incidence is given, and when only an off-axis point on the image ray is known:

a). the arm of the reference hyperbola used is drawn in air in the same quadrant as the refraction
b). the arm of the reference hyperbola used is drawn in air in the opposite quadrant as the refraction
c). the arm of the reference hyperbola used is drawn in glass in the same quadrant as the refraction
d). the arm of the reference hyperbola used is drawn in glass in the opposite quadrant as the refraction

Answer Key

1. B
2. A
3. A
4. B
5. D
6. A
7. C
8. A
9. B
10. C
11. D
12. C
13. D
14. B
15. B
16. D
17. B
18. C
19. B
20. D