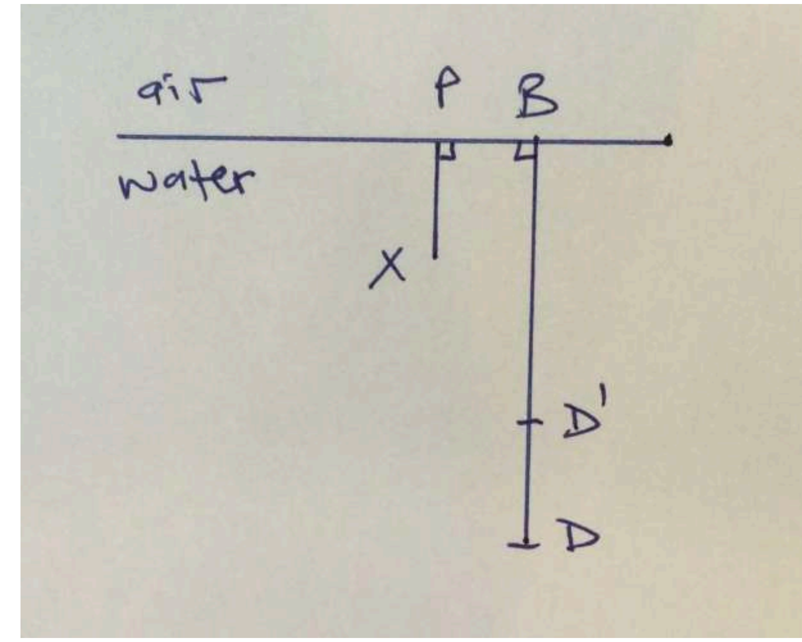
The “Coin-in-fountain” Example

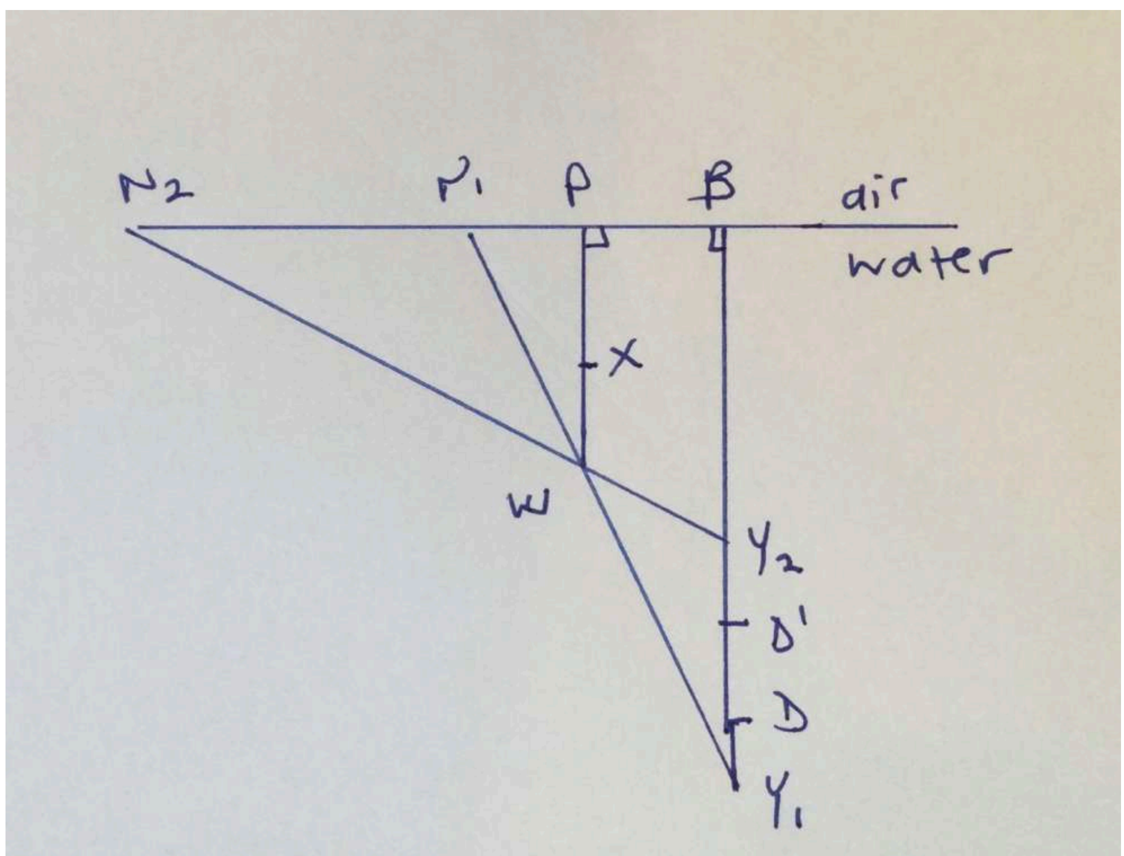
Imagine a coin at the bottom of a fountain in a city park. While sitting on the edge of the fountain looking down and forward towards the refracted image of the coin, we perceive its image location relative to the object using only the image rays within the plane connecting the two, the “tangential plane.” Isaac Barrow showed that this image does not lie directly above the object, but above and slightly towards us.

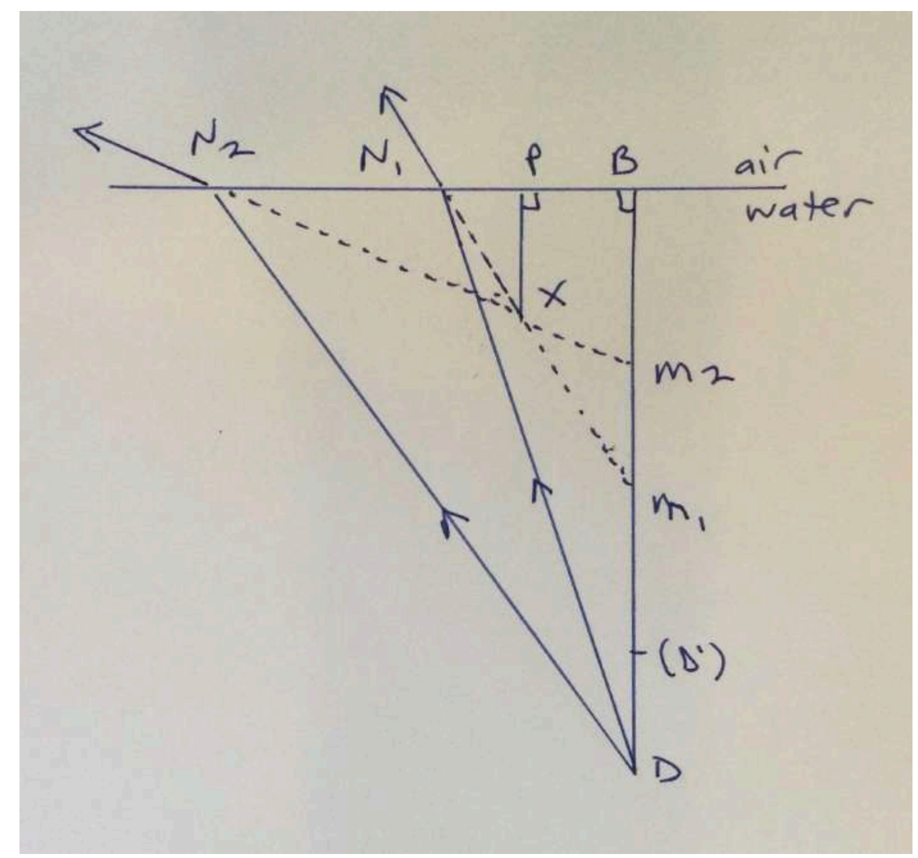
For example, let the object lie at point D; and its perpendicular distance from the water's surface, (DB), equal 4 inches. Let the object's image along the perpendicular be at D'; and its perpendicular distance from the water's surface, (D'B), equal 3 inches. This would occur since the index of refraction of water, (4/3), would have to equal DB/D'B.



Isaac Barrow described a method of finding all the possible image rays through a specific image point X, without knowing their points of refraction along the surface of the water, or their intersection with the perpendicular DB.

He also showed that there can be a maximum of *two* image rays through X, since only two segments equaling his calculated constant YWN fit through W within the right angle at B.

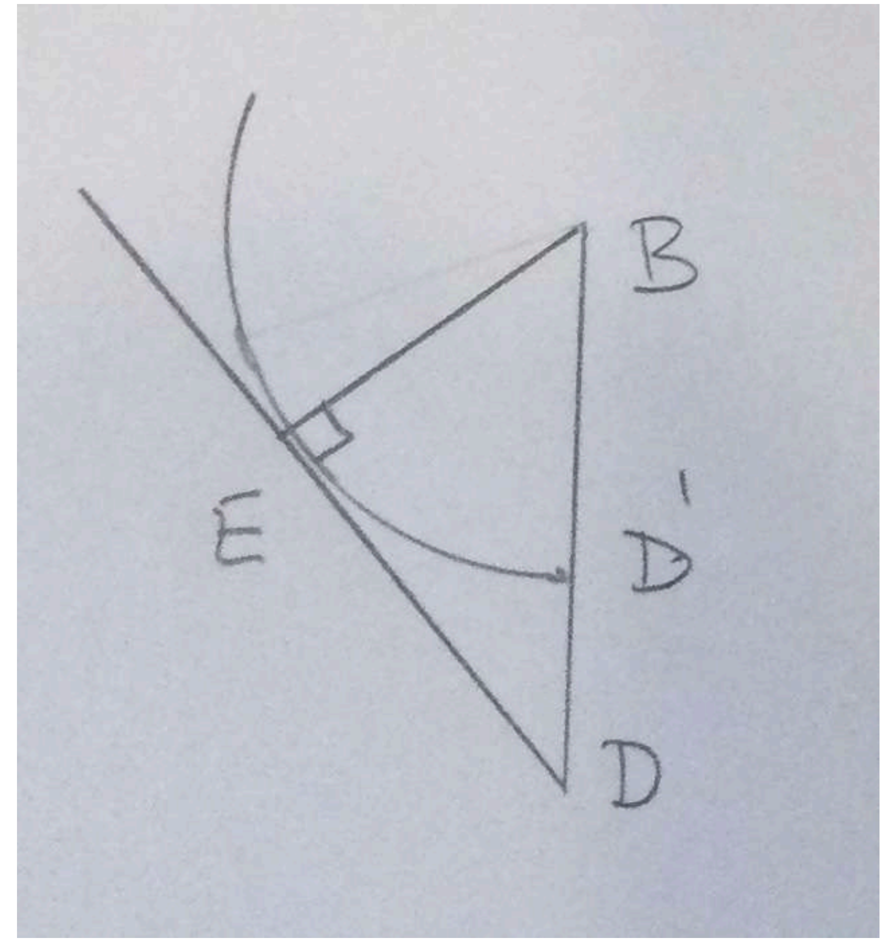


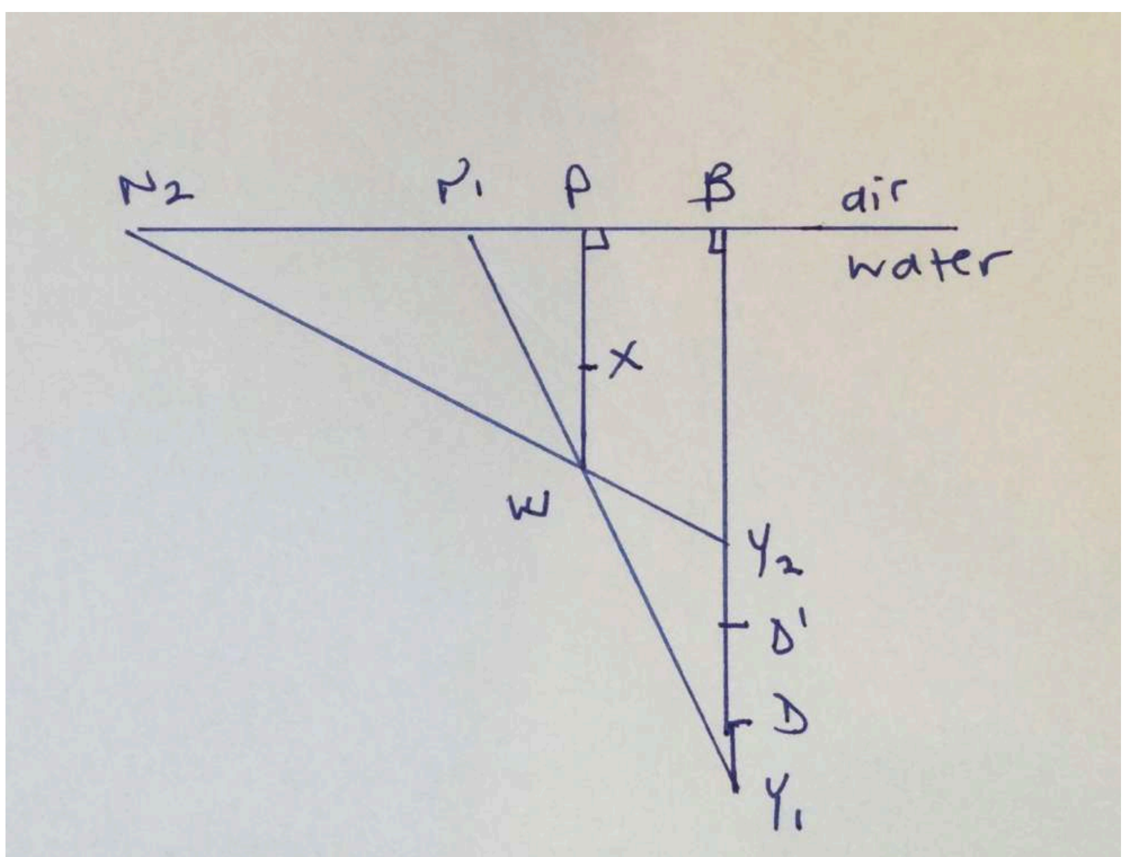


To locate the two possible image rays through X, we first locate point W using his calculation:

PW/PX = DB/DE = 1.5

When PX = 1 inch,
PW = 1.5 inches

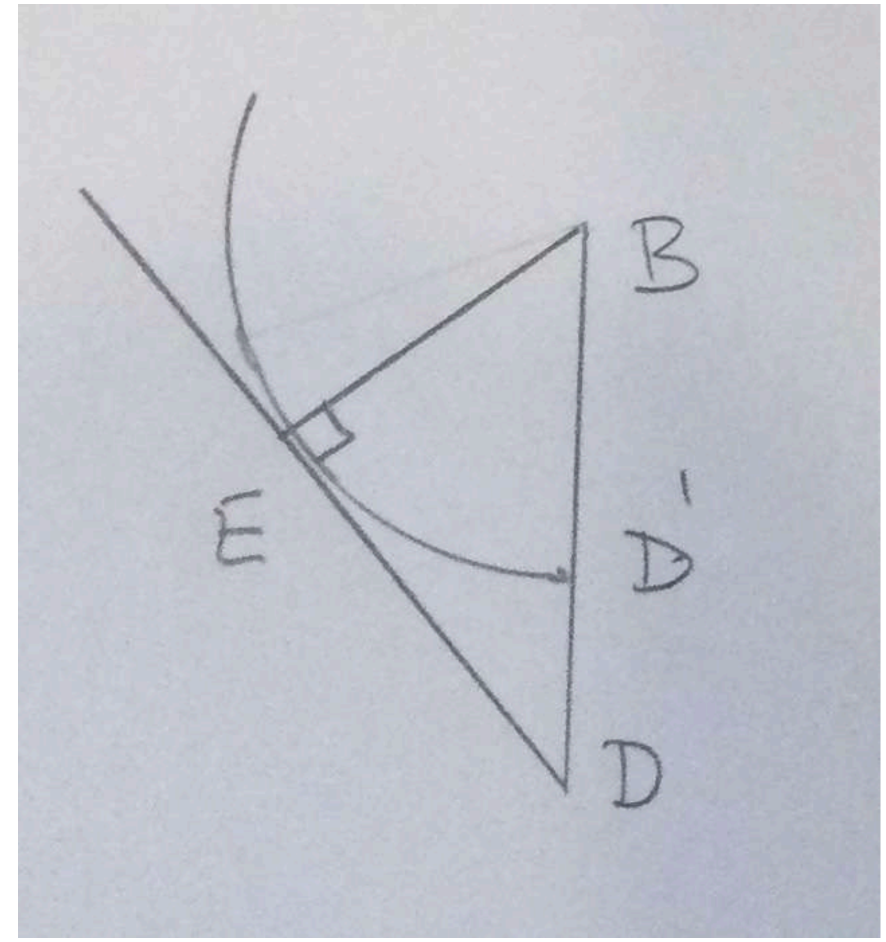




DB/DE = $4/\sqrt{7}$= 1.5

ED/EB = $\sqrt{7}/3$ = 0.88

We then calculate the constant reference line segment length YWN, (based solely on the distance DB and the index of refraction), using his formula:



DB/YWN = ED/EB

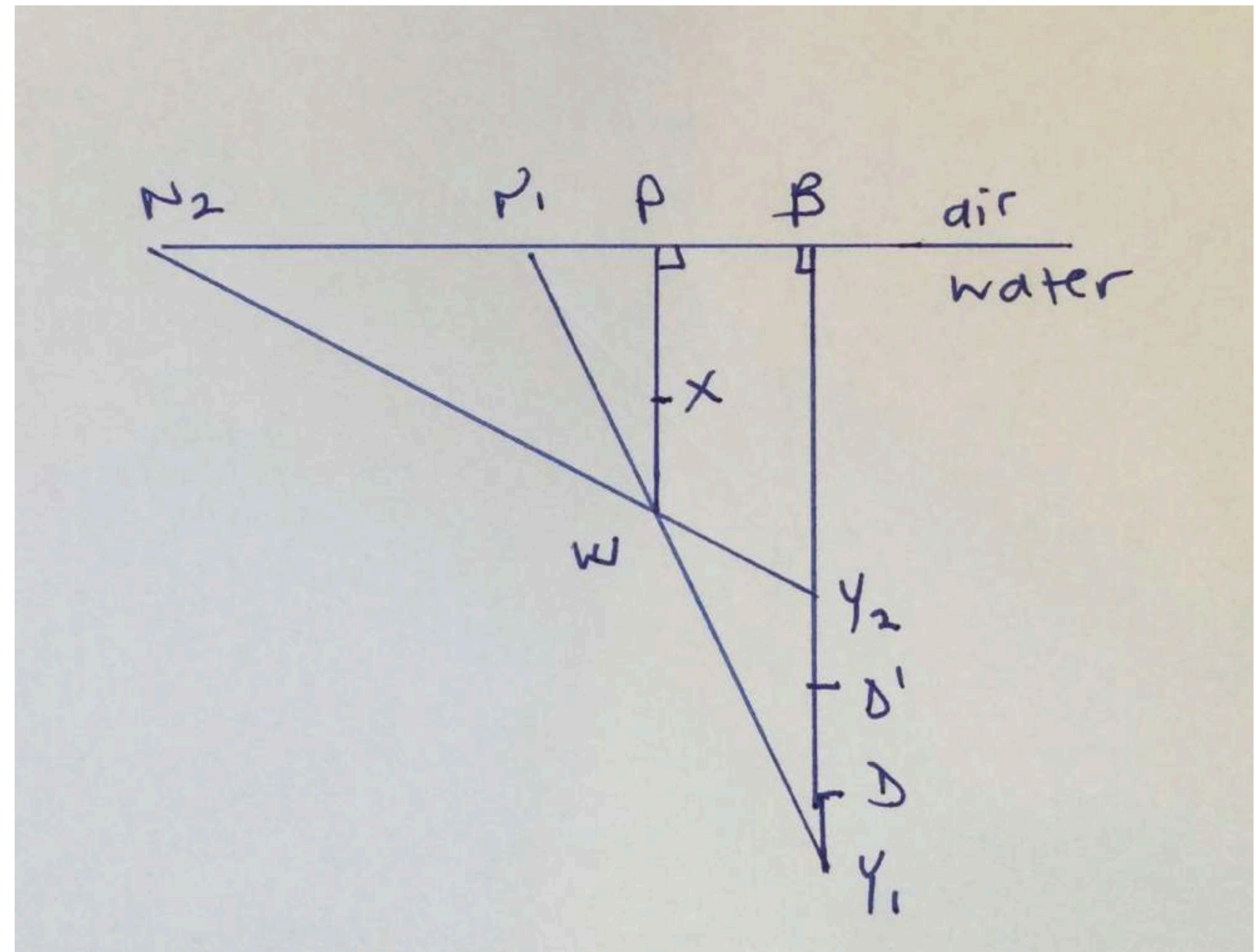
So YWN = DB/0.88

YWN = 4.54 inches

DB/DE = $4/\sqrt{7} = 1.5$

ED/EB = $\sqrt{7}/3 = 0.88$

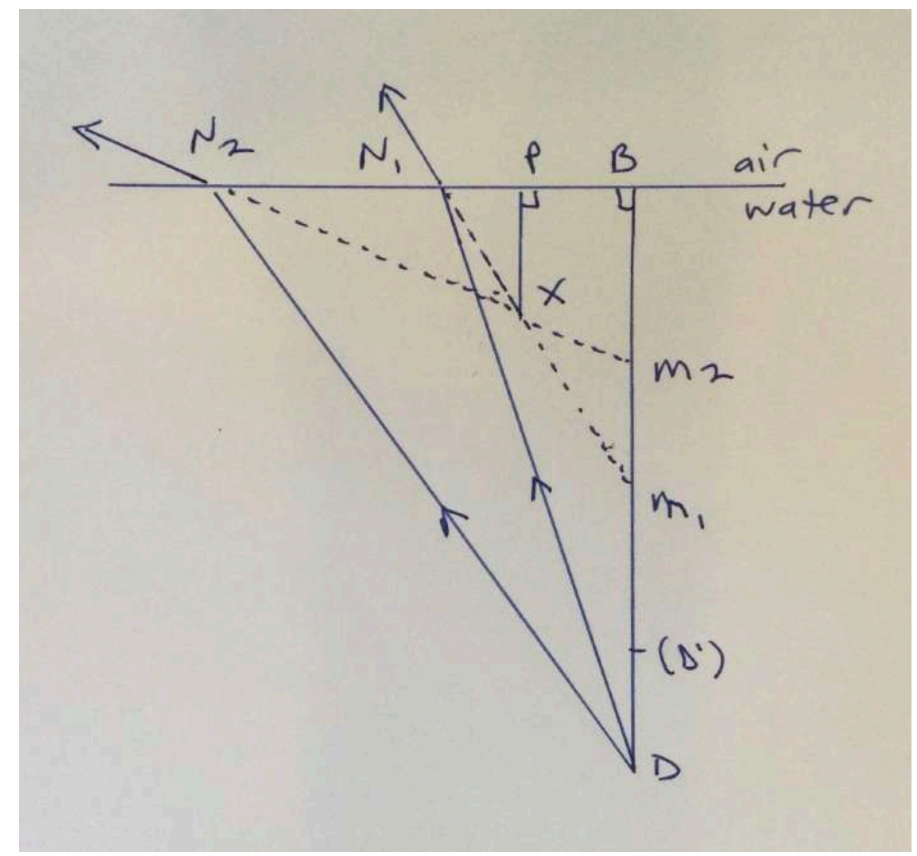
Remember that Y_1WN_1 and Y_2WN_2 are neither object or image rays, but rather reference line segments that simply allow for the determination of points N_1 and N_2, which are the points of refraction that produce image rays through X.



Once these two image rays are drawn, we can measure them in inches to confirm that:

$DB/D'B = DN_1/M_1N_1 = DN_2/M_2N_2 = 4/3$

$DB/D'B = 4/3$
$DN_1/M_1N_1 = 4.25/3.20 = 1.33$
$DN_2/M_2N_2 = 5.60/4.20 = 1.33$

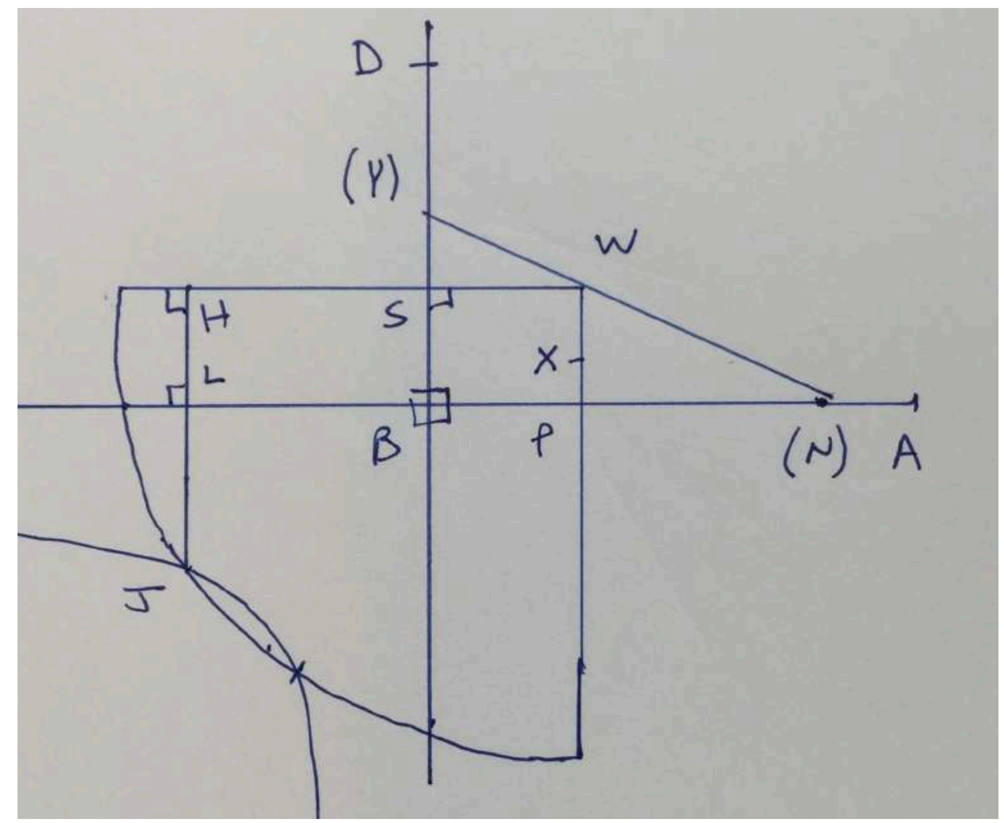


Isaac Barrow draws the line segments of calculated length YWN through the right angle at B by using a reference hyperbola where:

(LB)LJ = (BP)PW

and by making: PN = BL

so that: WJ = YWN



When the reference circle with radius WJ intersects the reference hyperbola at a *single* point J, the reference segment length YWN is a minimum, N_2 overlaps N_1, and X is a clear image. The position of the minimum YWN can be then found by simply making PN = BL.