Axial Magnification from Distance and Near Corrections

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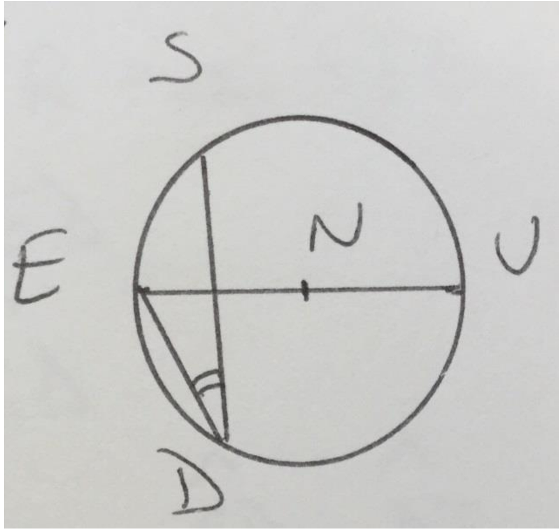
Reference:
Isaac Barrows Optical Lectures, 1667;
Translated by H.C. Fay
Edited by A.G. Bennett
Publisher: The Worshipful Company of Spectacle Makers;
London, England; 1987
ISBN # 0-951-2217-0-1

Introduction

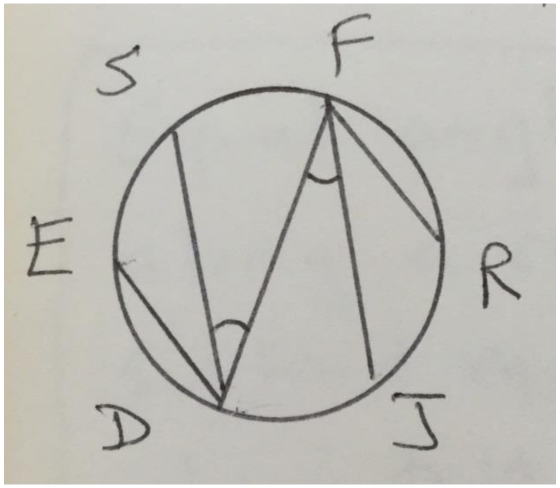
Equal arcs along a circle subtend equal angles along that circle. Therefore, certain triangles within a circle can be shown to have the same shape, with their sides forming ratio equalities. Cyclic quadrilaterals can then describe equalities with multiple ratios, and these multivariable relationships can be used to find triangles with other triangles. This plane geometry approach was used by Isaac Barrow in 1667 to describe tangential refraction along a line and at a circle, without trigonometry, algebra, or calculus. It is particularly suited for clinicians in the field of low vision and ophthalmic optics, since it requires no math background beyond high school plane geometry, and encourages a spatial understanding devoid of sign convention and jargon. For those clinicians wishing to have more than a working knowledge of the subject of axial magnification, I have drawn a progression of geometric figures to cover the necessary preliminary concepts, each building on the previous, with labeled points maintaining their significance until noted otherwise. Axial magnification is presented only after a thorough spatial representation of tangential refraction along a line and a circle. In order to visualize the relevant axial ratio equalities involved using triangles, the optic axis is then represented as a circle of infinite radius, and the sign convention remains unnecessary.

Figure 1:



given a circle with diameter EU and center N

Figure 2:



with *any* FJ || SD:

~SF = ~JD

∠FDS = ∠DFJ

when ∠JFR = ∠SDE:

FR || ED

~EF = ~RD

~EF - ~SF = ~RD - ~DJ

~ES = ~RJ

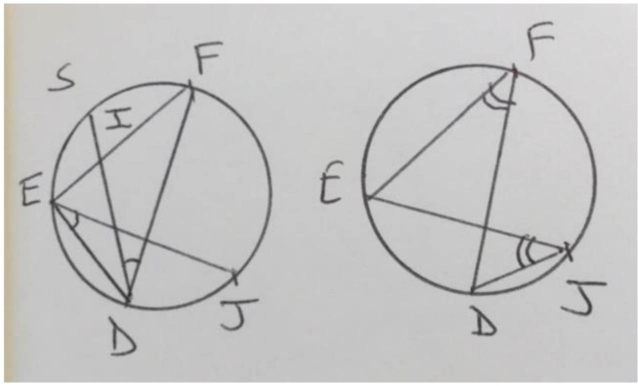
Equal angles along a circle subtend equal arcs along that circle

therefore, an angle along any circle can be defined in terms of subtended arc and diameter

$$\angle JFR = \sim\frac{RJ}{EU}$$

triangles need only two equal angles to be the same shape, (or ≅).

Figure 3:



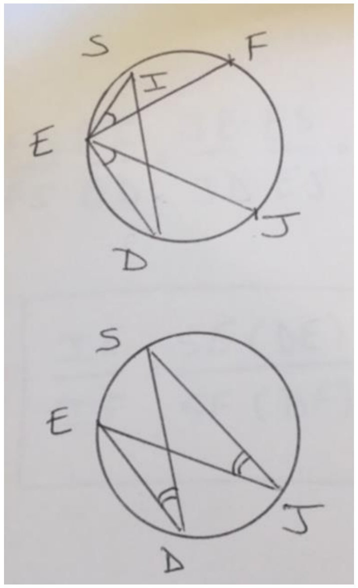
FJ || SD

~SF = ~JD

ΔEJD ≅ ΔDFI

$$\frac{FD}{FI} = \frac{JE}{JD}$$

Figure 4:



ΔEJS ≅ ΔEDI

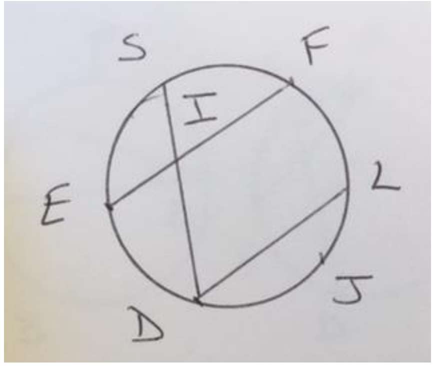
$$\frac{EI}{ED} = \frac{ES}{EJ}$$

$$\frac{FD.EI}{FI.ED} = \frac{JE.ES}{JD.EJ} = \frac{SE}{SF}$$

$$\mathbf{\frac{IE}{IF} = \frac{SE.DE}{SF.DF}}$$

which describes an important property of *any* cyclic quadrilateral SEDF

Figure 5:



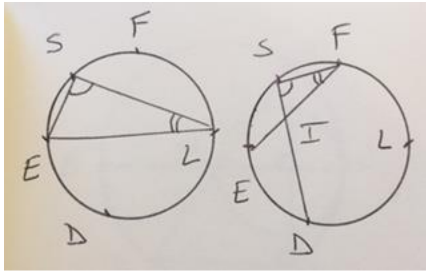
LD || FE

$$\frac{DE}{DF} = \frac{LF}{LE}$$

$$\frac{IE}{IF} = \frac{SE.LF}{SF.LE}$$

$$\frac{FE}{FI} = \frac{SE.LF + SF.LE}{SF.LE}$$

Figure 6:



LD || FE

~EL = ~FD

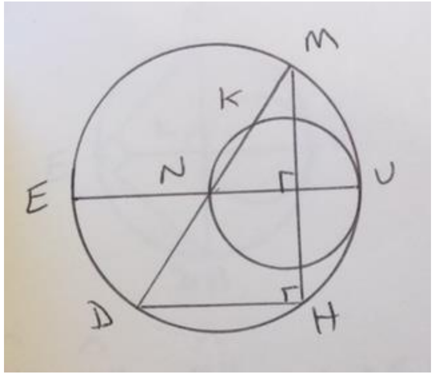
ΔLSE ≅ ΔFSI

$$LS = \frac{FS.LE}{FI}$$

FE.LS = SE.LF + SF.LE

which describes an important property
of *any* cyclic quadrilateral SELF

Figure 7:



$\angle KNU = \angle MDH$

$$\frac{\sim UK}{UN} = \frac{\sim MH}{MD} = \frac{\sim MH}{UE} =$$

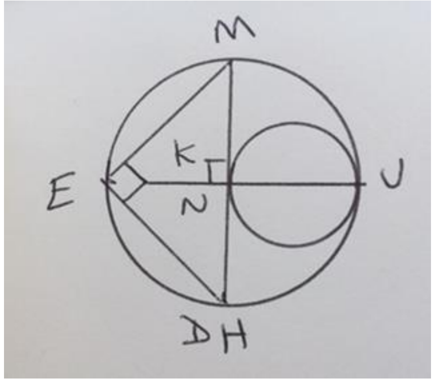
$$\frac{2(\sim UM)}{UE} = \frac{2(\sim UM)}{2(UN)}$$

$\mathbf{\angle KNU = 2\angle MEU}$

$\mathbf{\sim UK = \sim UM}$

let $K \Rightarrow N$ and $D \Rightarrow H$:

Figure 8:

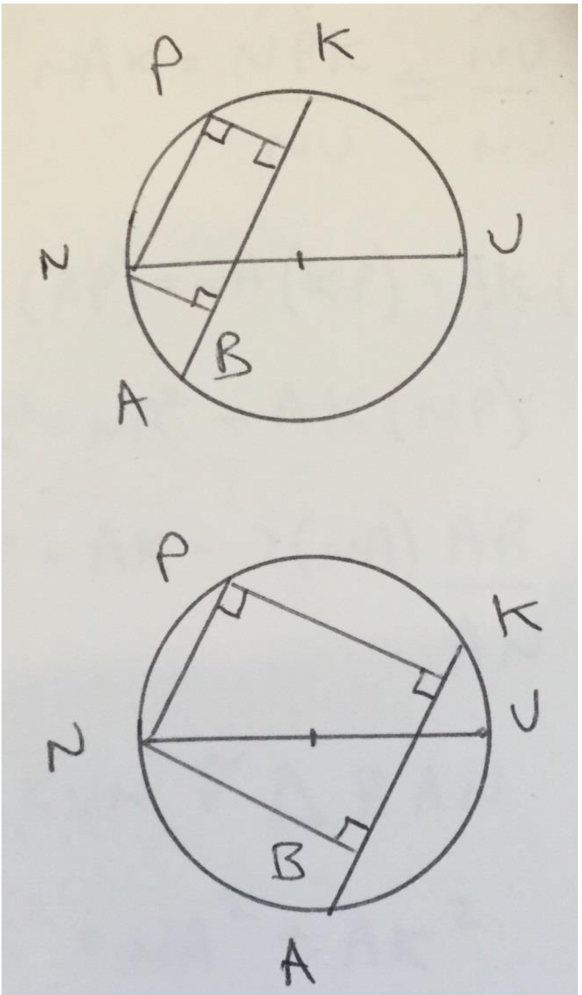


$$\sim\frac{UK}{UN} = \sim\frac{MH}{MD} = \sim\frac{MH}{UE} = \angle MEH$$

$$\sim\frac{UK}{UN} = \angle MNU$$

$$\mathbf{\frac{2(\sim UK)}{UN} = \angle MNH = \pi}$$

Figure 9:



AK ≥ NP || AK

$$\angle NAK = \frac{\sim NPK}{NU} \leq \frac{\sim NU}{NU} = \frac{\pi}{2}$$

$$NK.AP = NA.KP + AK.NP$$

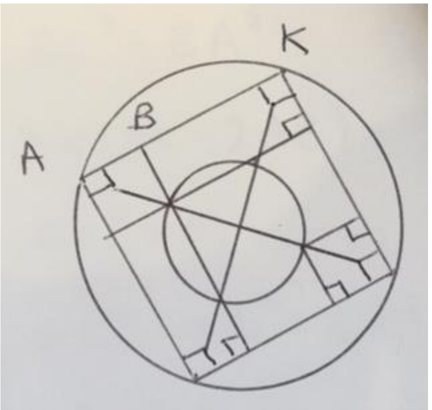
$$NK^2 = NA^2 + AK.NP$$

$$NP = AK - 2(NA)\frac{AB}{AN}$$

$$\Delta KUN \cong \Delta BAN$$

$$NK^2 = NA^2 + AK^2 - 2(AK)NA.\frac{UK}{UN}$$

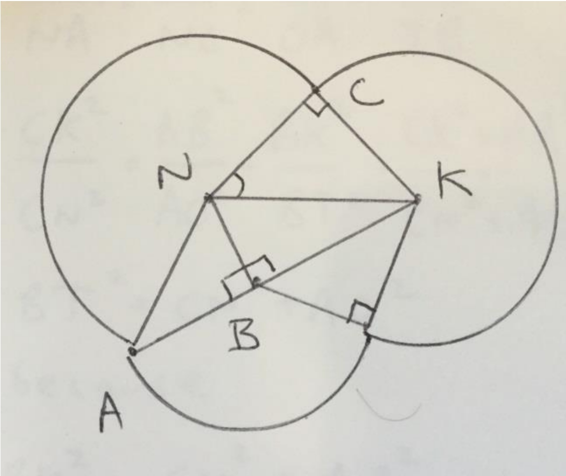
Figure 10:



$$BK^2 - BA^2 = AK^2 - 2(AK)AB =$$

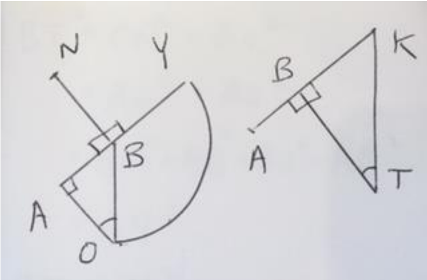
$$AK.NP = NK^2 - NA^2$$

Figure 11:



$BK^2 - BA^2 = NK^2 - NA^2 = CK^2$

Figure 12:



$$\frac{NK}{NA} = \frac{NK}{NC} = \frac{OB}{OA} = \frac{TK}{TB}$$

$$\frac{CK^2}{CN^2} = \frac{AB^2}{AO^2} = \frac{BK^2}{BT^2} = \frac{CK^2 + AB^2}{CN^2 + AO^2}$$

$BT^2 = CN^2 + AO^2$

because:

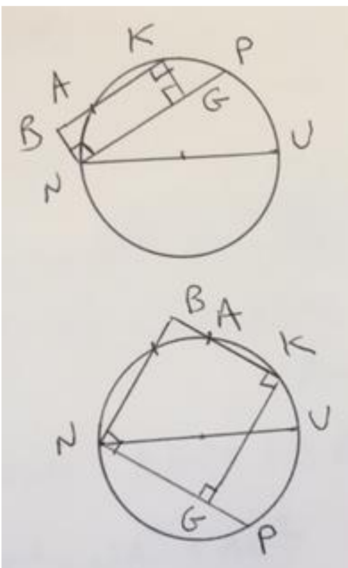
$BK^2 = CK^2 + AB^2$

$BT^2 = CN^2 + AO^2$
$= AN^2 + AO^2$
$= BN^2 + AB^2 + BO^2 - AB^2$
$= NY^2$

$BT = NY$

given ΔBAO
use ΔKBT to find ΔYBN
and use ΔYBN to find ΔKBT

Figure 13:



$NP \geq AK \parallel NP$

$\angle NAK = \sim\frac{NUK}{NU} \geq \sim\frac{NU}{NU} = \frac{\pi}{2}$

$NK.AP = NA.KP + AK.NP$

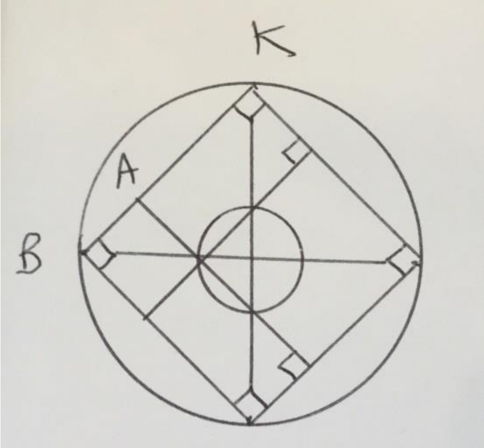
$NK^2 = NA^2 + AK.NP$

$$NP = AK + 2(NA)\frac{AB}{AN}$$

$\Delta KUN \cong \Delta GPK \cong \Delta BAN$

$$NK^2 = NA^2 + AK^2 + 2(AK)NA.\frac{UK}{UN}$$

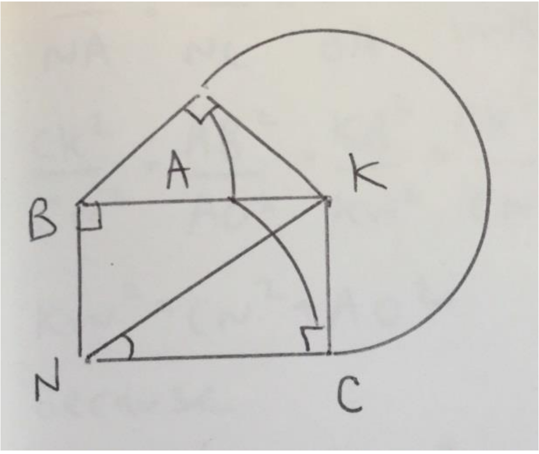
Figure 14:



$BK^2 - BA^2 = AK^2 + 2(AK)AB =$

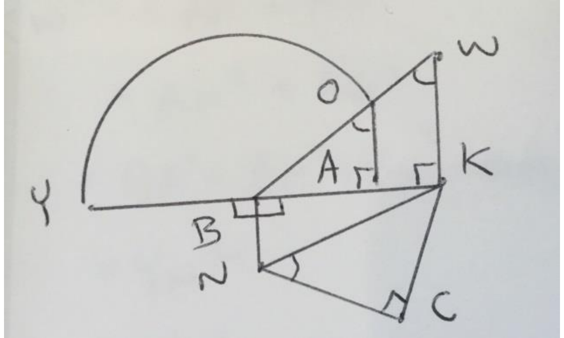
$AK.NP = NK^2 - NA^2$

Figure 15:



$BK^2 - BA^2 = NK^2 - NA^2 = CK^2$

Figure 16:



$$\frac{NK}{NA} = \frac{NK}{NC} = \frac{OB}{OA} = \frac{WB}{WK}$$

$$\frac{CK^2}{CN^2} = \frac{AB^2}{AO^2} = \frac{KB^2}{KW^2} = \frac{CK^2 + AB^2}{CN^2 + AO^2}$$

$KW^2 = CN^2 + AO^2$

because:

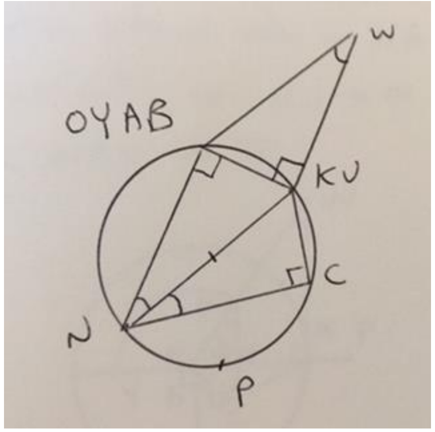
$KB^2 = CK^2 + AB^2$

$$\begin{aligned} KW^2 &= CN^2 + AO^2 \\ &= AN^2 + AO^2 \\ &= BA^2 + BN^2 + BO^2 - BA^2 \\ &= YN^2 \end{aligned}$$

$KW = YN$

given ΔBAO
use ΔBKW to find ΔYBN
and use ΔYBN to find ΔBKW

Figure 17:



with NK constant

let circle NPKA shrink
and rotate counter-clockwise around N
so that:

$$U \Rightarrow K, \text{ and } \angle NAK \Rightarrow \angle NBK = \frac{\pi}{2}$$

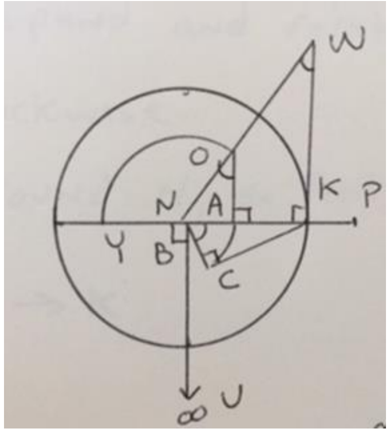
or, with NA constant
let circle NPKA expand
and rotate clockwise
around N
so that:

$$K \Rightarrow U, \text{ and } \angle NAK \Rightarrow \angle NBK = \frac{\pi}{2}$$

$$\frac{NK}{NA} = \frac{NK}{NC} = \frac{WB}{WK}$$

$$KW = YN$$

Figure 18:



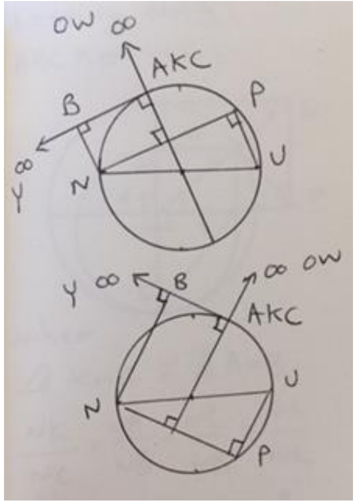
with either NK or NA constant

as NU $\Rightarrow \infty$
$\angle NAK \Rightarrow \pi$

$$\frac{(KW)}{(OA)} = \frac{NK}{NA} = \frac{NK}{NC} = \frac{OB}{OA} = \frac{WB}{WK}$$

KW (=OB) = YN

Figure 19:



with NK constant
let circle NPKA expand
and rotate clockwise
around N
so that:

A $\Rightarrow$ K

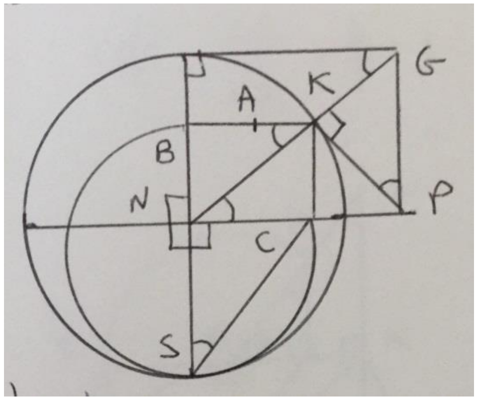
or, with NA constant
let circle NPKA shrink
and rotate counter-clockwise around N
so that:

K ⇒ A

$\frac{NK}{NA} = \frac{NK}{NC} = \frac{OB}{OA} = \frac{WB}{WK}$

KW = YN

Figure 20:



keeping only:
ABCKNOWY:

when ΔKNC ≅ ΔANB:

$$\frac{NK}{NC} = \frac{NA}{NB} = \frac{NC}{NB} = \frac{NS}{NC}$$

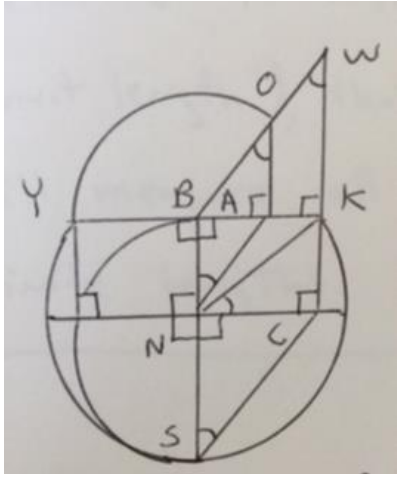
$$\frac{NK}{NB} = \frac{GN}{GP} = \frac{NK + NB}{NK}$$

$$\frac{NK}{NB} = 1 + \frac{1}{NK/NB} = 1 + \frac{1}{1 + \ldots}$$

$$\frac{NK}{NB} = \varphi$$

$$\frac{NB}{NK} = \varphi - 1$$

Figure 21:

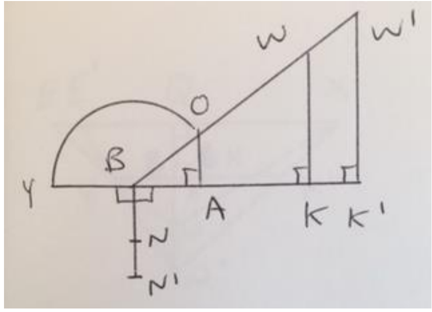


$$\frac{NK}{NA} = \frac{NK}{NC} = \frac{OB}{OA} = \frac{WB}{WK}$$

$$KW = YN$$

Note that there is no length relative to itself, ("unit length"), that will measure all finite lengths

Figure 22:

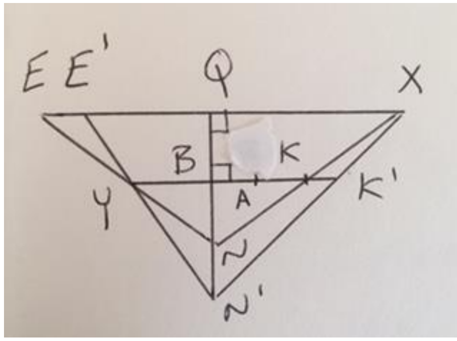


$$\frac{OB}{OA} = \frac{NK}{NA} = \frac{N'K'}{N'A}$$

KW = YN ; K'W' = YN'

$$\frac{KB}{YN} = \frac{K'B}{YN'}$$

Figure 23:



$$\frac{QX}{EN} = \frac{KB}{YN} = \frac{K'B}{YN'} = \frac{QX}{E'N'}$$

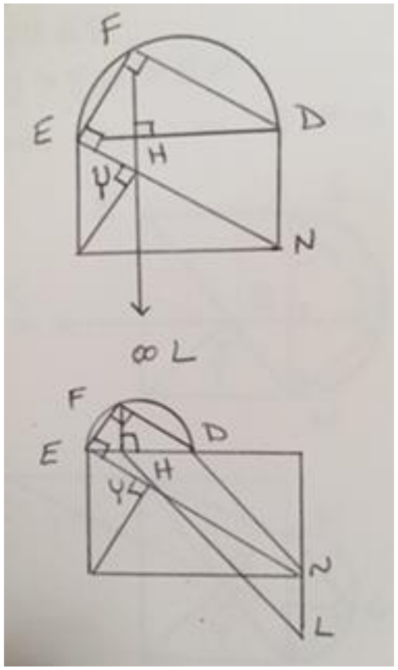
only one N'K'X exists for NKX
because only one E'N' equals EN

with ΔBAX constant
only one ΔXNN' exists for ΔOAB

in order for EN to equal E'N'
as N' approaches N
both EN and E'N' must rotate around Y until
they superimpose

therefore, with ΔBAX constant
as N' approaches N
ΔOAB (or $\frac{NK}{NA}$) must change

Figure 24:



keeping only:
ABEKK'NN'OWXY:

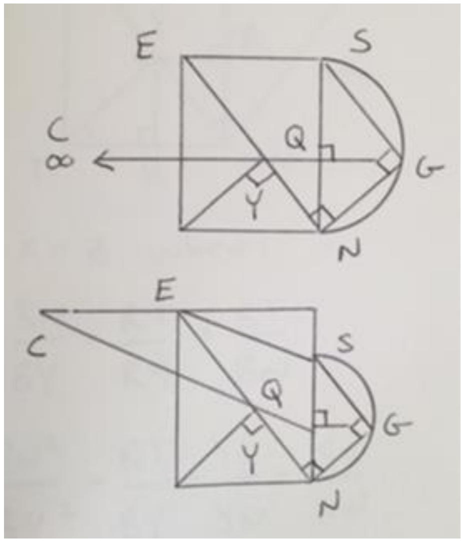
LH || ND

LH > NF > NE

holds true as:

H ⇒ E

Figure 25:

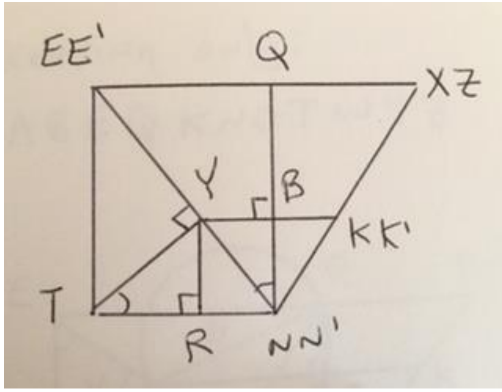


CQ || ES

CQ > EG > EN

holds true as Q ⇒ N

Figure 26:

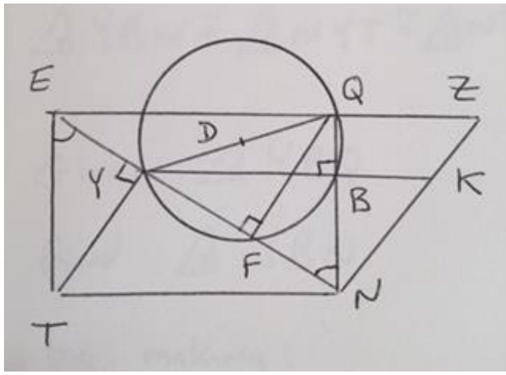


X = Z when:

$$\frac{BN}{BY} = \frac{RT}{RY} = \frac{RT}{BN}$$

$$\frac{BN^2}{BY^2} = \frac{RT}{BY} = \frac{YE}{YN} = \frac{KX}{KN}$$

Figure 27:

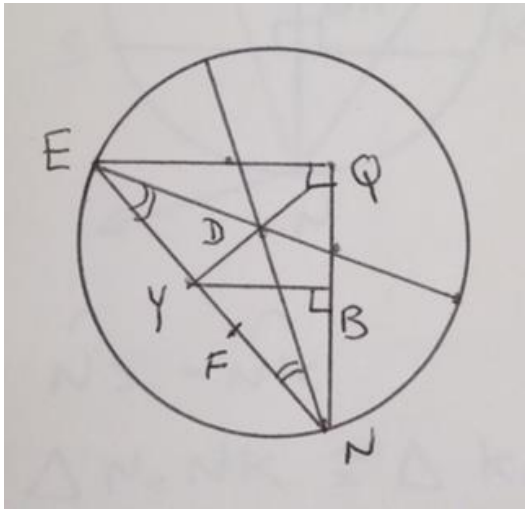


keeping only:
ABEQKNOTWYZ:

given ΔYBN, find ΔYBQ using:

ΔYBN ≅ ΔNYT ≅ ΔNTE

Figure 28:



given ΔYBQ, find ΔYBN by making:

EY = NF

which occurs when ~EN lies on a circle concentric with circle YFBQ

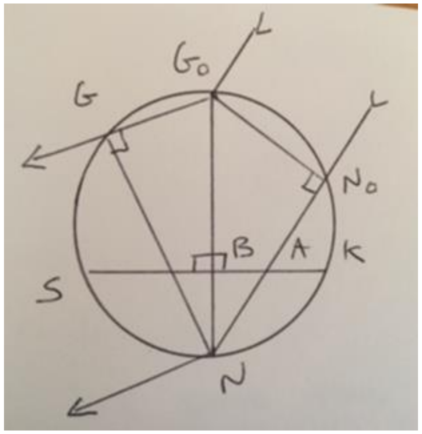
because:

DY = DF

ΔEDY = ΔNDF

EY = NF

Figure 29:



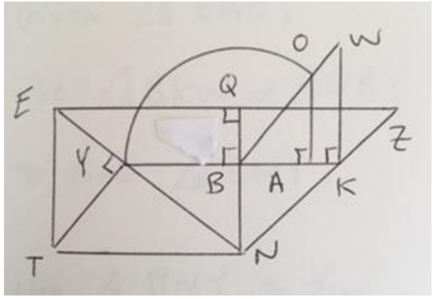
$\sim NS = \sim NK$

$\Delta N_o NK \cong \Delta KNA$

$$\mathbb{R} = \frac{NN_o}{GG_o} = \frac{NN_o}{NK} = \frac{NK}{NA}$$

wavefront G_oN_o refracts into
wavefront GN along G_oN,
because it travels G_oG
in the same time it travels N_oN

Figure 30:



If $\mathbb{R} = \frac{OB}{OA}$ and KW = YN:

$\mathbb{R} = \frac{NK}{NA}$

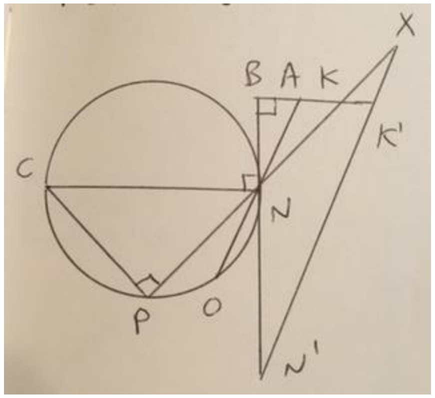
and Z is the clear image of object A refracted at N along BN

given ΔBAO:

use ΔBKW or ΔQBY
to find ΔBNY

use ΔBNY to find
ΔBKW or ΔQBY

Figure 31:

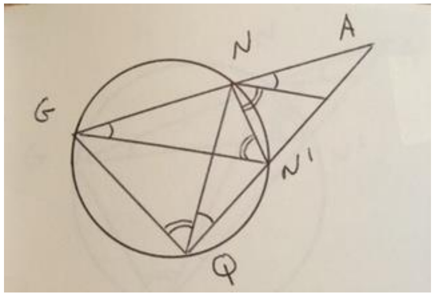


keeping only:
ABKNXZ:

ΔKNA ≅ ΔOCP

$$\mathbb{R} = \frac{NK}{NA} = \frac{N'K'}{N'A} = \frac{CO}{CP}$$

Figure 32:



ΔANN' ≅ ΔAQG

Figure 33:

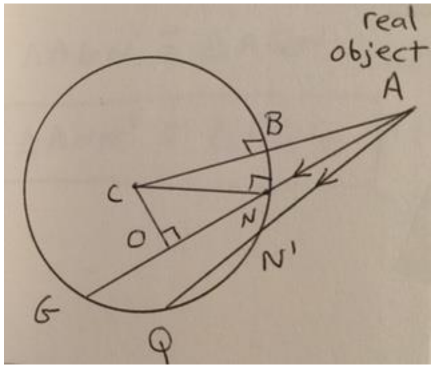
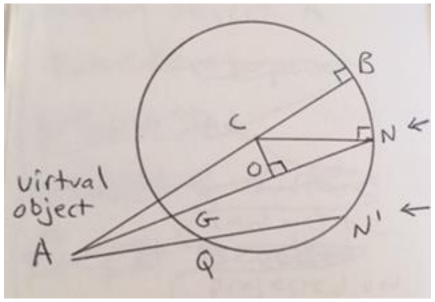
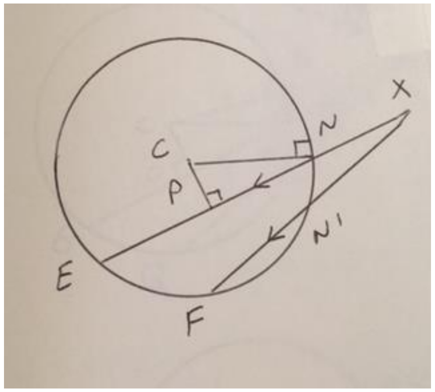


Figure 34:



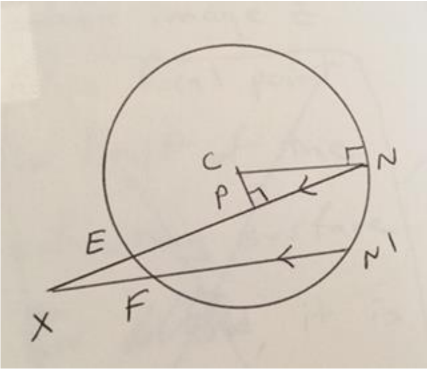
the virtual object A can not be
projected on a screen
due to refraction at BN

Figure 35:



ΔXNN' ≅ ΔXFE
the virtual image (Z) can not
be projected on a screen

Figure 36:



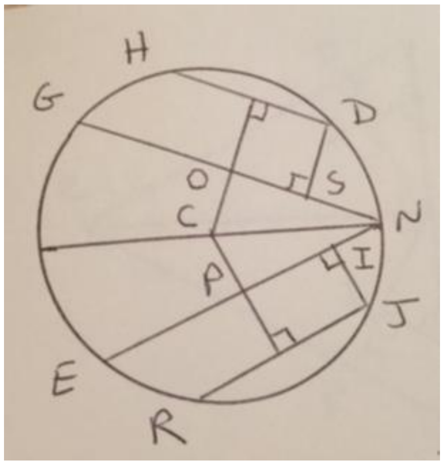
the real image (Z) can be
projected on a screen

$$\frac{AG + AN'}{2AN'} = \frac{QG + NN'}{2NN'}$$

$$\frac{XE + XN'}{2XN'} = \frac{EF + NN'}{2NN'}$$

$$\frac{QG + NN'}{EF + NN'} = \frac{(AG + AN')}{2AN'} \frac{2XN'}{(XE + XN')}$$

Figure 37:



HD = QN’

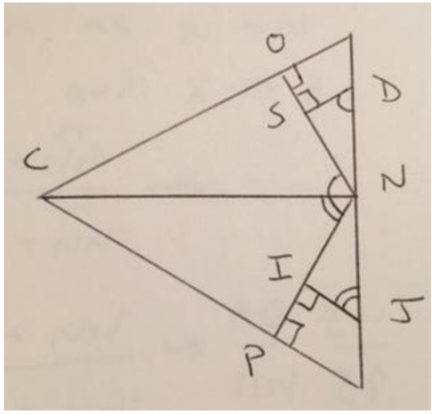
RJ = FN’

as N’ ⇒ N:

X ⇒ Z, and ~DJ ⇒ DJ

so that:

Figure 38:



$$\frac{DS}{JI} \Rightarrow \frac{CO}{CP}$$

$$\frac{JI}{JN} \Rightarrow \frac{NP}{NC}$$

$$\frac{DN}{DS} \Rightarrow \frac{NC}{NO}$$

$$\frac{ND}{NJ} \Rightarrow \frac{NP}{NO}\ \frac{CO}{CP}$$

thus, as N’ ⇒ N and X ⇒ Z:

$$\frac{\sim QG + \sim NN'}{\sim EF + \sim NN'} \Rightarrow \frac{QG + NN'}{EF + NN'} \Rightarrow$$

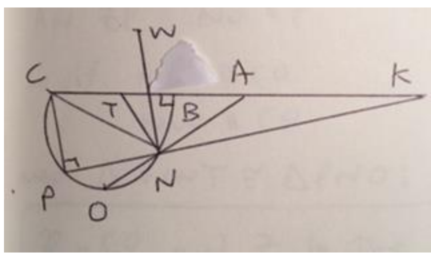
$$\frac{AO}{AN}\ \ \frac{ZN}{ZP}$$

and:

$$\frac{\sim QG + \sim NN'}{\sim EF + \sim NN'} = \frac{2\,(\sim ND)}{2\,(\sim NJ)} \Rightarrow$$

$$\frac{ND}{NJ} \Rightarrow \frac{NP}{NO}\ \ \frac{CO}{CP}$$

Figure 39:



keeping only:
ABKNXZCPO and $\mathbb{R}$:

$NT \parallel CO$

$NW \parallel CP$

when $X = Z$ lies along

both NP and CW:

$$\frac{AO}{AN}\ \frac{ZN}{ZP} = \frac{CO}{NT}\ \frac{NW}{CP}$$

when $\Delta WNT \cong \Delta PNO$, $NW > NT$

and

$$\frac{AO}{AN}\ \frac{ZN}{ZP} = \frac{NP}{NO}\ \frac{CO}{CP}$$

so if:

$NT \parallel CO$

$NW \parallel CP$

and $\Delta WNT \cong \Delta PNO$:

$$\mathbb{R} = \frac{CO}{CP}$$

and Z is the clear image of object
A refracted at N along ~BN

Figure 40:

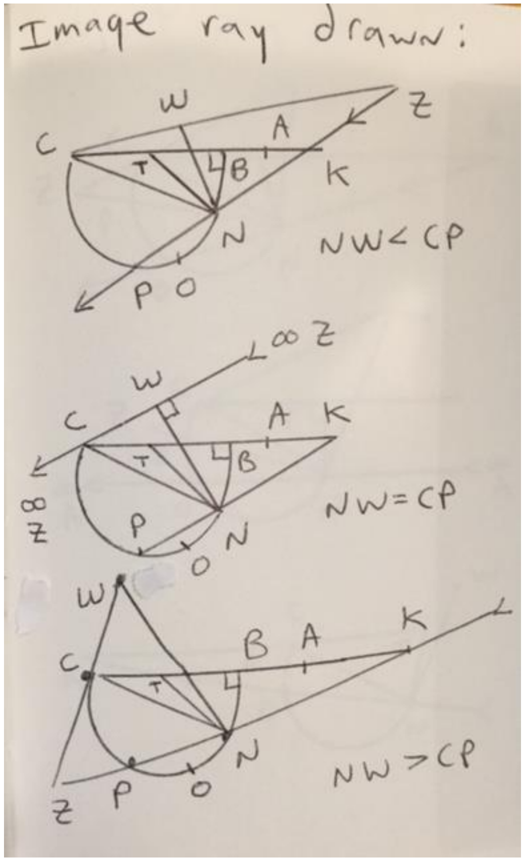


Image ray drawn:
W
Z
C
A
T
B
K
N
NW< CP
P O
∞ Z
W
C
A K
T
B
NW=CP
∞ Z
P
N
O
W
B A K
C
T
N
NW >CP
Z P O

Figure 41:

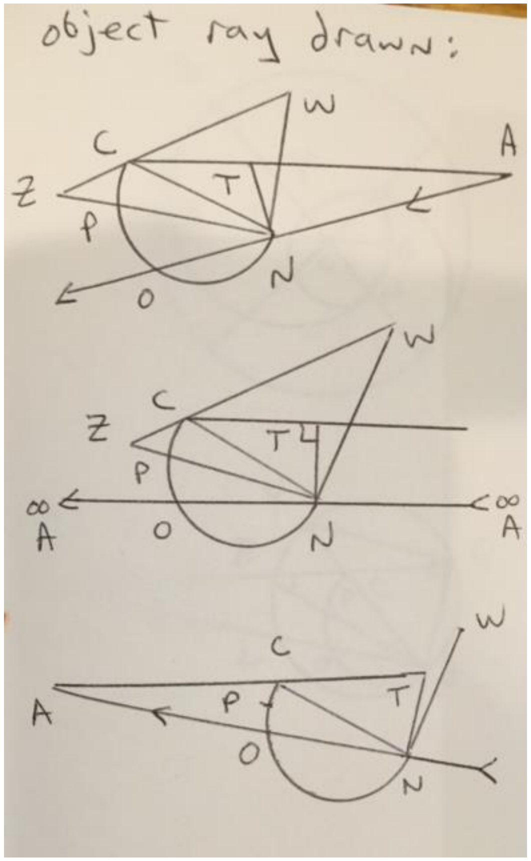
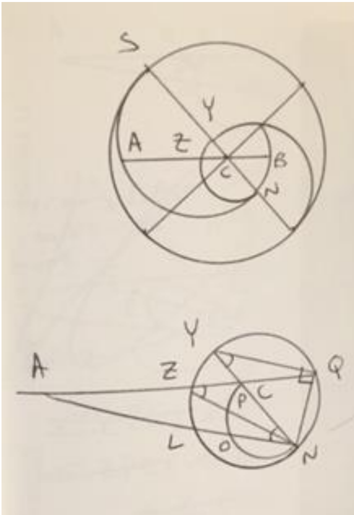


Figure 42:



$$\frac{CY}{CN} = \frac{CN}{CS} = \frac{CY + CN}{CN + CS} = \frac{NY}{NS}$$

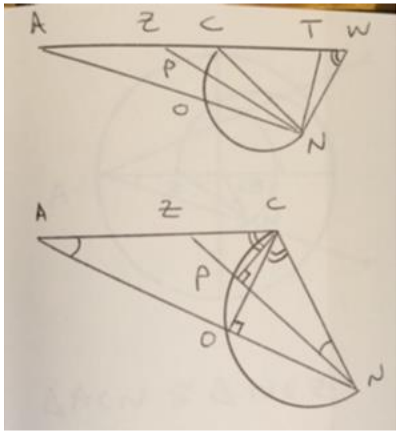
$$\frac{AO}{AN}\ \frac{ZN}{ZP} = \frac{SC}{SN}\ \frac{ZN}{ZP} = \frac{NC}{NY}\ \frac{ZN}{ZP} =$$

$$\frac{NC}{NY}\ \frac{YN}{YC} = \frac{CN}{CY}$$

$$\frac{CO}{CP}\ \frac{NP}{NO} = \frac{LY}{LN}\ \frac{PN}{PC} = \frac{QN}{QY}\ \frac{PN}{PC} =$$

$$\frac{QN\ (ZN)}{QY\ (ZY)} = \frac{CN}{CY}$$

Figure 43:



NT ∥ CO

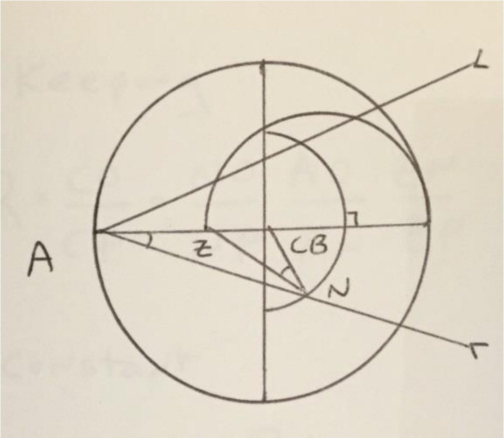
NW ∥ CP

ΔWNT ≅ ΔPNO

∠NWT = ∠NPO = ∠NCO

ΔCPN ≅ ΔCOA

Figure 44:



$\Delta ACN \cong \Delta NCZ$ for all N

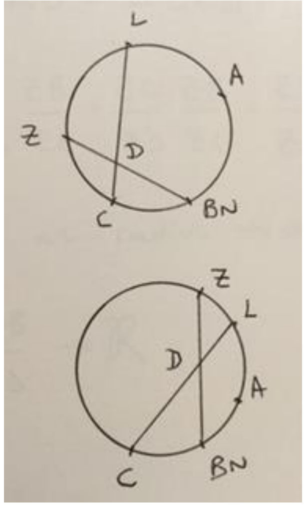
keeping:

$$\mathbb{R} = \frac{CO}{CP} = \frac{NO}{NP}\ \frac{AO}{AN}\ \frac{ZN}{ZP}$$

constant as N ⇒ B:

$$\frac{BC}{BC}\ \frac{AC}{AB}\ \frac{ZB}{ZC} \Rightarrow \mathbb{R}$$

Figure 45:



"axial" refraction can be described along a circle of infinite radius

draw CDL so:

AL || ZB so:

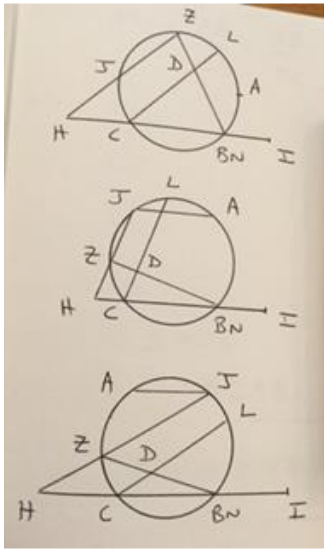
ΔACB ≅ ΔZCD and:

$$\frac{AC}{AB}\,\frac{ZB}{ZC} = \frac{ZC}{ZD}\,\frac{ZB}{ZC} = \frac{ZB}{ZD}$$

so as the radius ⇒ ∞

$$\frac{ZB}{ZD} \Rightarrow \mathbb{R}$$

Figure 46:



$HZ \parallel CL$

$$\frac{ZB}{ZD} = \frac{HB}{HC}$$

~AZ = ~BL

~ZC = ~LJ

~AC = ~BJ

$AJ \parallel CB$

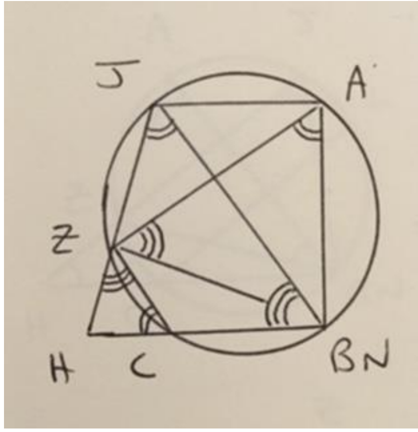
$\Delta HBZ \cong \Delta HJC$

when $\Delta HJC = \Delta IAB$:

$$\frac{BI}{ZH} = \frac{AI}{BH}$$

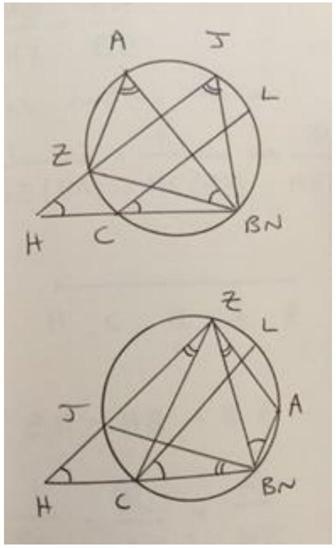
BI (BH) = ZH (AI)

Figure 47:



$\Delta HCZ \cong \Delta HJB \cong \Delta BAZ$

Figure 48:



$\Delta HCZ \cong \Delta HJB \cong \Delta BAZ$

$$\frac{HC}{HZ} = \frac{BA}{BZ}$$

as the radius $\Rightarrow \infty$

$$\frac{1}{HZ\,(BA)} = \frac{1}{HC\,(BZ)} \Rightarrow \frac{\mathbb{R}}{HB\,(BZ)}$$

|________|________|_________|_________|
H C B A Z
N

ZH = HB + BZ

$$\frac{1}{BA} = \frac{\mathbb{R}}{BZ} + \frac{\mathbb{R}}{HB}$$

|_________|_________|__________|__________|
Z H C B A
N

ZB = HZ + HB

$$\frac{\mathbb{R}}{HB} = \frac{1}{BA} + \frac{\mathbb{R}}{BZ}$$

|_________|_________|__________|__________|
A H Z C BN
|_________|_________|__________|__________|

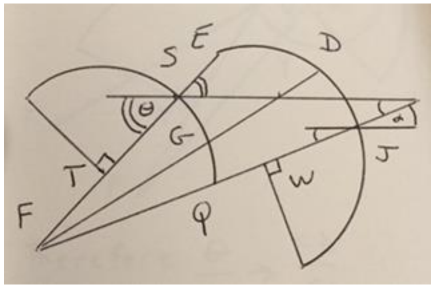
H A Z C BN

HB = HZ + ZB

$$\frac{\mathbb{R}}{BZ} = \frac{1}{BA} + \frac{\mathbb{R}}{HB}$$

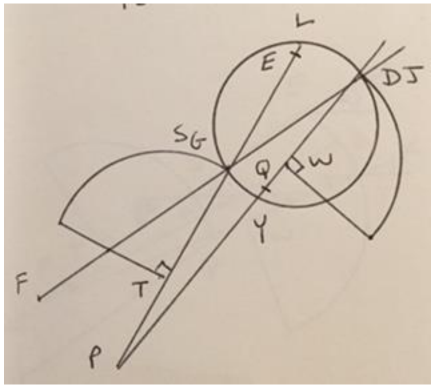
Figure 49:

keeping only:
ZAHCBNI and ℝ:



as ∠SFG = ∠GFJ ⇒ 0

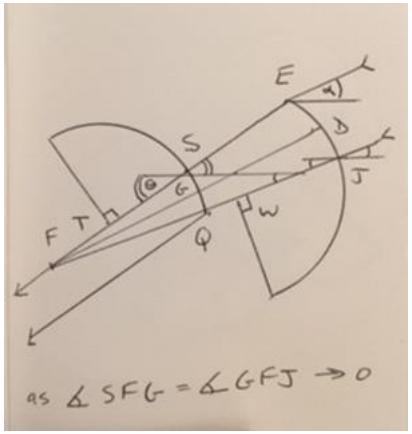
Figure 50:



$\frac{\theta}{\alpha} \Rightarrow \frac{\sim LD/GD}{\sim YG/GD}$ as P ⇒ F therefore, $\frac{\theta}{\alpha} \Rightarrow \frac{FD}{FG}$

changing TS = TG = TQ
so rays remain afocal:

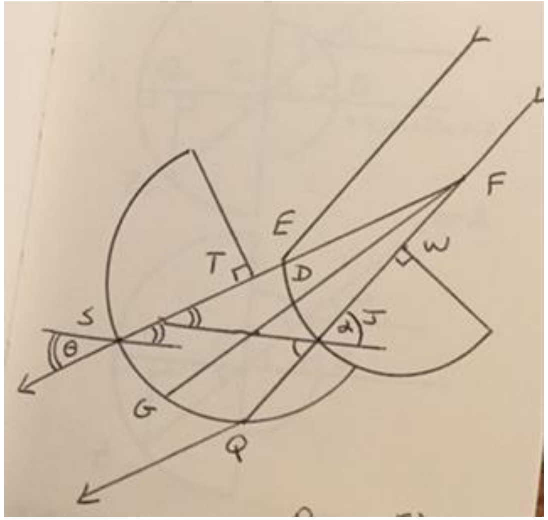
Figure 51:



as ∠SFG = ∠GFJ ⇒ 0

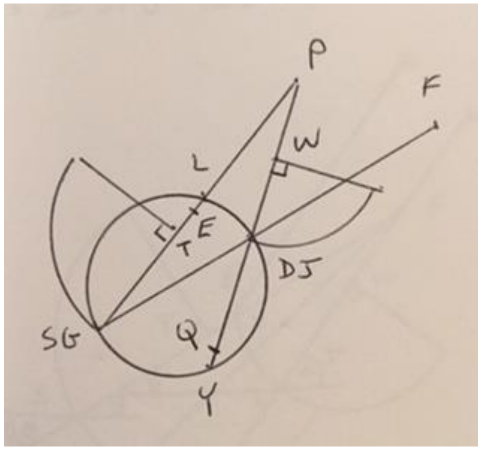
$\frac{\theta}{\alpha} \Rightarrow \frac{FD}{FG}$

Figure 52:



as ∠SFG = ∠GFJ ⇒ 0

Figure 53:

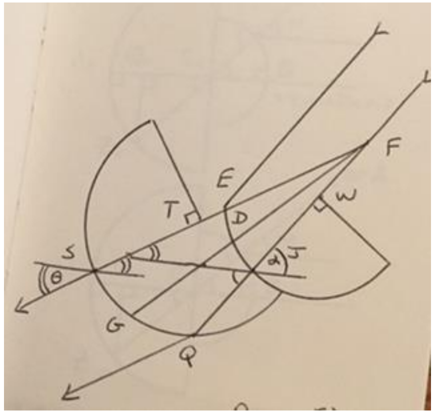


$\frac{\theta}{\alpha} \Rightarrow \frac{\sim LD/GD}{\sim YG/GD}$ as P $\Rightarrow$ F

therefore, $\frac{\theta}{\alpha} \Rightarrow \frac{FD}{FG}$

changing TS = TG = TQ
so rays remain afocal:

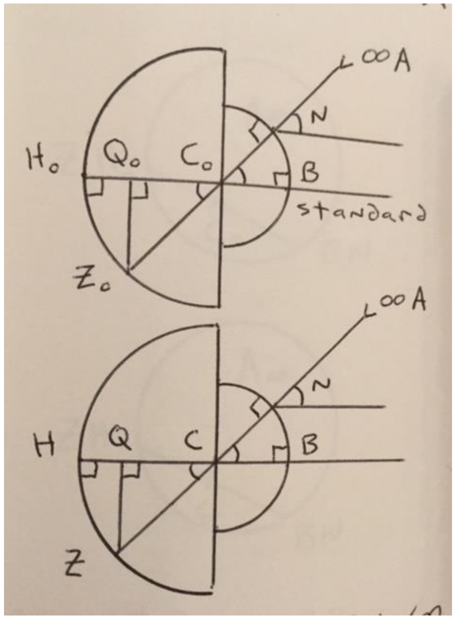
Figure 54:



as ∠SFG = ∠GFJ ⇒ 0

$\frac{\theta}{\alpha} \Rightarrow \frac{FD}{FG}$

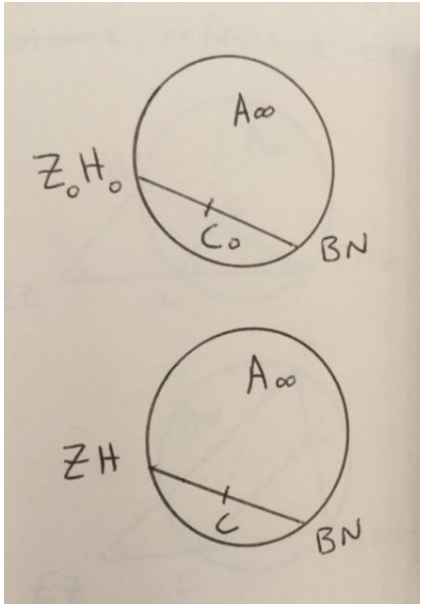
Figure 55:



keeping only:
ZAHCBNIDGF and ℝ:

$$\frac{ZQ}{Z_{\circ}Q_{\circ}} = \frac{ZC}{Z_{\circ}C_{\circ}} = \frac{HC}{H_{\circ}C_{\circ}} = \frac{BH/\mathbb{R}}{BH_{\circ}/\mathbb{R}}$$

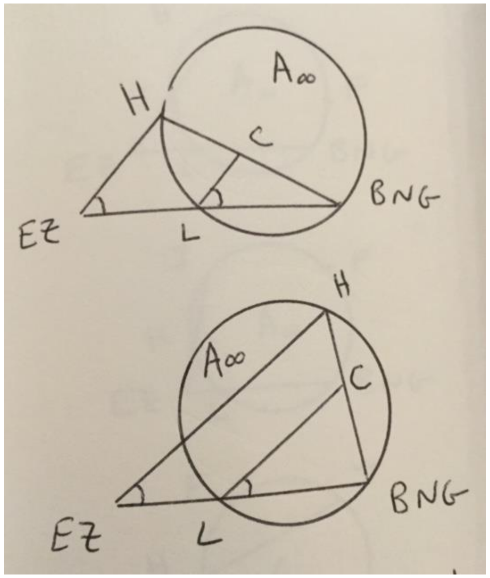
Figure 56:



as N ⇒ B:

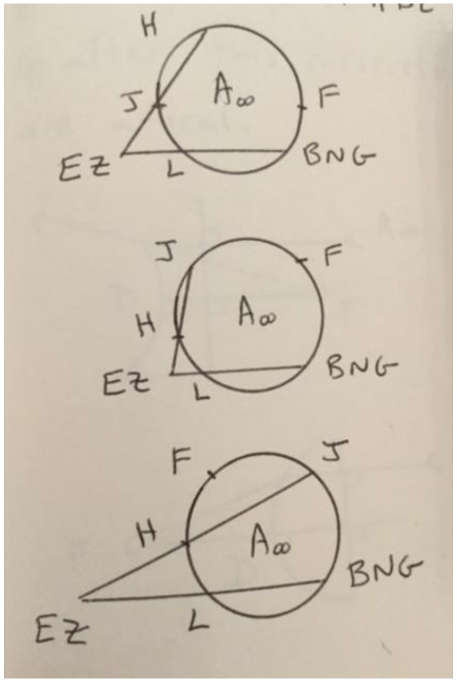
$$M \Rightarrow \frac{ZQ}{Z_\circ Q_\circ} = \frac{BH}{BH_\circ}$$

Figure 57:



additional refraction at G (at B)
creates distance refractive error
with combined curvature
of radius BL

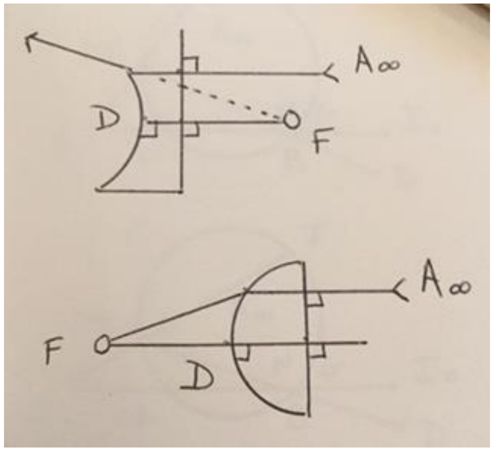
Figure 58:



the distance correction must focus (A∞) at F

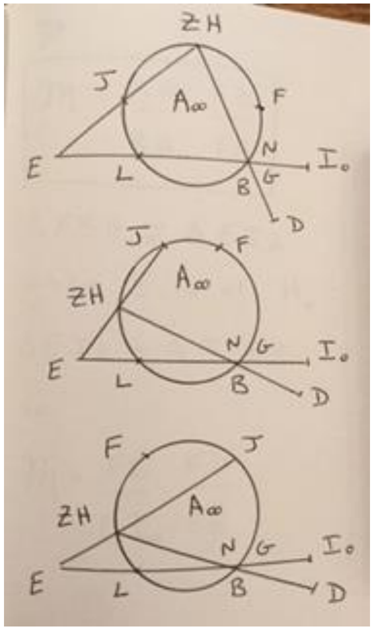
so that JF || BE

Figure 59:



since the distance correction
at D moves Z to H
rays leaving G after this correction are afocal

Figure 60:



$$\boldsymbol{M} = \frac{BH}{BH_\circ}\ \frac{FD}{FB}$$

$\Delta EBH \cong \Delta EJL$

when E is at $H_\circ$:

$\Delta EJL = \Delta I_\circ FB$ so:

$$\boldsymbol{M} = \frac{FB}{FI_\circ}\ \frac{FD}{FB}$$

measure $\boldsymbol{M} = \frac{BH}{BH_\circ}\ \frac{FD}{FB}$

by measuring FD and BD
to find FB,

and by measuring BL to find

$$\frac{\mathbb{R}}{BE} = \frac{1}{EL} = \frac{\mathbb{R} - 1}{BL}$$

in order to calculate BH using:

E B
|________|________|_________|_________|
H Z L G F

$$\frac{\mathbb{R}}{BE} = \frac{1}{BF} + \frac{\mathbb{R}}{BH}$$

---

or

E G
F Z H L B
|________|________|_________|_________|

E G
Z F H L B
|________|________|_________|_________|

$$\frac{\mathbb{R}}{BH} = \frac{1}{BF} + \frac{\mathbb{R}}{BE}$$

---

note that the condition producing a virtual image at H:

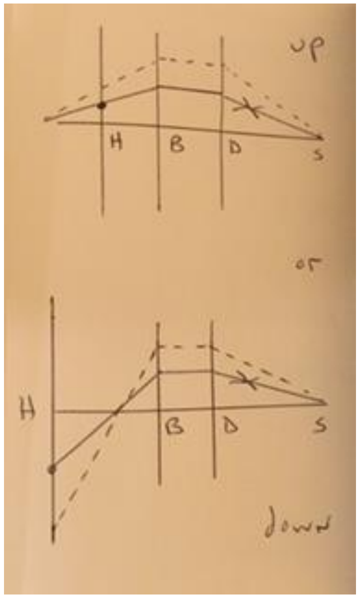
```
E                   B
|_________|_________|__________|__________|
Z        L         G          F          H
```

$$\frac{1}{BF} = \frac{\mathbb{R}}{BE} + \frac{\mathbb{R}}{BH}$$

is meaningless when considering the focused axial image size magnification $\frac{BH}{BH_{\circ}}$

when the standard image is real.

Figure 61:



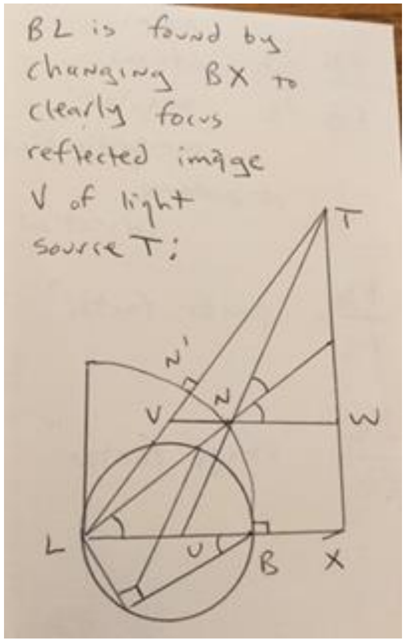
keeping only:
ZAHCBNDGFEL and ℝ:

when a light source at S
focused towards ∞ at D
tilts up
the reflection off a surface at H observed at S
of its unfocused off-axis image
moves up or down

adding possible distance corrections with known values
of FD at D
the proper distance correction

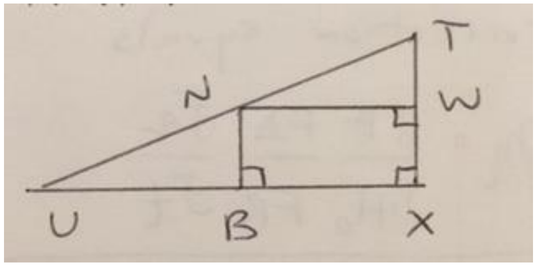
can be found
which moves the focused image
of S on axis at H
and eliminates this movement

Figure 62:



BL Is found
by changing BX
to clearly focus
the reflected image V
of light source T

Figure 63:

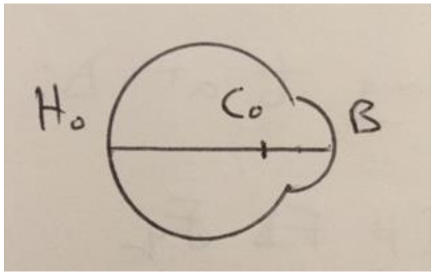


make T $\Rightarrow$ X
so that 2BU $\Rightarrow$ BL
and $\angle$NBU $\Rightarrow$ $\frac{\pi}{2}$
so that:

$$\frac{XT}{XW} \rightarrow \frac{UX}{UB} \rightarrow \frac{2UX}{BL} \leftarrow \frac{2VW}{BL}$$

with a very small XT
measure XW and VW
to approximate BL

Figure 64:



keeping only:
ZAHCBNDFGEL and ℝ:

using BH_o as the chosen ocular standard where:

$$\mathbb{R} = \frac{H_oB}{H_oC_o} = \frac{HB}{HC} = \frac{EB}{EL} = \frac{4}{3}$$

and $\frac{\mathbb{R}}{BH_o}$ = 60 diopters

(where a diopter is a unit of inverse meter length)

only the corneal component ***K*** of $\frac{\mathbb{R}}{BE}$ can be approximated with BL from the reflection off B

when its deviation from the standard 42
is assumed to equal the deviation
of the total $\frac{\mathbb{R}}{BE}$

from its standard of 60:

$$\boldsymbol{K} + (42 - \boldsymbol{K}) = 42$$

$$\frac{\mathbb{R}}{BE} + (42 - \boldsymbol{K}) = 60$$

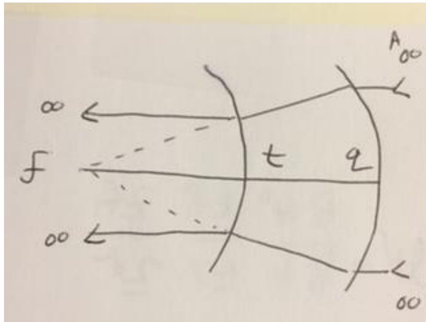
$$\frac{\mathbb{R}}{BE} = \boldsymbol{K} + 18$$

and since:

$$\boldsymbol{M} = \frac{\mathbb{R}}{BH_o} \quad \frac{BH}{\mathbb{R}} \quad \frac{FD}{FB}$$

$$\boldsymbol{M} = \frac{60}{\frac{\mathbb{R}}{BE} \pm \frac{1}{BF}} \quad \frac{(FD)}{(FB)}$$

Figure 65:



when the front surface of a spectacle lens that
corrects distance refractive error is not flat
it is convex
and produces additional
axial afocal angular magnification

placing t at D:

$$\boldsymbol{M} = \frac{BH}{BH_{\circ}} \; \frac{FD}{FB} \; \frac{fq}{ft}$$

In summary:

axial magnification of distance
correction equals:

$$\boldsymbol{M} = \frac{BH}{BH_{\circ}} \; \frac{FD}{FB} \; \frac{fq}{ft}$$

where:

$\frac{BH}{BH_\circ}$ = axial corrected image size magnification

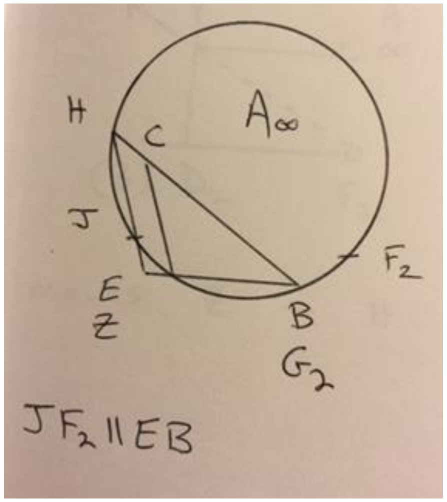
and:

$\frac{FD}{FB} \frac{fq}{ft}$ = axial afocal angular magnification of distance correction

$\frac{FD}{FB}$ = “power factor”

$\frac{fq}{ft}$ = “shape factor”

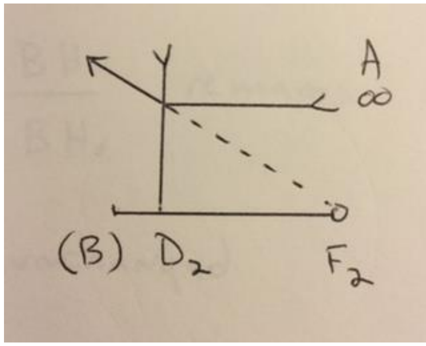
Figure 66:



adding new myopic
distance error
at G_2 (at B)

JF_2 || EB

Figure 67:

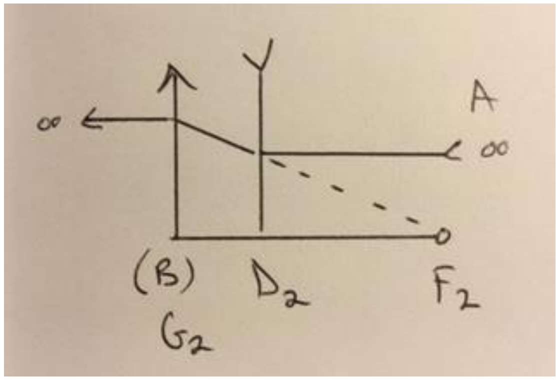


the new myopic distance correction at D_2 moves Z to H

and retinal image size magnification remains unchanged:

$$\frac{BH}{BH_{\circ}}$$

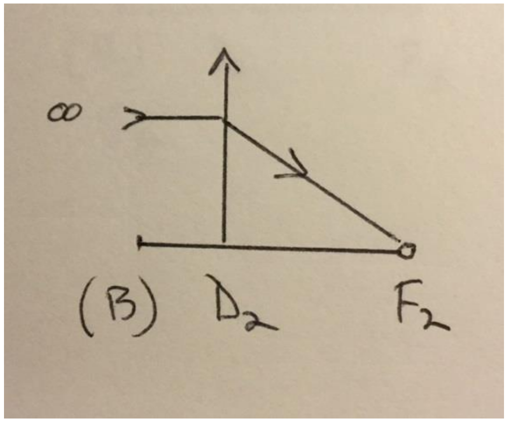
Figure 68:



the new myopic
distance correction at D_2
produces the additional
axial afocal angular magnification
factor

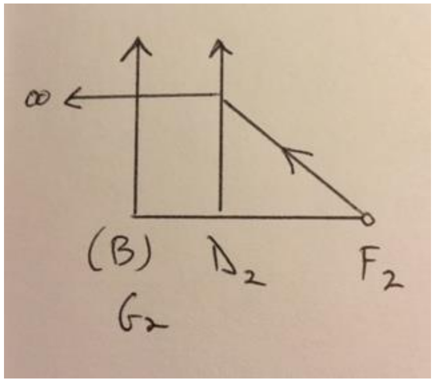
$$\frac{F_2 D_2}{F_2 B}$$

Figure 69:



removing the new
myopic distance correction
at D_2 using a magnifier
(converging lens)
creates a near correction for F_2
(shown with reversed light)

Figure 70:



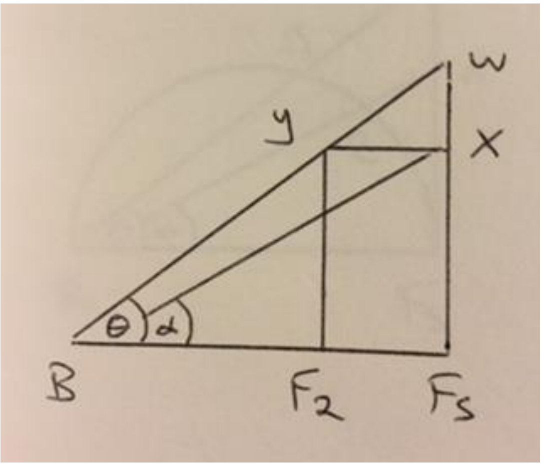
this near correction removes the axial afocal angular magnification of distance correction factor of

$$\frac{F_2D_2}{F_2B}$$

by the addition of the axial magnification of near correction factor of

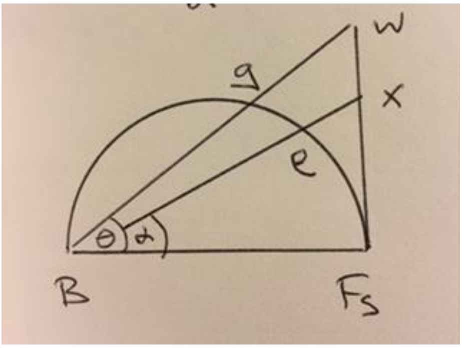
$$\frac{F_2B}{F_2D_2}$$

Figure 71:



when an object at
a standard distance Fs
is moved to F_2

Figure 72:



the near object subtense magnification equals

$$\frac{\theta}{\alpha} = \frac{\sim gFs/BFs}{\sim eFs/BFs}$$

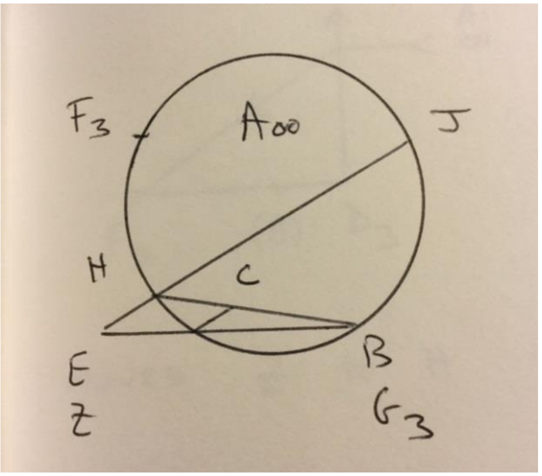
as $yF_2 = xFs \Rightarrow 0$:

$$\frac{\theta}{\alpha} \Rightarrow \frac{wFs}{xFs} = \frac{wFs}{yF_2} = \frac{BFs}{BF_2}$$

multiplying this factor
by the axial magnification
of near correction for F_2 produces:

$$\frac{F_2B}{F_2D_2}\,\frac{BFs}{BF_2} = \frac{BFs}{F_2D_2}$$

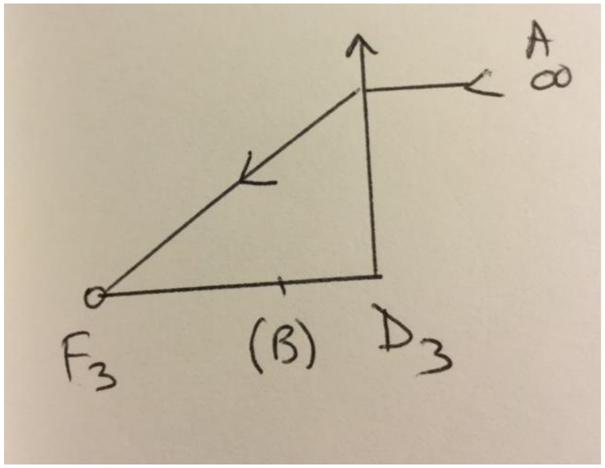
Figure 73:



adding new hyperopic
distance error
at G_3 (at B)

$JF_3 \parallel EB$

Figure 74:

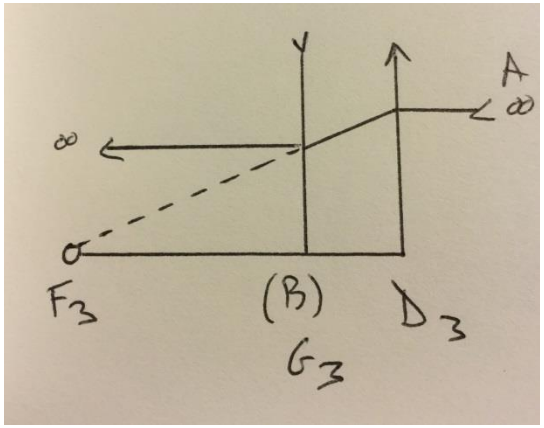


the new hyperopic
distance correction
at D_3 moves Z to H

and retinal image size magnification
remains unchanged:

$\frac{BH}{BH_o}$

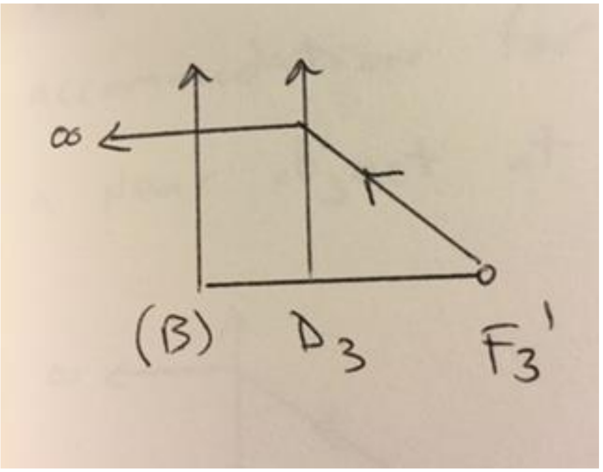
Figure 75:



the new hyperopic
distance correction at D_3
produces the additional
axial afocal angular magnification
factor

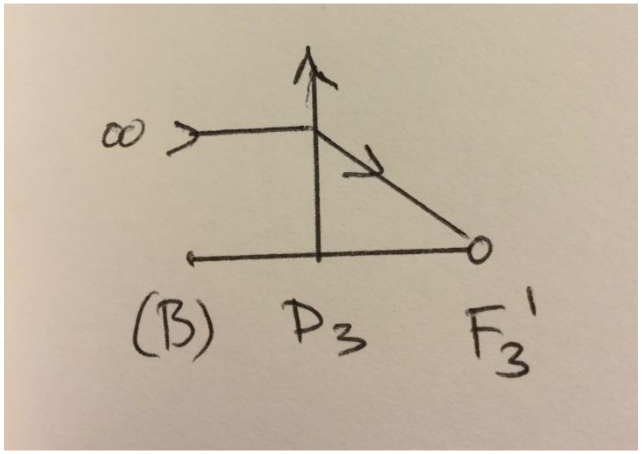
$$\frac{F_3 D_3}{F_3 B}$$

Figure 76:



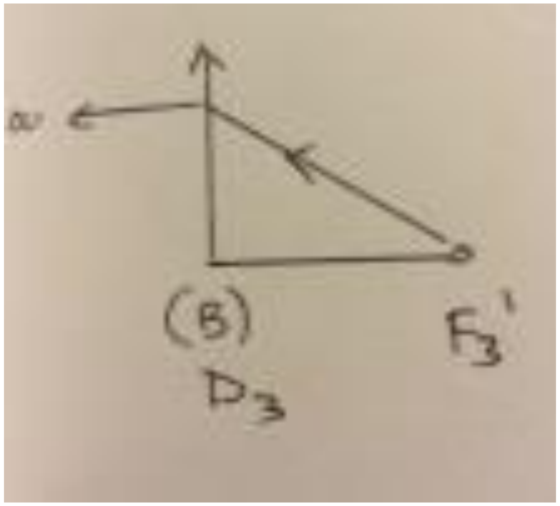
removing the new hyperopic
distance ***error*** at G_3
without removing
its correction at D_3
creates a near correction for F_3'

Figure 77:



the new hyperopic
distance correction at D_3
shown with reversed light
as a magnifier (converging lens)

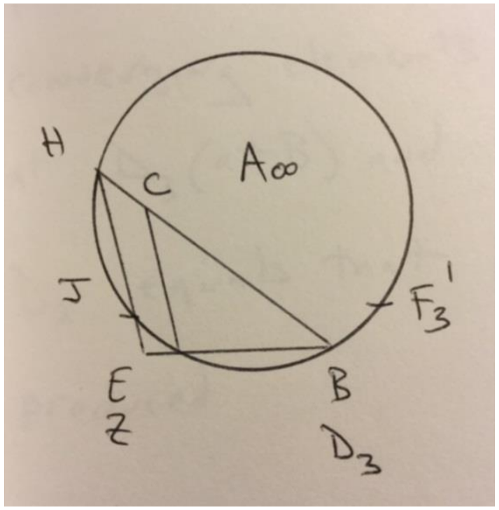
Figure 78:



removing the new hyperopic
distance ***error*** at G_3
without removing
its correction at D_3
creates a near correction for F_3'

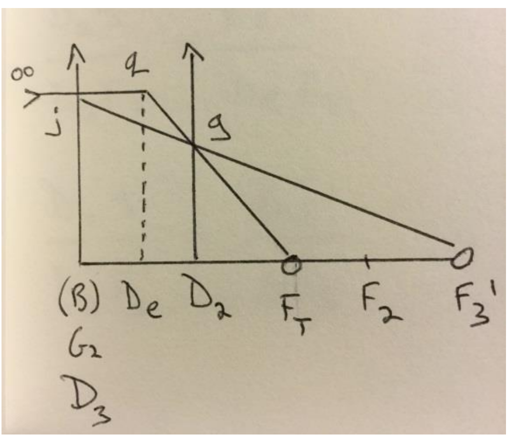
when this near correction
lies at B
this can represent
a new myopic distance error
at B
or "ocular accommodation"
at B

Figure 79:



$JF_3' \parallel EB$

Figure 80:



the total axial magnification
of near correction
produced by both
converging elements
at D_3 (at B) and D_2
equals that produced ***as if***
all convergence occurred
at the single axial point D_e
so that the axial magnification
of near correction factor equals

$$\frac{F^T B}{F^T D_e}$$

$$\frac{D_2 g}{D_2 F^T} = \frac{D_e q}{D_e F^T}$$

$$\frac{D_2 g}{D_2 F_3'} = \frac{Bj}{BF_3'}$$

$$D_2 F^T \frac{(D_e q)}{(D_e F^T)} = D_2 F_3' \frac{(Bj)}{(BF_3')}$$

$$\frac{D_e q}{D_e F^T} = \frac{D_2 F_3'}{D_2 F^T} \frac{Bj}{BF_3'}$$

$$\frac{1}{D_e F^T} = \frac{D_2 F_3'}{D_2 F^T} \frac{Bj}{BF_3'} \frac{1}{D_e q}$$

$$= \frac{D_2 F_3'}{D_2 F^T} \frac{1}{BF_3'}$$