Images Seen Through Water

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2021

Reference:

Isaac Barrow's Optical Lectures
1667

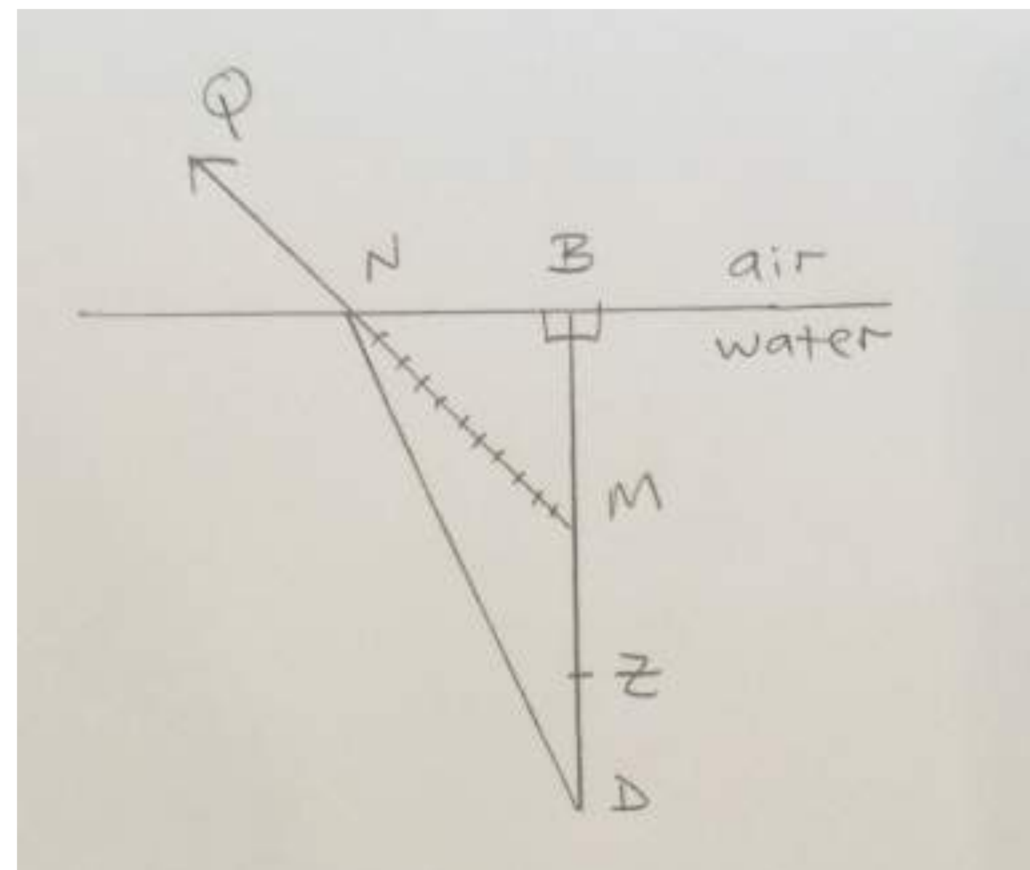
Translated by H.C. Fay
Edited by A.G. Bennett and D.F. Edgar

Published by
"The Worshipful Co. of Spectacle Makers"
1987

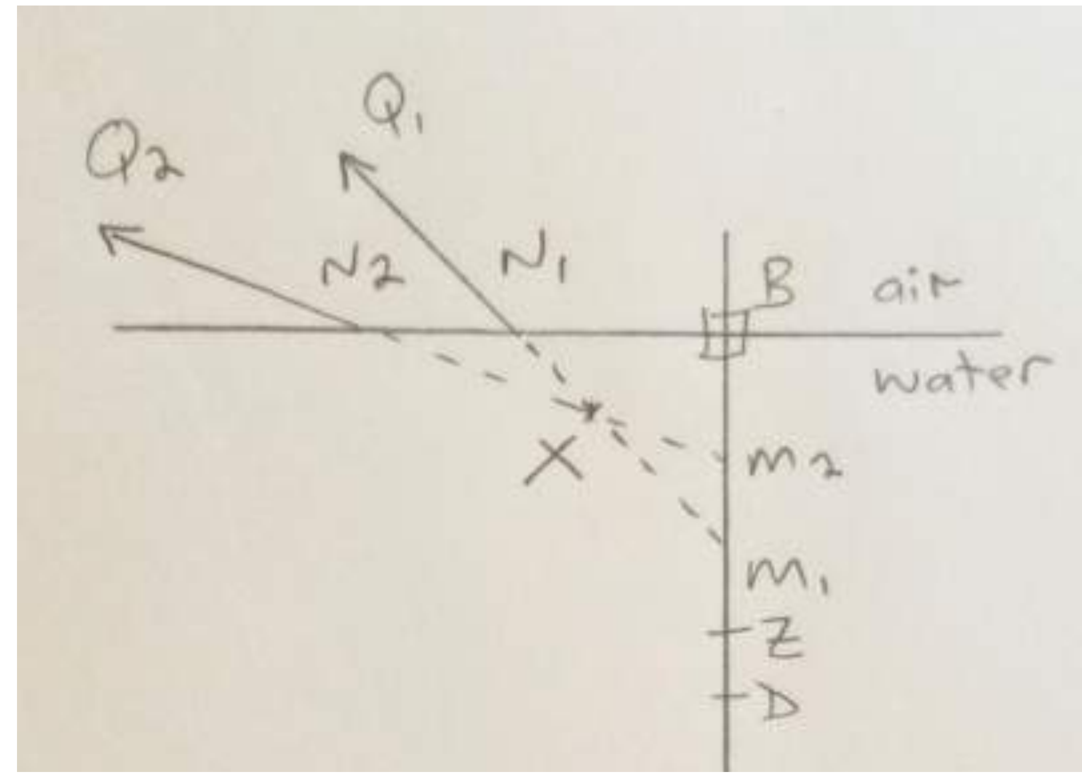
Lectures 4 & 5

If an underwater object D is at a perpendicular distance DB from the plane of the water's surface in all radial directions, the image of object D along that perpendicular, (when seen from directly above in air), is at Z, and BD/BZ = 4/3.

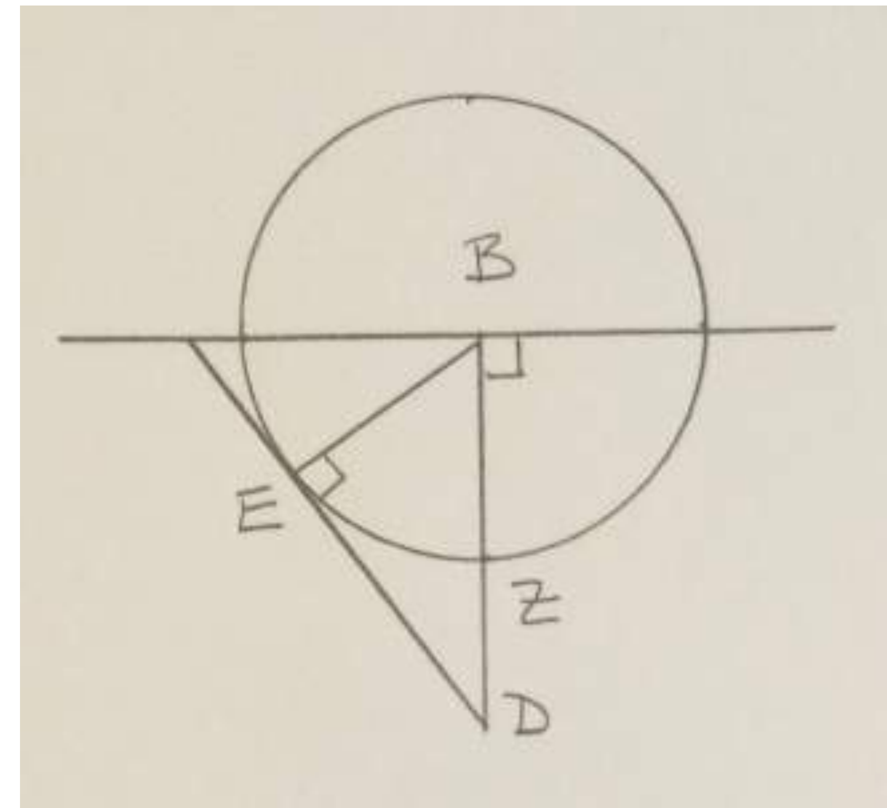
Isaac Barrow showed that the image of object D, (when seen from Q *obliquely* along image ray MNQ), lies above the object, but also towards the observer relative to DB, (the object's perpendicular to the surface).



As the first step in finding an oblique image ray XNQ, along which the image of object D is seen at a designated point, Isaac Barrow described a method of finding *all* possible oblique image rays through a designated point X, without knowing their points of refraction (N) along the surface of the water, or their intersections (M) with the perpendicular DB.



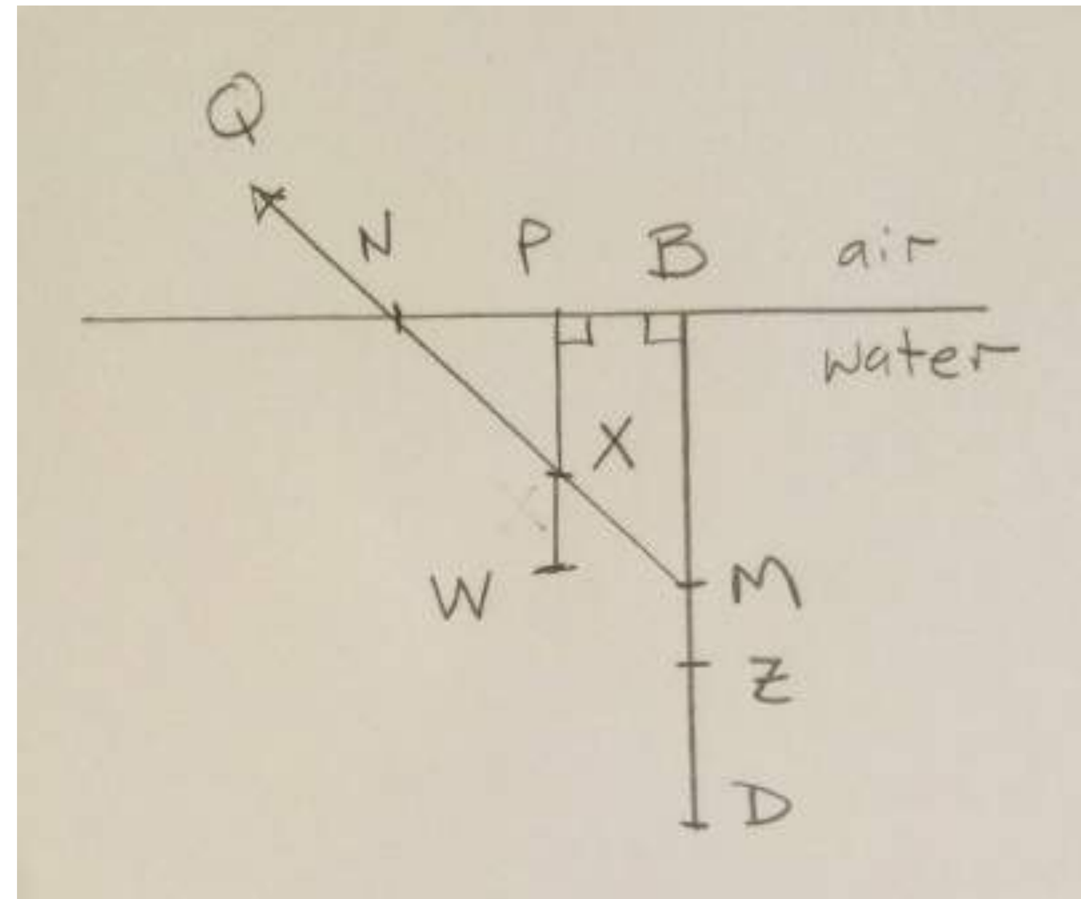
To do this, he first drew a *reference right triangle* created by drawing BE = BZ as shown, which created the following constant ratios for air/water refraction:



BD/BZ = BD/BE = 4/3
DB/DE = $4/\sqrt{(16-9)}$ = 1.5
ED/EB = $\sqrt{(16-9)}/3$ = 0.87

He showed that, given a designated desired clear image location X, if we draw PW as shown, where:

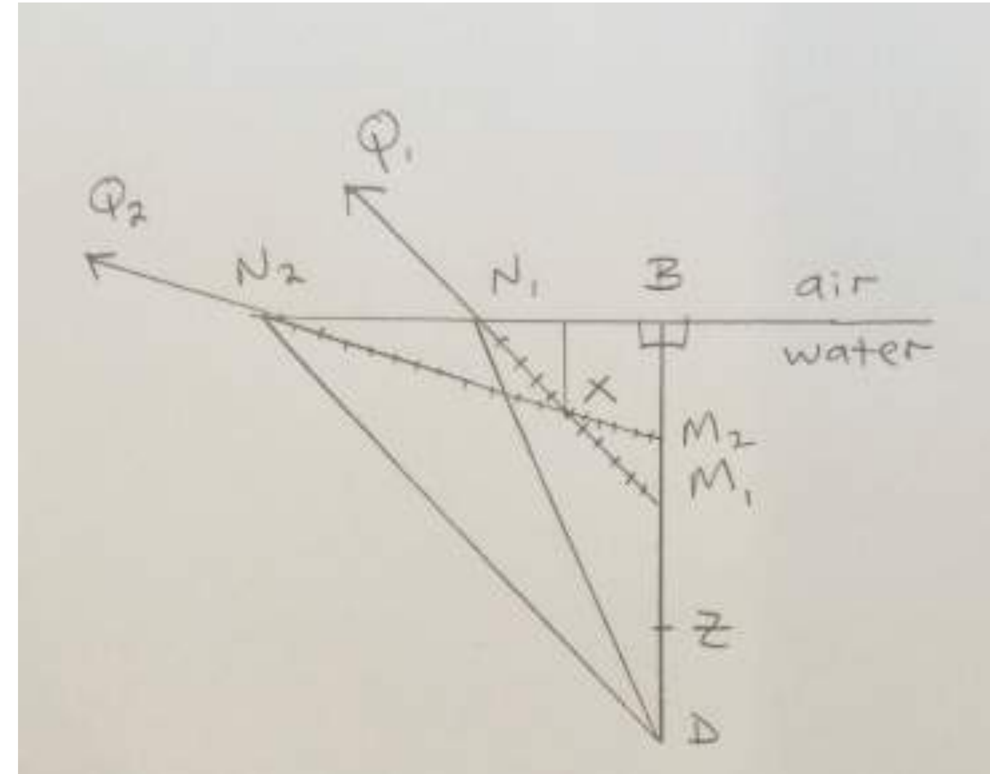
PW/PX = DB/DE = 1.5

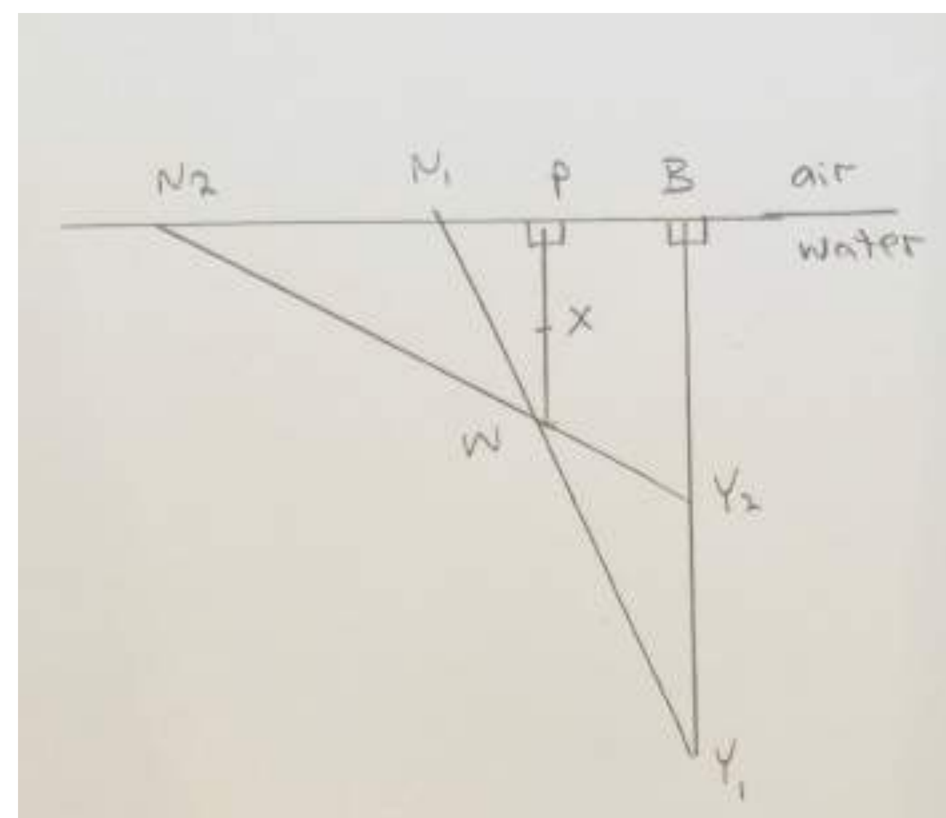


all possible image rays through X, (MXNQ) are found using:

DB/YN = ED/EB = 0.87

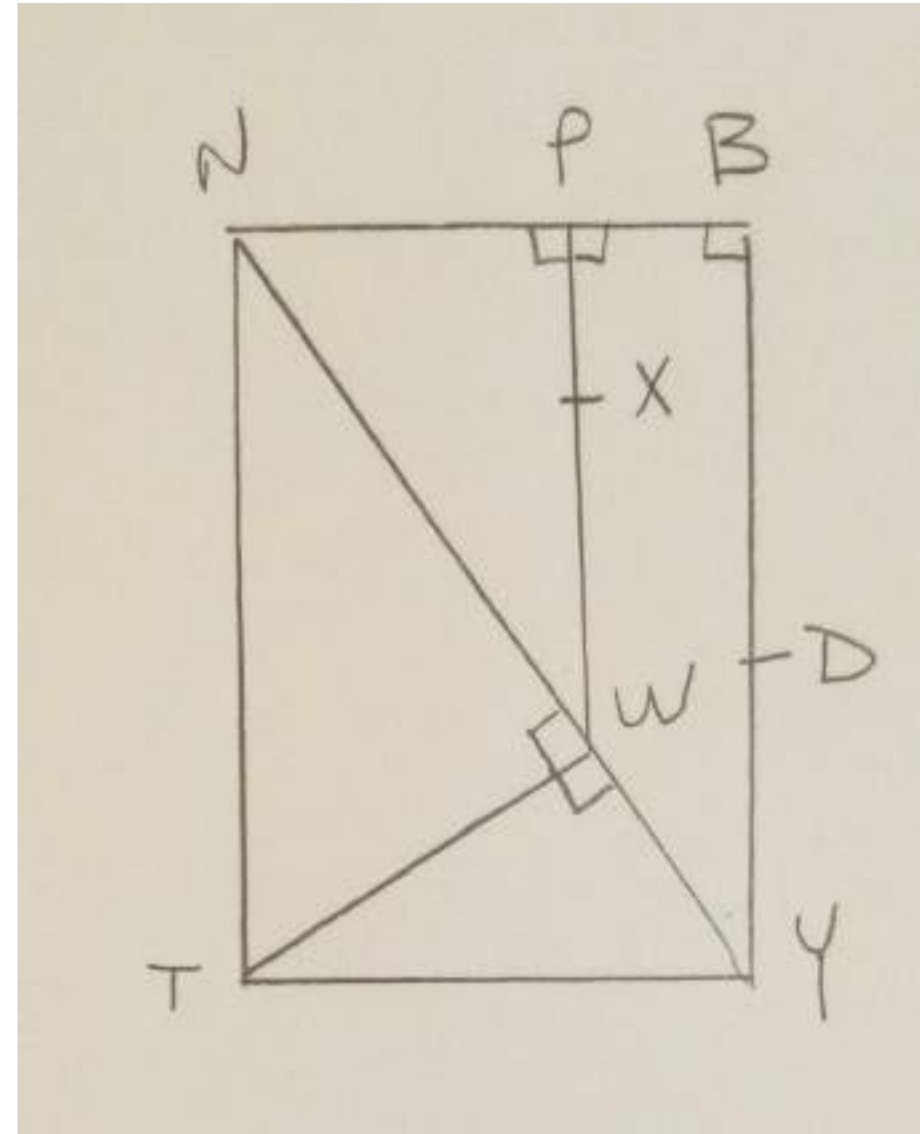
by drawing all possible reference lines of length YN = DB/0.87 through W, in order to locate the required positions of N.





He showed that there can be a maximum of *two* image rays through a designated point X, since only two reference line segments within the right angle $\angle$(Y)B(N), and equaling his calculated constant YN, can fit through point W. This is true since $Y_2N_2 = Y_1N_1$ means that the right triangle ΔY_2BN_2 must equal the right triangle ΔN_1BY_1.

Isaac Barrow showed that YN can be drawn as the shortest segment through W bounded by the right angle ∠(Y)B(N) when right triangles ΔYBN, ΔNWT, and ΔTWY are all drawn as similar.

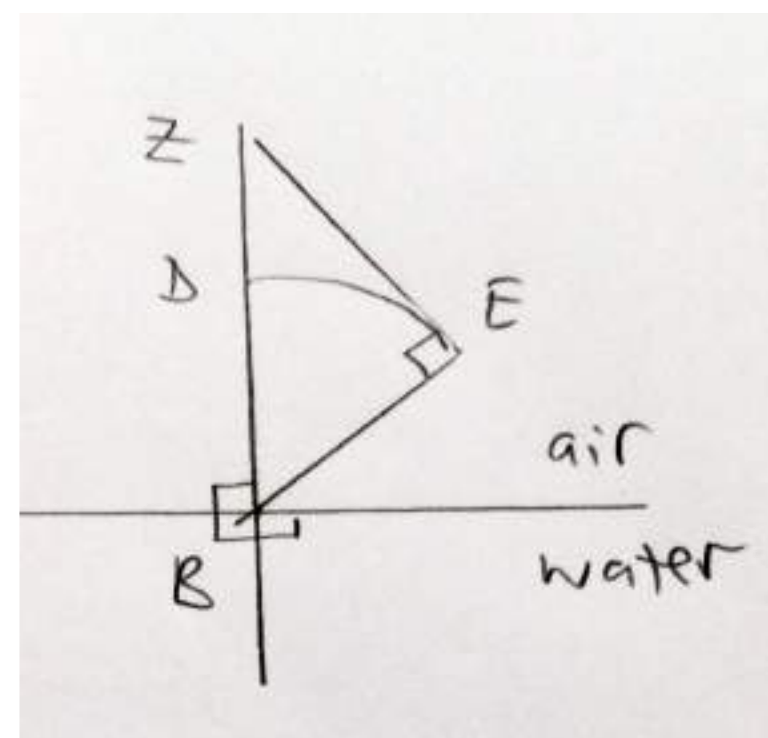


The *length* of YN through a designated W and bounded by the right angle ∠(Y)B(N) must be varied as it is rotated about W to find the position of its minimum length. Therefore, the position of N and Y must change to find N that corresponds to an image ray QNXM with its clear image at the designated (unchanging) point X. Furthermore, since:
PW/PX = DB/DE is constant,
ED/EB = DB/YN is also constant,
so DB varies with the length YN as a constant proportion.

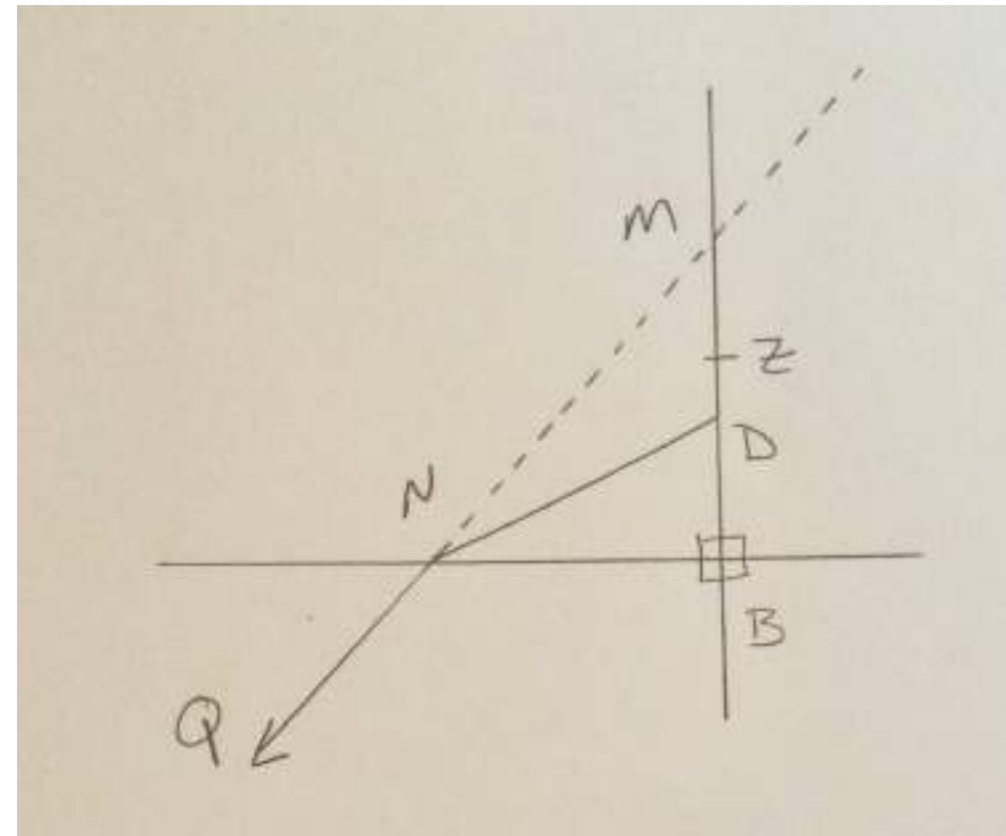
With an object underwater, Isaac Barrow's method does not allow for finding the location of the image ray on which a designated clear image is seen, while keeping both the image location *and the object position* constant. It does, however, allow for a geometric understanding of the conditions required to provide a clear image. As will be now demonstrated, with an object in air, Isaac Barrow's method actually *does* allow for finding the location of the image ray on which a designated clear image is seen, while keeping both the image location and the object position constant.

If object D is in air, and at a perpendicular distance DB from the surface of the water in all radial directions, the image of the object along that perpendicular when seen from underwater is at Z, and BZ/BD = 4/3.
A *reference right triangle* created by drawing BE = BD as shown, creates the following additional constant ratios:

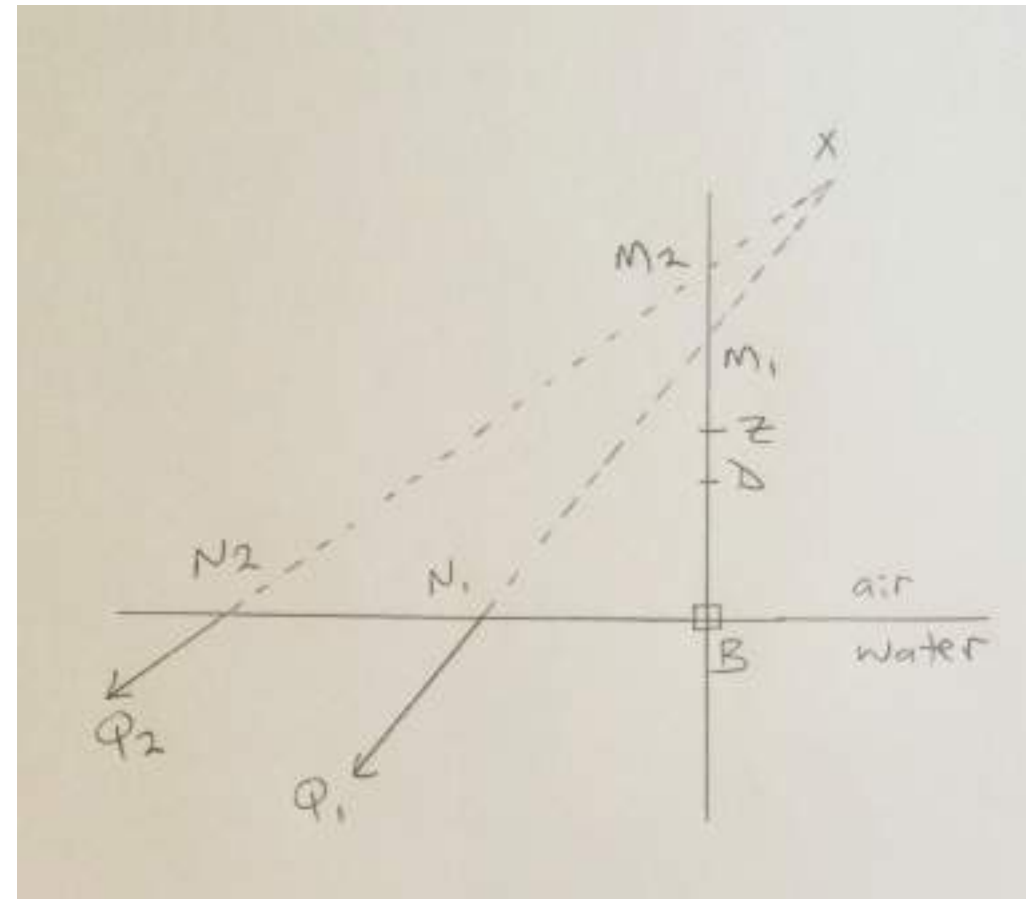
BZ/BE = 4/3
ZB/ZE = 4/$\sqrt{}$(16-9) = 1.5
EZ/EB = $\sqrt{}$(16-9)/3 = 0.87



Isaac Barrow showed that the image of object D, (when seen from Q *obliquely* along image ray MNQ), lies above the object, but also away from the observer relative to DB, (the object's perpendicular to the surface).

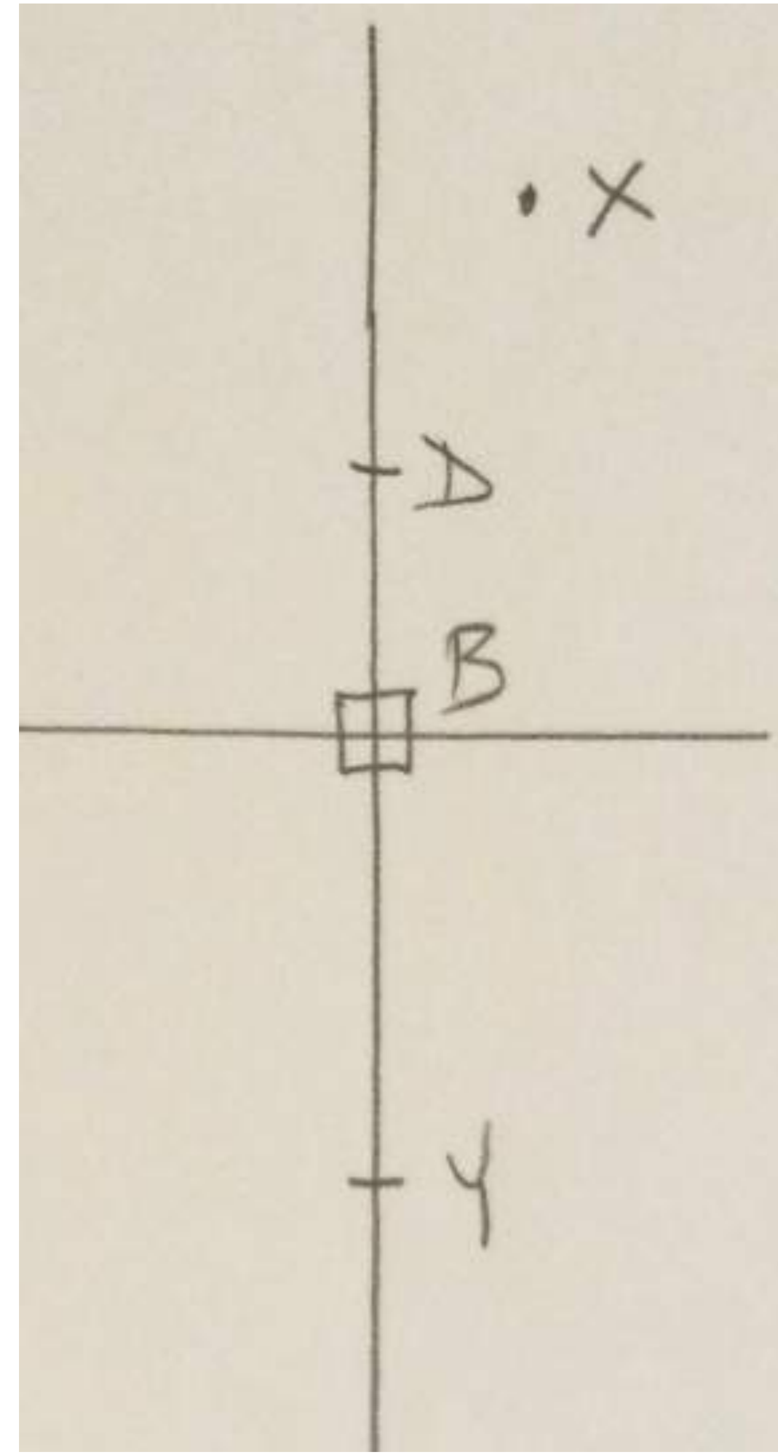


As the first step in finding an oblique image ray XMNQ, along which the image of object D is seen at a designated point X, Isaac Barrow described a method of finding *all* possible oblique image rays through point X, without knowing their points of refraction (N) along the surface of the water, or their intersections (M) with the perpendicular DB.

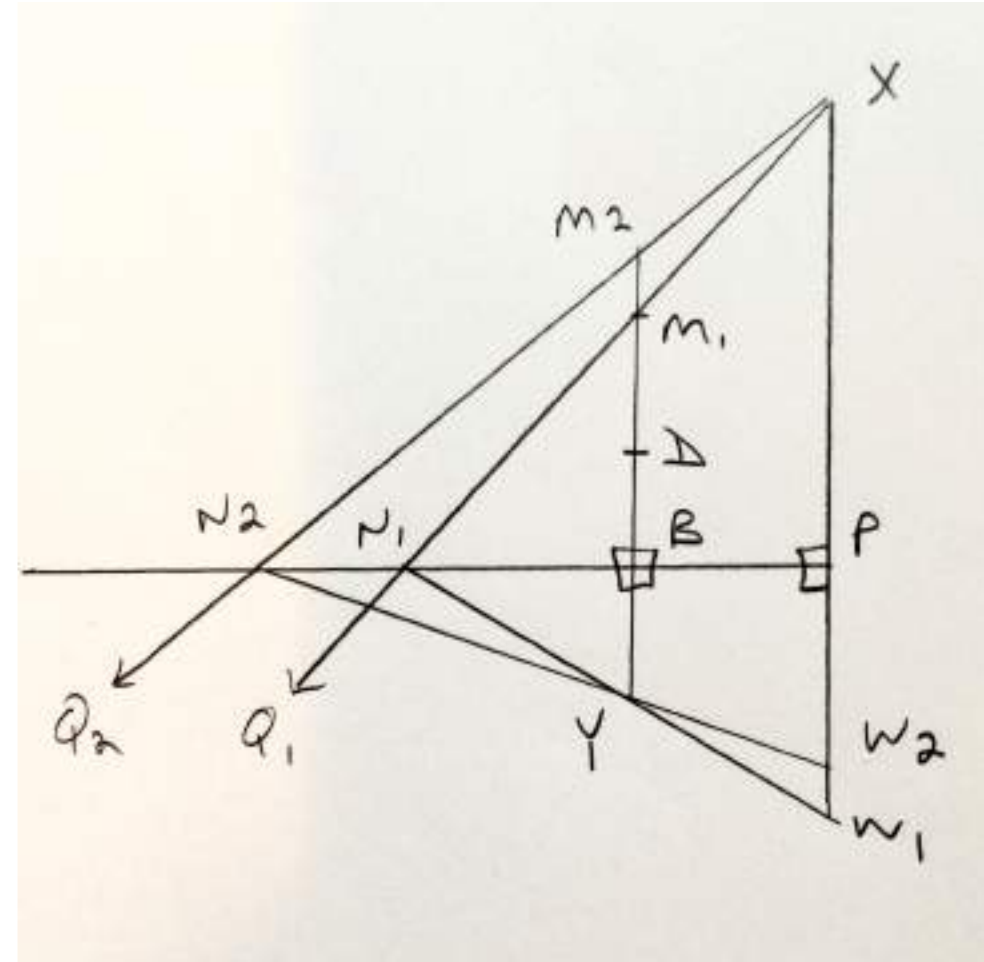


If we draw BY as shown, where:

BY/BD = ZB/ZE = 1.5

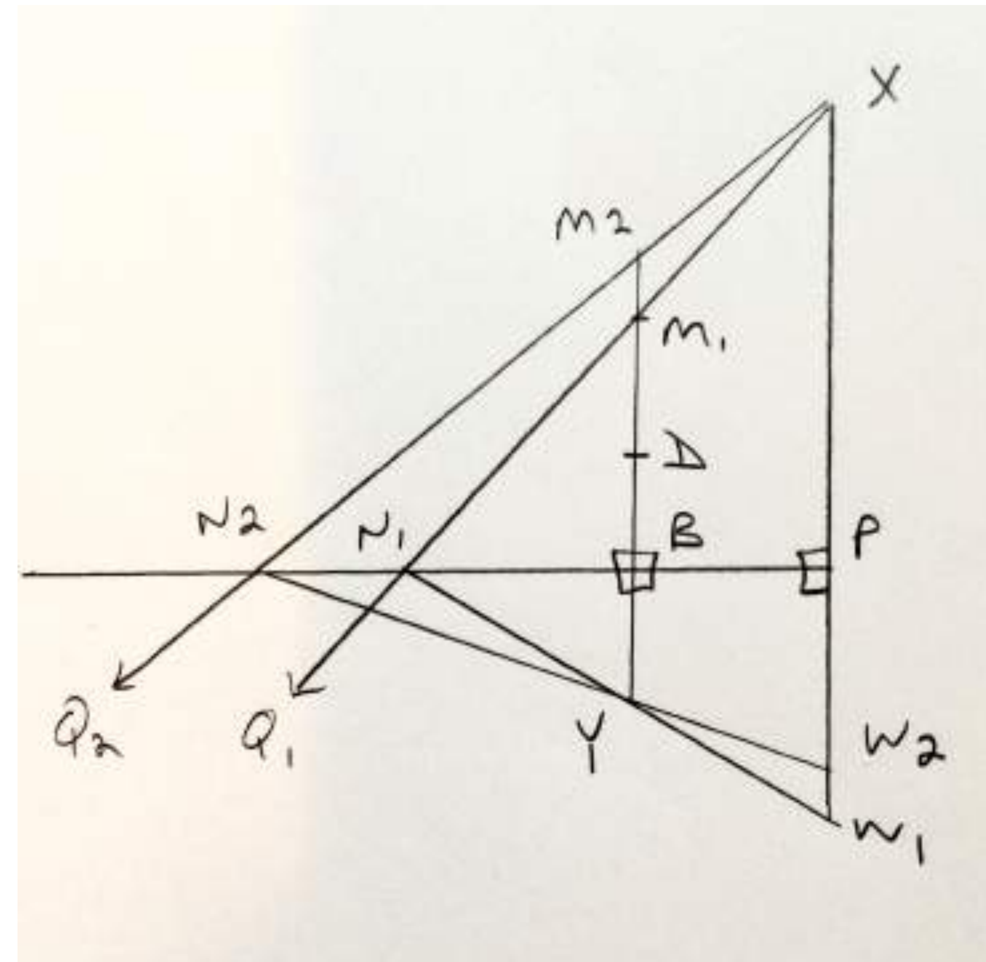


Isaac Barrow showed that all possible image rays through X, (XMNQ) are found using:

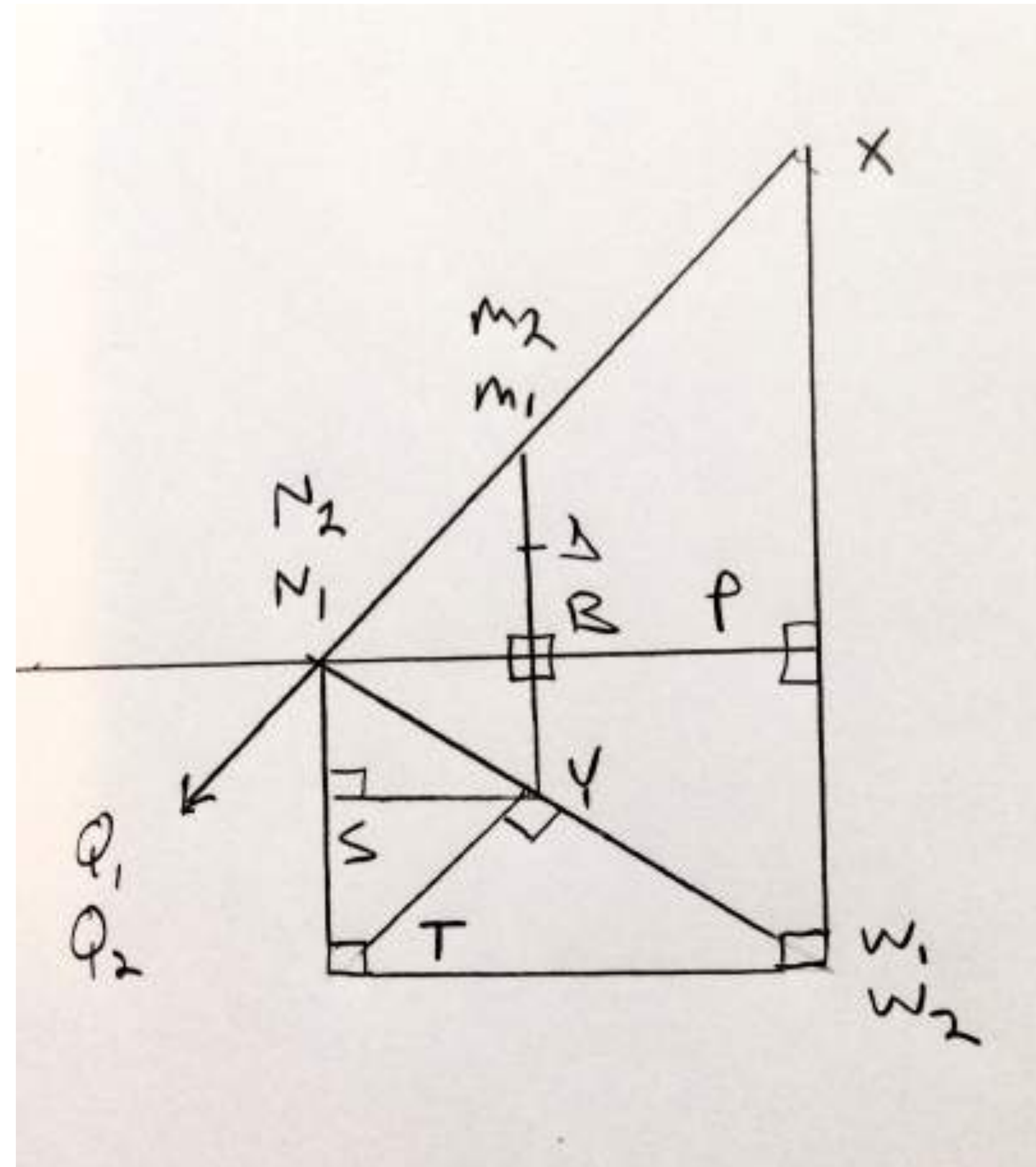


XP/WN = MB/YN = EZ/EB = 0.87
by drawing all possible reference lines of length WN = XP/0.87 through Y.

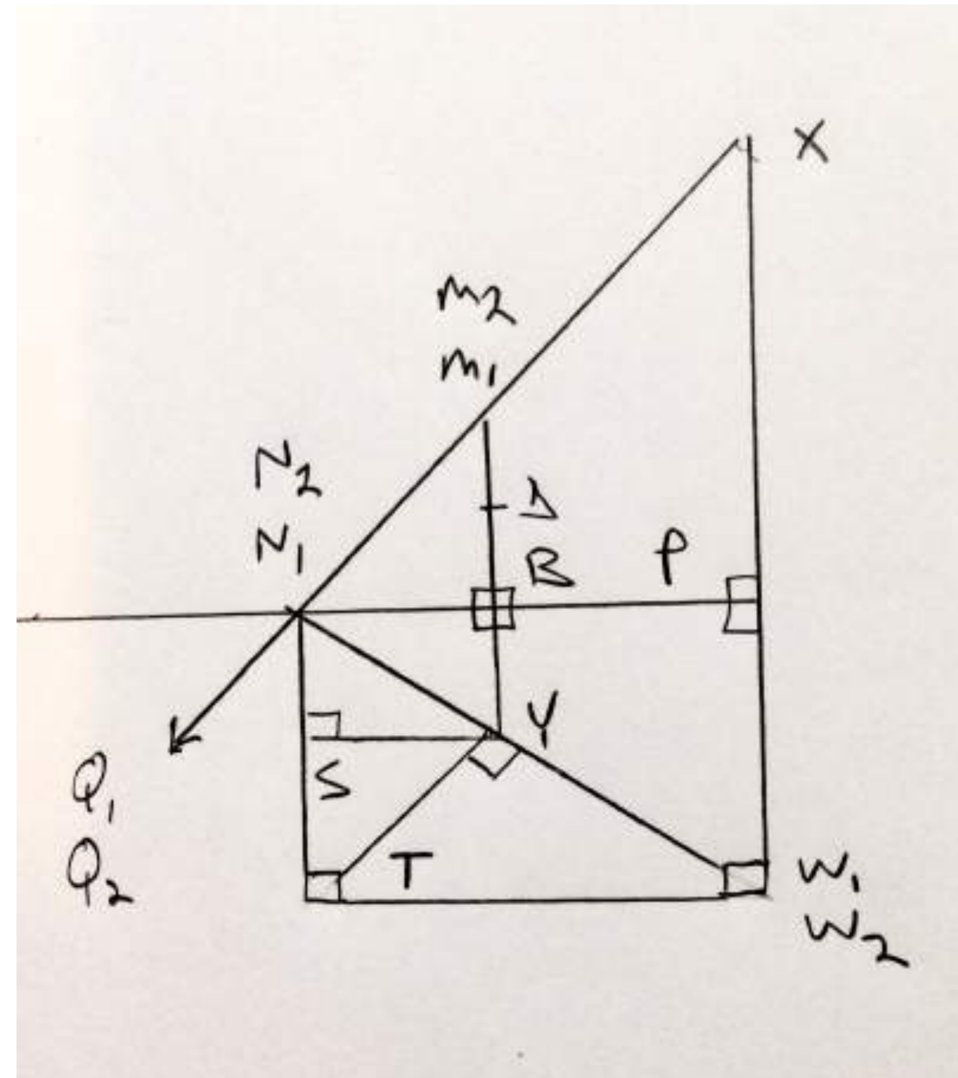
He showed that there can be a maximum of *two* image rays through any designated point X, since only two reference line segments within the right angle ∠(W)P(N), and equaling his calculated constant WN, can fit through point Y.



The point X that is the clear image of object D seen along a to-be-determined XMNQ is found using the *minimum* reference line segment length (W)Y(N) through Y, that is bounded by the right angle ∠(W)P(N).



Isaac Barrow showed that WN can be drawn as the shortest segment through Y bounded by the right angle ∠(W)P(N) when right triangles ΔWPN, ΔNYT, and ΔWYT are all drawn as similar.



As any two equal segments W_1YN_1 and W_2YN_2 are rotated about Y in order to approach their single common minimum length, N_2 approaches N_1, and ΔN approaches zero. Both the positions of N_2 *and* N_1 must change during this process of finding the point N associated with a designated clear image X.

Since Y (not W) is the pivot point as segments W_1YN_1 and W_2YN_2 rotate, BY remains unchanged. Therefore, BD also remains unchanged because BY/BD = BZ/BE. Therefore, unlike when the object is in water, when the object is in air, this method can find an image ray XMNQ that will produce a designated clear X, while holding the object position constant.